Senior Cycle

TEACHER HANDBOOK

Senior Cycle Higher Level

5th & 6th year

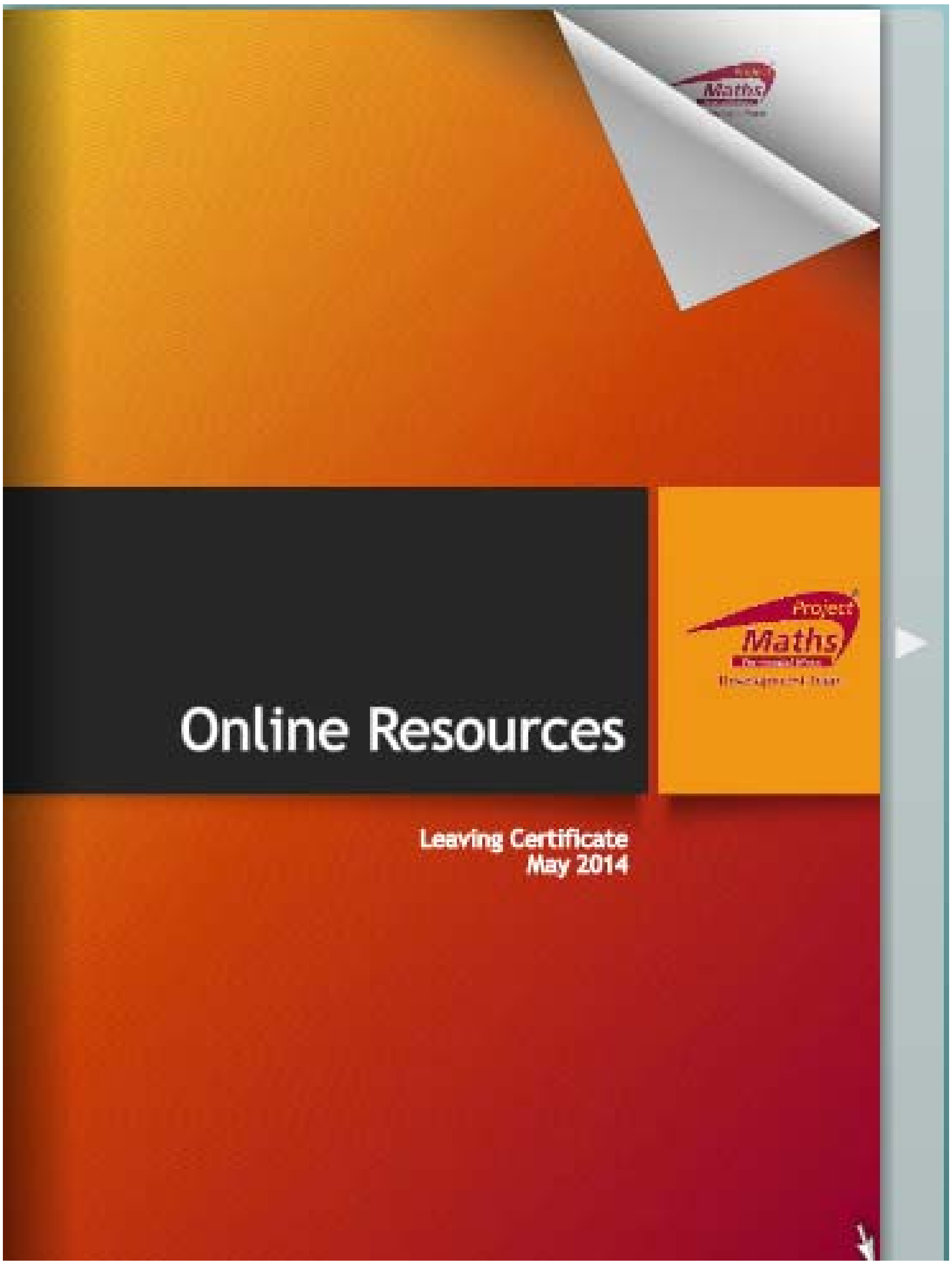
DRAFT

Based on the 2015 Syllabus

Contents

Page

The strand structure of the syllabus should not be taken to imply that topics are to be studied in isolation. Where appropriate, connections should be made within and across the strands and with other areas of learning. (NCCA JC syllabus page 10 and LC syllabus page 8)



This is a comprehensive reference for resources which will allow teachers plan lessons, easy access specific learning outcomes in the syllabus and relevant support material such as, Teaching & Learning and suggested activities to support learning and teaching. Click here.

Introduction: Student Learning

While this is a handbook for teachers, it must be emphasised that student learning and the process of mathematical thinking and building understanding are the main focus of this document.

Information and Communications Technologies are used whenever and wherever appropriate to help to support student learning. It is also envisaged that, at all levels, learners will engage with a dynamic geometry software package.

Students with mild general learning disabilities

Teachers are reminded that the NCCA Guidelines on mathematics for students with mild general learning disabilities can be accessed at

http://www.ncca.ie/uploadedfiles/PP_Maths.pdf

This document includes

- Approaches and Methodologies (from Page 4)
- Exemplars (from page 20)

Note: Synthesis and problem solving listed below must be incorporated into all of the Strands.
The list of skills below is taken from Strand 1of the syllabus but an identical list is given at the beginning of each strand in the syllabus.

At each syllabus level students should be able to

- explore patterns and formulate conjectures
- explain findings
- justify conclusions
- communicate mathematics verbally and in written form
- apply their knowledge and skills to solve problems in familiar and unfamiliar contexts
- analyse information presented verbally and translate it into mathematical form
- devise, select and use appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions.

Useful websites

	http://www.projectmaths.ie/ http://ncca.ie/en/Curriculum_and_Assessment/Post-Primary_Education/Project_Maths/ http://www.examinations.ie/

Literacy and Numeracy Strategy

The National Strategy to Improve Literacy and Numeracy among Children and Young People 2011-2020

Numeracy encompasses the ability to use mathematical understanding and skills to solve problems and meet the demands of day-to-day living in complex social settings. To have this ability, a young person needs to be able to think and communicate quantitatively, to make sense of data, to have a spatial awareness, to understand patterns and sequences, and to recognise situations where mathematical reasoning can be applied to solve problems.

Literacy includes the capacity to read, understand and critically appreciate various forms of communication including spoken language, printed text, broadcast media, and digital media.

Colour coding used in the suggested sequence below:

Strand 1 **Statistics and probability**	**Strand 2** **Geometry & Trigonometry**	**Strand 3** **Number**	**Strand 4** **Algebra**	**Strand 5** **Functions**

* Indicates proof of this theorem is required for JCHL
** Indicates proof of this theorem is required for LCHL

Suggested Sequence of topics -

Section number	Strand (Syllabus section)	Corresponding Lesson Number	Title of lesson idea	Page number	
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	3,4&5	LCHL.4	Relations approach to algebra - revision and extension of JC material	16	
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Section number	Strand (Syllabus section)	Corresponding Lesson Number	Title of lesson idea	Page number	
			point to a line		
	2.2	LCHL.23	Angle between two lines	29	
Section 7	2.1	LCHL.24	Revision - Plane and points, Axioms 1,2, 3,&5, Theorem 1, Constructions 8,9,5,	33	
	2.1	LCHL.25	Revision - Constructions 6,7,10,11,12 Axiom 4, Theorem 2	34	
	2.1	LCHL.26	Revision: Theorems 3, *4,5,*6, 7 & 8	34	
	2.1	LCHL.27	Revision: Constructions 1,2, 3& 4	35	
	2.1	LCHL.28	Revision: JC Transformation geometry	36	
	2.1	LCHL.29	Revision: quadrilaterals, parallelograms, Theorems *9 & 10, Corollary 1, Construction 20	37	
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Section number	Strand (Syllabus section)	Corresponding Lesson Number	Title of lesson idea	Page number	
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	2.1	LCHL.33	Constructions 13,14,& 15, Theorems *14 & 15: Pythagoras' Theorem & converse, Proposition 9	41	
	2.1	LCHL.34	Introduction to area, Theorem 16,Definition 38, Theorems 17&18	42	
	2.4	LCHL.35	Enlargements	43	
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	1.2&1.3	LCHL.37	Concepts of Probability	44	
	1.2&1.3	LCHL.38	Rules of probability	44	
	1.2&1.3	LCHL.39	Use of tree diagrams, set theory and counting method in probability	45	
	1.4 &1.5	LCHL.40	Data handling cycle	45	
	1.6 & 1.7	LCHL.41	Analysing data graphically and numerically, interpreting and drawing inferences from data	47	

Section number	Strand (Syllabus section)	Corresponding Lesson Number	Title of lesson idea	Page number	
Proposed beginning of 6th year programme					
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	5.2	LCHL.43	Concept of a limit, Limits as $n \to \infty$, recurring decimals, sum to infinity	49	
	5.2	LCHL.44	Differential Calculus	50	
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Section number	Strand (Syllabus section)	Corresponding Lesson Number	Title of lesson idea	Page number	
	1.3	LCHL.53	Normal distribution and standard normal	58	
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Section number	Strand (Syllabus section)	Corresponding Lesson Number	Title of lesson idea	Page number	
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The Lesson Ideas

Section 1: Number

Lesson Idea LCHL.1

Title
Number systems

Resources
Online Resources Leaving Certificate document

Content
These lessons will involve the students in investigating and understanding:

- $\mathbf{N}, \mathbf{Z}, \mathbf{Q}$ and representing these numbers on a number line
- Factors, multiples and prime numbers in $\mathbf{N}$
- How to express numbers in terms of their prime factors
- Highest Common Factor and Lowest Common Multiple
- How to make and justify estimates and approximations of calculations
- Make estimates of measures in the physical world around them
- How to calculate percentage error and tolerance
- How to calculate accumulated error (due to addition or subtraction only)
- How to calculate costing: materials, labour and wastage
- Metric system; change of units; everyday imperial units (conversion factors provided for imperial units)
- Terminating and non-terminating decimals
- Irrational numbers $\mathbf{R} \setminus \mathbf{Q}$
- The number system $\mathbf{R}$, appreciating that $\mathbf{R} \setminus \mathbf{Q}$ and representing $\mathbf{R}$ on a number line
- How to geometrically construct $\sqrt{2}$ and $\sqrt{3}$
- Proof (by contradiction) that $\sqrt{2}$ is an irrational number

Lesson Idea LCHL.2

Title
Rules for indices and scientific notation

Resources
Online Resources Leaving Certificate document

Content
These lessons will involve the students in investigating and understanding:

- The rules for indices (where $a,b \in \mathbf{R},\ p,q \in \mathbf{Q};\ a^p, a^q \in \mathbf{Q}$)

  $\boldsymbol{a^p a^q = a^{p+q}}$

  $\boldsymbol{\frac{a^p}{a^q} = a^{p-q}}$

  $\boldsymbol{a}^0 = 1$

  $\boldsymbol{(a^p)^q = a^{pq}}$

  $a^{-p} = \frac{1}{a^p}$

  $(ab)^p = a^p b^p$

  $\left(\frac{a}{b}\right)^p = \frac{a^p}{b^p}$

  $a^{\frac{1}{q}} = \sqrt[q]{a}, \quad q \in \mathbf{Z}, q \neq 0, a > 0$

  $a^{\frac{p}{q}} = \sqrt[q]{a^p} = \left(\sqrt[q]{a}\right)^p, \quad p,q \in \mathbf{Z}, q \neq 0, a > 0$

- How to express non - zero positive rational numbers in the form $a \times 10^n$, where $n \in \mathbf{Z}$ and $1 \leq a < 10$
- How to perform arithmetic operations on numbers in scientific notation

Lesson Idea LCHL.3

Title

Logarithms

Resources

Online Resources Leaving Certificate document

Content

These lessons will involve the students in investigating and understanding:

- How to solve problems using the rules of logarithms:

$$\log_a(xy) = \log_a x + \log_a y$$

$$\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$$

$$\log_a x^q = q\log_a x$$

$$\log_a a = 1$$

$$\log_a 1 = 0$$

$$\log_a x = \frac{\log_b x}{\log_b a}$$

Lesson Idea LCHL.4

Title

Relations approach to algebra- revision and extension of Junior Cycle material

Resources

Online Resources Leaving Certificate document

Dynamic software package

Content

These lessons will involve the students in investigating and understanding:

- That processes can generate sequences of numbers or objects
- How to investigate and discover patterns among these sequences
- How to use patterns to continue the sequence
- How to develop generalising strategies and ideas, present and interpret solutions, in the following:
    - The use of tables, diagrams, graphs and formulae as tools for representing and analysing **linear** patterns and relations
        - Discuss rate of change and the y - intercept. Consider how these relate to the context from which the relationship is derived and identify how they can appear in a table, in a graph and in a formula
        - Decide if two linear relations have a common value. (Decide if two lines intersect and where the intersection occurs.)
        - Recognise that the distinguishing feature of a linear relationship is a constant rate of change
        - Recognise discrete linear relationships as arithmetic sequences

- The use of tables, diagrams, graphs and formulae as tools for representing and analysing **quadratic** patterns and relations
    - Recognise that a distinguishing feature of quadratic relations is that the rate of change of the rate of change is constant
- The concept of a function as a relationship between a set of inputs and a set of outputs where each input is related uniquely to just one output
- **Exponential** relations
    - Recognise that a distinguishing feature of exponential relations is a constant ratio between successive outputs
    - Recognise discrete exponential relationships as geometric sequences

Lesson Idea LCHL.5

Title
Arithmetic and geometric sequences and series

Resources
Online Resources Leaving Certificate document
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

- The link between linear relations and the formula for the general term (T_n) of an arithmetic sequence
- How to find the sum (S_n) of n terms of an arithmetic series
- How to apply the formula for the *n*th term of an arithmetic sequence and the formula for the sum to *n* terms of an arithmetic series to different contexts.
- Geometric sequences and series
- Recognise discrete exponential relationships as geometric sequences
- Recognise whether a sequence is arithmetic, geometric or neither

Section 2: Functions

Lesson Idea LCHL.6

Title
Functions - interpreting and representing linear, quadratic and exponential functions in graphical form

Resources
Online Resources Leaving Certificate document
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

- That a function assigns a unique output to a given input
- Make use of function notation $f(x) =$, $f : x \rightarrow$, and $y =$
- Domain, co-domain and range
- How to graph functions of the form:
    - $ax + b$ where $a, b \in \mathbf{Q}, x \in \mathbf{R}$
    - $ax^2 + bx + c$, where $a, b, c \in \mathbf{Q}, x \in \mathbf{R}$
    - $ax^3 + bx^2 + cx + d$, where $a, b, c, d \in \mathbf{Z}, x \in \mathbf{R}$
    - $ab^x$where $a, b \in \mathbf{R}$
    - Logarithmic
    - Exponential
- How to find the inverse of a function
- How to sketch the graph of the inverse of a function given the graph of the function
- How to recognise surjective, injective, and bijective functions
- That the inverse of a bijective function is a function
- The concept of the limit of a function where it arises
- How to interpret equations of the form $f(x) = g(x)$ as a comparison of the above functions
- Use graphical methods to find approximate solutions to
    - $f(x) = 0$
    - $f(x) = k$

$f(x) = g(x)$

- where $f(x)$ and $g(x)$ are of the above form
  or where graphs of $f(x)$ and $g(x)$ are provided.

- Express quadratic functions in complete square form
- The relationship between x^2 , $ax^2, x^2 + c$, $(x-h)^2 + k$, and $a(x-h)^2 + k$
- How to use the complete square form of a quadratic function to find the roots and turning points
- How to use the complete square form of a quadratic function to sketch the function

Lesson Idea LCHL.7

Title
Composition of functions

Resources
Online Resources Leaving Certificate document
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

- How to form composite functions (including notation used)
- Composite functions in context

Section 3: Functions

Lesson Idea LCHL.8

Title
Revision of JC algebraic expressions and extension to HL LC

Resources
Online Resources Leaving Certificate document

Content
These lessons will involve the students in investigating and understanding:

- Factorising as listed on the JCHL syllabus
- The addition of expressions such as:
  $\frac{ax+b}{c} \pm \ldots \pm \frac{dx+e}{f}$ where $a,b,c,d,e,f \in \mathbf{Z}$
  $\frac{a}{bx+c} \pm \frac{q}{px+r}$ where $a,b,c,p,q,r \in \mathbf{Z}$

- How to perform the arithmetic operations of addition, subtraction, multiplication and division on polynomials and rational algebraic expressions paying attention to the use of brackets and surds

- The binomial theorem and how to apply the binomial theorem

Lesson Idea LCHL.9

Title
Rearranging formulae

Resources
Online Resources Leaving Certificate document
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

- How to rearrange formulae

Lesson Idea LCHL.10

Title

Solving equations and the *Factor Theorem*

Resources

Online Resources Leaving Certificate document

Dynamic software package

Content

These lessons will involve the students in investigating and understanding:

- The selection and use of suitable strategies (graphic, numerical, algebraic and mental) for finding solutions to equations of the form:
  - $f(x)=g(x)$, with $f(x)=ax+b$, $g(x)=cx+d$ and where $a,b,c,d\in\mathbf{Q}$
  - $f(x)=g(x)$ with $f(x)=\dfrac{a}{bx+c}\pm\dfrac{p}{qx+r}$, $g(x)=\dfrac{e}{f}$ where $a,b,c,e,f,p,q,r\in\mathbf{Z}$
  - $f(x)=g(x)$ with $f(x)=\dfrac{ax+b}{ex+f}\pm\dfrac{cx+d}{qx+r}$, $g(x)=k$ where $a,b,c,d,e,f,q,r\in\mathbf{Z}$
  - $f(x)=k$ with $f(x)=ax^2+bx+c$ (and not necessarily factorisable), $a,b,c\in\mathbf{Q}$ and interpret the results
  - Simultaneous linear equations with two unknowns and interpret the results
  - One linear and one equation of order two with two unknowns and interpret the results
  - Simultaneous linear equations with three unknowns and interpret the results
- How to use the *Factor Theorem* for polynomials
- The selection and use of suitable strategies (graphic, numerical, algebraic and mental) for finding solutions to cubic equations with at least one integer root and interpret the results
- How to form polynomial equations given the roots
- How to sketch polynomials given the polynomial in the form of factors - some of which may be repeated

Lesson Idea LCHL.11

Title
Inequalities - linear, quadratic and rational

Resources
Online Resources Leaving Certificate document
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

Selecting and using of suitable strategies (graphic, numeric, algebraic, mental) for finding solutions to inequalities of the form:

$g(x) \le k,\ g(x) \ge k$

$g(x) < k,\ g(x) > k$ where

$g(x) = ax + b$ or

$g(x) = ax^2 + bx + c$ or

$g(x) = \dfrac{ax+b}{cx+d}, a, b, c, d, k \in \mathbf{Q}, x \in \mathbf{R}$

Lesson Idea LCHL.12

Title
Modulus inequalities

Resources
Online Resources Leaving Certificate document
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

- Use notation $|x|$
- How to select and use suitable strategies (graphic, numeric, algebraic, mental) for finding solutions to inequalities of the form: $|x-a| < b, |x-a| > b$ and combinations of these, where $a, b \in \mathbf{Q}, x \in \mathbf{R}$

Section 4: Nets, Length, Area and Volume

Lesson Idea LCHL.13

Title
Nets, length, area and volume

Resources
Online Resources Leaving Certificate document
Dynamic software package

A mathematical instruments set

Content
These lessons will involve the students in investigating and understanding:

- How to solve problems involving the length of the perimeter and the area of a disc, triangle, rectangle, square, parallelogram, trapezium, sectors of discs and figures made from combinations of these

- The nets of prisms (polygonal bases), cylinders and cones

- How to solve problems involving the surface area and volume of the following solid figures: rectangular block, cylinder, right cone, triangular based prism (right angle, isosceles and equilateral), sphere, hemisphere and solids made from combinations of these

- How to use the trapezoidal rule to approximate area
- How to calculate percentage error involved in using trapezoidal rule for area in, for example, the circle

Section 5: Trigonometry

Lesson Idea LCHL.14

Title
Revision of JC Trigonometry with extension to LC HL (radian measure)

Resources

Online Resources Leaving Certificate document
Dynamic software package
A mathematical instruments set
Clinometers (can be home- made)

Content
These lessons will involve the students in investigating and understanding:

- The use of Pythagoras' Theorem to solve problems
- Trigonometric ratios in a right-angled triangle
- The use of the ratios to solve problems involving right angled triangles
- The use of similar triangles to find unknowns in right-angled triangles
- The use of the clinometer
- How to work with trigonometric ratios in surd form and solve problems involving surds
- How to manipulate measure of angles in both decimal and DMS forms
- The use of radians as a unit of measurement of angles
- The area of sectors and arc length using degrees and radians

Lesson Idea LCHL.15

Title
The unit circle and graphs of trigonometric functions

Resources

Online Resources Leaving Certificate document
Dynamic software package
A mathematical instruments set

Content
These lessons will involve the students in investigating and understanding:

- The properties and uses of the unit circle
- Radian and degree metrics of the unit circle

- The trigonometric ratios for angles in each of the four quadrants
- Graphs of the trigonometric functions : $\sin x, \cos x, \tan x$
- The period and range of these trigonometric functions
- That the inverse of these trigonometric functions is not a function for all values of x
- Graphs of trigonometric functions of the type $f(\theta) = a + b\sin(c\theta)$,

  $g(\theta) = a + b\cos(c\theta)$ for $a, b, c \in \mathbf{R}$
- Given the graph of a trigonometric function, sketch the graph of its inverse
- The period and range of the above trigonometric functions and the effect of changing the values of a, b and c
- Solutions of equations of the form $\sin\theta = 0$ and $\cos\theta = \pm\frac{1}{2}$ giving all solutions for specified values of θ
- Solutions to trigonometric equations such as $\sin n\theta = 0$ and $\cos n\theta = \pm\frac{1}{2}$ giving all solutions for specified values of θ
- Solutions of equations of the type $a\sin(bx) = \frac{1}{2}, a\cos(bx) = \frac{1}{2}$ for the domain used in the graph
- Solve equations for example of the form $15\cos^2 x = 13 + \sin x$ for all values of x where
  $0° \le x \le 360°$.

Lesson Idea LCHL.16

Title
Area of triangle, sine rule & cosine rule

Resources
Online Resources Leaving Certificate document
Dynamic software package
A mathematical instruments set

Content
These lessons will involve the students in investigating and understanding:

- The area of a triangle using Area = ½ ab sin C and use of this formula
- The connection between this formula and the geometric approach to the area of a triangle
- How to derive the sine rule

- Uses of the sine rule to solve real life problems
- How to derive the cosine rule
- The uses of the cosine rule to solve real life problems
- Use of the clinometer

Lesson Idea LCHL.17

Title
3D trigonometry

Resources
Online Resources Leaving Certificate document
Dynamic software package
A mathematical instruments set

Content
These lessons will involve the students in investigating and understanding:

- Problems involving 3D diagrams using trigonometry

Lesson Idea LCHL.18

Title
Trigonometric formulae and proofs

Resources
Online Resources Leaving Certificate document
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

- The derivation of the trigonometric formulae 1, 2, 3,4, 5, 6, 7,9 (Appendix in syllabus)
  (Derivations of formulae 2 & 3 in the previous lesson)
- How to apply the trigonometric formulae 1 - 24 (Appendix in syllabus)

Section 6: Coordinate Geometry

Lesson Idea LCHL.19

Title
Review of Junior Cycle co-ordinate geometry

Resources
Online Resources Leaving Certificate document
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

- The coordination of the plane
- The distance formula
- The midpoint formula
- The idea of slope as $\frac{Rise}{Run}$
- The slope formula
- The meaning of *positive, negative, zero and undefined slope*
- The use of slopes to investigate if two lines are parallel
- The use of slopes to investigate if two lines are perpendicular or not
- That 3 points on the coordinate plane are collinear if and only if the slope between any two of them is the same
- The equation of a line in the forms :

  $$y - y_1 = m(x - x_1)$$
  $$y = mx + c$$
  $$ax + by + c = 0$$

  (The significance of the variables m and c)
- Whether or not a point is on a line
- Where a line intersects the axes and why these points might be of interest to someone trying to interpret or plot a graph
- The interpretation of the intercepts in context
- How to find the slope of a line given its equation
- How to solve problems involving slopes of lines
- How to solve problems involving the intersection of two lines

Lesson Idea LCHL.20

Title
Area of a triangle given the coordinates of the vertices

Resources
Online Resources Leaving Certificate document
Dynamic software package

Content
This lesson will involve the students in investigating and understanding:

- How to calculate the area of a triangle using coordinates
- The connection between this formula ,the geometric approach to the area of a triangle and the formula used in trigonometry for finding the area of a triangl**e**

Lesson Idea LCHL.21

Title
Divide a line segment internally in the ratio $m:n$

Resources
Online Resources Leaving Certificate document
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

- How to divide a line segment internally in the ratio $m:n$ (link to similar triangles)

Lesson Idea LCHL.22

Title
The perpendicular distance from a point to a line

Resources
Online Resources Leaving Certificate document
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

- How to solve problems involving the perpendicular distance from a point to a line

Lesson Idea LCHL.23

Title
The angle between two lines

Resources
Online Resources Leaving Certificate document
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

- How to solve problems involving the angle between two lines

Section 7: Synthetic Geometry

Concepts: Set, plane, point, line, ray, angle, real number, length, degree, triangle, right-angle, congruent triangles, similar triangles, parallel lines, parallelogram, area, tangent to a circle, subset, segment, collinear points, distance, midpoint of a line segment, reflex angle, ordinary angle, straight angle, null angle, full angle, supplementary angles, vertically-opposite angles , acute angle, obtuse angle, angle bisector, perpendicular lines, perpendicular bisector of a line segment, ratio, isosceles triangle, equilateral triangle, scalene triangle, right-angled triangle, exterior angles of a triangle, interior opposite angles, hypotenuse, alternate angles, corresponding angles, polygon, quadrilateral, convex quadrilateral, rectangle, square, rhombus, base and corresponding apex and height of triangle or parallelogram, transversal line, circle, radius, diameter, chord, arc, sector, circumference of a circle, disc, area of a disc, point of contact of a tangent, concurrent lines. Vertex, vertices (of angle, triangle, triangle, polygon), endpoints of segment, arms of an angle, equal segments, equal angles, adjacent sides, angles, or vertices of triangles or quadrilaterals, the side opposite an angle of a triangle, opposite sides or angles of a quadrilateral, centre of a circle.

The following is a suggested sequence for teaching the Leaving Cert. Course. In teaching these lessons, teachers and students can draw from the Teaching and learning Plans and student activities on the website at www.projectmaths.ie

As outlined at the workshops, the use of learning materials such as “geostrips”, “anglegs”, geo-boards etc. can make the learning so much more enjoyable for students of all perceived abilities.

While proofs are not the issue as regards informal introduction, it is important that students are kept aware that the theorems build logically.

The lesson divisions which follow are for guidance only. The initial lesson ideas give the students a chance to revisit the material they met in the Junior Cycle. This can be done at a pace that is appropriate to the student’s needs. It is recommended that new activities and challenges be introduced during this revision so that students do not see it as too much repetition and that they can see new ways of investigating familiar situations.

Useful websites

	www.projectmaths.ie http://ncca.ie/en/Curriculum_and_Assessment/Post-Primary_Education/Project_Maths/ http://www.examinations.ie/

Note on experimentation and experimental results:
With experimentation, involving measurement, the results are only approximations and won't agree exactly. It is important for students to report faithfully what they find e.g. for a triangle they could find the sum of the angles to be 179^0 or 181^0 etc. The conclusion is that the angles appear to add up to 180^0 .This is a plausible working assumption. There is a distinction between what you can discover and what you can prove.
See below Section 8.2 (From Discovery to Proof) of *Geometry for Post-primary School Mathematics*"

8.2 From Discovery to Proof

It is intended that all of the geometrical results on the course would first be encountered by students through investigation and discovery. As a result of various activities undertaken, students should come to appreciate that certain features of certain shapes or diagrams appear to be independent of the particular examples chosen. These apparently constant features therefore seem to be general results that we have reason to believe might always be true. At this stage in the work, we ask students to accept them as true for the purpose of applying them to various contextualised and abstract problems, but we also agree to come back later to revisit this question of their truth. Nonetheless, even at this stage, students should be asked to consider whether investigating a number of examples in this way is sufficient to be convinced that a particular result always holds, or whether a more convincing argument is required. Is a person who refuses to believe that the asserted result will always be true being unreasonable? An investigation of a statement that appears at first to be always true, but in fact is not, may be helpful, (e.g. the assertion that $n^2 + n + 41$ is prime for all $n \in \mathbb{N}$). Reference might be made to other examples of conjectures that were historically believed to be true until counterexamples were found.

Informally, the ideas involved in a mathematical proof can be developed even at this investigative stage. When students engage in activities that lead to closely related results, they may readily come to appreciate the manner

in which these results are connected to each other. That is, they may see for themselves or be led to see that the result they discovered today is an inevitable logical consequence of the one they discovered yesterday. Also, it should be noted that working on problems or "cuts" involves logical deduction from general results.

Later, students at the relevant levels need to proceed beyond accepting a result on the basis of examples towards the idea of a more convincing logical argument. Informal justifications, such as a dissection-based proof of Pythagoras' theorem, have a role to play here. Such justifications develop an argument more strongly than a set of examples. It is worth discussing what the word "prove" means in various contexts, such as in a criminal trial, or in a civil court, or in everyday language. What mathematicians regard as a "proof" is quite different from these other contexts. The logic involved in the various steps must be unassailable. One might present one or more of the readily available dissection-based "proofs" of fallacies and then probe a dissection-based proof of Pythagoras' theorem to see what possible gaps might need to be bridged.

As these concepts of argument and proof are developed, students should be led to appreciate the need to formalise our idea of a mathematical proof to lay out the ground rules that we can all agree on. Since a formal proof only allows us to progress logically from existing results to new ones, the need for axioms is readily identified, and the students can be introduced to formal proofs.

Lesson Idea LCHL.24

Title
Revision of preliminary concepts - Plane and points, Axioms 1, 2 , 3 & 5 , Theorem 1, Constructions 8,9 & 5

Resources
Online Resources Leaving Certificate document
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

- Plane, points, lines, rays, line segments, , collinear points, length of a line segment
- Terms: theorem, proof, axiom, implies, is equivalent to, if and only if
- Axiom 1: [Two Points Axiom] There is exactly one line through any two given points.
- Axiom 2: [Ruler Axiom]: The properties of the distance between points
- Angle as a rotation, angles in different orientations
- Terms: Perpendicular, parallel, vertical, horizontal
- Axiom 3: [Protractor Axiom]
- That a straight angle has 180^0
- Supplementary angles
- Vertically opposite angles
- Theorem 1: Vertically opposite angles are equal in measure.
- Construction 8: Line segment of a given length on a given ray
- Construction 9: Angle of a given number of degrees with a given ray as one arm
- Axiom 5**:** Given any line *l* and a point P, there is exactly one line through P that is parallel to *l*.
- Construction 5: Line parallel to a given line, through a given point

Lesson Idea LCHL.25

Title
Revision of JC synthetic geometry - Constructions 6,7,10,11,12, Axiom 4, Theorem 2

Resources
Online Resources Leaving Certificate document
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

- Construction 6: Division of a line segment into 2, 3 equal segments without measuring it
- Construction 7: Division of a line segment into any number of equal segments, without measuring it
- Triangles: scalene, isosceles, equilateral, right-angled
- Construction 10: Triangle given SSS data (Axiom 4 -Congruent triangles)
- Construction 11: Triangle given SAS data (Axiom 4 -Congruent triangles)
- Construction 12: Triangle given ASA data (Axiom 4 -Congruent triangles)
- More constructions of triangles with SSS, SAS and ASA
- By construction, show that AAA and ASS are not sufficient conditions for congruence.
- Theorem 2**:** (i) In an isosceles triangle the angles opposite the equal sides are equal.
- What is meant by the term "converse"
- (ii) Conversely, if two angles are equal, then the triangle is isosceles

Lesson Idea LCHL.26

Title
Revision of JC synthetic geometry - Theorems 3, *4, 5, *6, 7 & 8

Resources
Online Resources Leaving Certificate document
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

- Theorem 3: (i) If a transversal makes equal alternate angles on two lines, then the lines are parallel (and converse)
- *Theorem 4: The angles in any triangle add to 180^0.
- Theorem 5: Two lines are parallel, if and only if, for any transversal, the corresponding angles are equal.
- *Theorem 6: Each exterior angle of a triangle is equal to the sum of the interior opposite angles.
- Theorem 7: In a triangle, the angle opposite the greater of two sides is greater than the angle opposite the lesser. Conversely, the side opposite the greater of two angles is greater than the side opposite the lesser angle.
  (Students might engage with proof by contradiction in proving the converse of Theorem 7)
- Theorem 8: Two sides of a triangle are together greater than the third.

Lesson Idea LCHL.27

Title
Revision - Constructions 1, 2, 3 & 4

Resources
Online Resources Leaving Certificate document
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

- Construction 1: Bisector of a given angle, using only compass and straight edge
- Construction 2: Perpendicular bisector of a line segment, using only compass and straight edge
- Construction 3: Line perpendicular to a given line *l*, passing through a given point not on *l*.
- Construction 4: Line perpendicular to a given line *l*, passing through a given point on *l*

Lesson Idea LCHL.28

Title
Revision: Junior Certificate transformation geometry

Resources
Online Resources Leaving Certificate document
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

- Translations
- Axial symmetry
- Central symmetry
- Rotations

Suggested class activities
Students might engage in the following investigations:

Does a translation preserve length?
Does a translation preserve angle size?
Does a translation map a line onto a parallel line?
Does a translation map a triangle onto a congruent triangle?

Does an axial symmetry preserve length?
Does an axial symmetry preserve angle size?
Does an axial symmetry maps a line onto a parallel line?
Does an axial symmetry map a triangle onto a congruent triangle?
How many axes of symmetry does an isosceles triangle have?
How many axes of symmetry does an equilateral triangle have?
How many axes of symmetry does a circle have?
(Draw examples of the above.)

Does a central symmetry preserve length?
Does a central symmetry preserve angle size?
Does a central symmetry map a line onto a parallel line?
Does a central symmetry map a triangle onto a congruent triangle?
Does an isosceles triangle have a centre of symmetry?
Does an equilateral triangle have a centre of symmetry?
Which types of triangle have a centre of symmetry?
Does a circle have a centre of symmetry?

Note: quadrilaterals are investigated in the lessons following.

Lesson Idea LCHL.29

Title
Revision of quadrilaterals, parallelograms, Theorems *9 & 10, Corollary 1 and Construction 20

Resources
Online Resources Leaving Certificate document
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

- Properties of parallelograms

- *Theorem 9*: In a parallelogram, opposite sides are equal, and opposite angles are equal
  Conversely, (1) if the opposite angles of a convex quadrilateral are equal, then it is a parallelogram; (2) if the opposite sides of a convex quadrilateral are equal, then it is a parallelogram.

- Remark 1 of *Geometry Course for Post-Primary School Mathematics*: Sometimes it happens that the converse of a true statement is false. For example, it is true that if a quadrilateral is a rhombus, then its diagonals are perpendicular. But it is not true that a quadrilateral whose diagonals are perpendicular is always a rhombus.
- Remark 2: The converse of Corollary 1 is false: it may happen that a diagonal divides a convex quadrilateral into two congruent triangles, even though the quadrilateral is not a parallelogram.
- Further properties of parallelograms
- Use of the term “corollary”
- Corollary 1: A diagonal divides a parallelogram into two congruent triangles
- Theorem 10: The diagonals of a parallelogram bisect one another.

  Conversely, if the diagonals of a quadrilateral bisect one another, then the quadrilateral is a parallelogram.
- Construction 20: Parallelogram, given the length of the sides and the measure of the angles.

- The properties of different quadrilaterals

Suggested class activities

Students might engage in the following activities which lead to an informal proof of theorem 9:

Draw a parallelogram ABCD which is not a rectangle or a rhombus
Draw in one diagonal BD
Mark in all the alternate angles – they should find 2 pairs
Establish that triangles ABD and BCD are congruent and explain their reasoning
Establish what this means about the opposite sides of parallelogram ABCD
Make a deduction about the opposite angles of parallelogram ABCD

The students can determine:

If the diagonal bisects the angles at the vertex

The sum of the four angles of parallelogram ABCD
The result if two adjacent angles of the parallelogram are added together

Students might engage in the following activities which lead to an informal proof of theorem 10 (In all instances they should be encouraged to explain their reasoning):

Draw a parallelogram ABCD which is not a rectangle or a rhombus
Draw in the two diagonals AC and BD intersecting at E
Determine if the two diagonals equal in length. (Measure)
Mark in all the equal sides and angles in the triangles AED and BEC
Explain why triangles ADE and BEC are congruent (Give a reason.)

Possible further investigations:

The students can determine:

If the triangles AEB and DEC are congruent
If the diagonals perpendicular
If the parallelogram contains 4 two pairs of congruent triangles
If the diagonals bisect the vertex angles of the parallelogram
The number of axes of symmetry the parallelogram has
If the parallelogram has a centre of symmetry and its location if it does exist

Students might engage in the following activities about a square, rhombus, parallelogram and rectangle: (In all instances they should be encouraged to explain their reasoning.)

Describe each of them in words.
Draw three examples of each in different orientations.
Determine which sides are equal in length.
Determine the sum of the angles in each case.
Determine which angles are equal.
Determine the sum of two adjacent angles in each case.
Establish whether or not a diagonal bisects the angles it passes through.
Establish whether or not the diagonals are perpendicular.
Determine whether or not a diagonal divides it into two congruent triangles.
Calculate the length of a diagonal given the length of its sides, where possible.
Establish whether or not the two diagonals are equal in length.
Determine whether or not the diagonals divide the different shapes into 4 congruent triangles.
Establish if the diagonals bisect each other.

The students should determine the number of axes of symmetry each of the shapes has and which ones have a centre of symmetry.
An interesting option would be to conduct the activities above on a KITE.

Lesson Idea LCHL.30

Title
Revision: More on quadrilaterals – investigating a Square

Resources
Online Resources Leaving Certificate document
Dynamic software package

Content

Students might engage in the following activities relating to a square: (In all instances they should be encouraged to explain their reasoning.)

Draw a square ABCD.
Draw in the two diagonals AC and BD intersecting at E.
Determine whether or not the two diagonals are equal in length.
Mark in all the equal sides and angles in the triangles AED and BEC.
Establish that triangles ADE and BEC are congruent.
Determine if the triangles AEB and DEC are congruent.
Determine if there are two pairs of congruent triangles in the square.
Show that the diagonals perpendicular. Give a reason.
Establish whether or not the diagonals bisect the vertex angles of the square.
Find how many axes of symmetry the square has.

Determine whether or not the square has a centre of symmetry and if it does, what is its location.

Students might engage in the following activities about a rectangle:

(In all instances they should be encouraged to explain their reasoning.)

Draw a rectangle ABCD which is not a square
Draw in the two diagonals AC and BD intersecting at E and establish if the two diagonals are equal in length
Mark in all the equal sides and angles in the triangles AED and BEC.
Establish that triangles ADE and BEC are congruent.
Determine whether or not the triangles AEB and DEC are congruent.
Determine whether or not there are two pairs of congruent triangles in the rectangle.
Show that the diagonals are perpendicular.
Determine whether or not the diagonals bisect the vertex angles of the rectangle.
Find how many axes of symmetry the rectangle has.
Determine whether or not the rectangle has a centre of symmetry and if it does, find its location.

Possible extra activity:
Repeat these activities for the rhombus ABCD

Lesson Idea LCHL.31

Title
**Theorem 11

Resources
Online Resources Leaving Certificate document
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

- ****Theorem 11**:If three parallel lines cut off equal segments on some transversal line, then they will cut off equal segments on any other transversal.
- Proof of this theorem

Lesson Idea LCHL.32

Title
**Theorem 12 and **Theorem 13

Resources
Online Resources Leaving Certificate document
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

- Theorem 12: Let ΔABC be a triangle. If a line l is parallel to BC and cuts [AB] in the ratio
  s: t, then it also cuts [AC] in the same ratio.
- Converse to Theorem 12: Let ΔABC be a triangle. If a line l cuts the sides AB and AC in the same ratio, then it is parallel to BC.
- Proof of Theorem 12
- The meaning of similar triangles and the difference between similar and congruent triangles.

- Theorem 13: If two triangles ΔABC and $\Delta A'B'C'$' are similar, then their sides are proportional, in order:
$$\frac{|AB|}{|A'B'|} = \frac{|BC|}{|B'C'|} = \frac{|CA|}{|C'A'|}$$
- Converse to Theorem 13: If
  $\frac{|AB|}{|A'B'|} = \frac{|BC|}{|B'C'|} = \frac{|CA|}{|C'A'|}$,then the two triangles ΔABC and $\Delta A'B'C'$' are similar .
  .

- Proof of Theorem 13

Lesson Idea LCHL.33

Title
Constructions 13, 14, 15, *Pythagoras' Theorem (*Theorem 14), converse of Pythagoras' Theorem (Theorem 15)
Proposition 9

Resources
Online Resources Leaving Certificate document
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

- Construction 13: Right-angled triangle, given length of hypotenuse and one other side.
- Construction 14: Right-angled triangle, given one side and one of the acute angles (several cases)
- Construction 15: Rectangle given side lengths
- *Theorem 14: [Theorem of Pythagoras]
- Theorem 15: [Converse to Pythagoras' Theorem] If the square of one side of a triangle is the sum of the squares of the other two, then the angle opposite the first side is a right angle.
- Proposition 9: (RHS) If two right-angled triangles each have hypotenuse and another side equal in length respectively, then they are congruent.

Lesson Idea LCHL.34

Title
Introduction to area, Theorem 16, Definition 38, Theorem 17 and Theorem 18

Resources
Online Resources Leaving Certificate document
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

- Theorem 16: For a triangle, base times height does not depend on the choice of base.
- Definition 38: The **area** of a triangle is half the base by the height.
- Theorem 17: A diagonal of a parallelogram bisects the area.
- Theorem 18: The area of a parallelogram is the base by the height.

Suggested class activities
Students might engage in the following activities:

In the case of each of these types of triangles: equilateral, isosceles, right-angled and obtuse-angled:

draw three diagrams for each type of triangle showing each side as a base and the corresponding perpendicular height.
Students investigate the validity of the following statement and its converse: “Congruent triangles have equal areas”.

Lesson Idea LCHL.35

Title

Enlargements

Resources

Online Resources Leaving Certificate document
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

- Enlargements, paying attention to
    - Centre of enlargement
    - Scale factor k, $0 < k < 1$, $k > 1$,$k \in \mathbf{Q}$
    - How to draw an enlargement of a figure given a scale factor when the centre of enlargement is outside the figure to be enlarged
    - How to draw an enlargement of a figure given a scale factor when the centre of enlargement is inside the figure to be enlarged
    - How to draw an enlargement of a figure given a scale factor when the centre of enlargement is a vertex of the figure to be enlarged or is a point on the figure
    - How to find the scale factor
- That when a figure is enlarged by a scale factor k, the area of the image figure is increased by a factor k^2
- How to solve problems involving enlargements

Section 8: Probability and Statistics 1

Lesson Idea LCHL.36

Title
Fundamental Principle of Counting, arrangements and combinations

Resources
Online Resources Leaving Certificate document
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

- The Fundamental Principle of Counting
- How to count the arrangements of n distinct objects (n!)
- How to count the arrangements of n distinct object taking r at a time
- How to count the number of ways of selecting r objects from n distinct objects (combinations of r objects from n distinct objects)
- How to compute binomial coefficients

Lesson Idea LCHL.37

Title
Concepts of probability

Resources

Online Resources Leaving Certificate document
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

- JC learning outcomes for probability
-

Lesson Idea LCHL.38

Title
Rules of probability

Resources
Online Resources Leaving Certificate document
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

- *The use of set theory to discuss experiments, outcomes, sample spaces*
- The basic rules of probability (AND/OR), mutually exclusive events, through the use of Venn diagrams
- The use of the formulae:
    1. Addition Rule (for mutually exclusive events only): $P(A \cup B) = P(A) + P(B)$
    2. Addition Rule: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
    3. Multiplication Rule(for independent events): $P(A \cap B) = P(A) \times P(B)$
- The use of tree diagrams

Lesson Idea LCHL.39

Title
Use of the counting method to evaluate probabilities

Resources

Online Resources Leaving Certificate document
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

- The use of the counting method (combinations) to evaluate probabilities

Lesson Idea LCHL.40

Title
The purpose of Statistics and the Data Handling Cycle

Resources
Online Resources Leaving Certificate document

Content
These lessons will involve the students in investigating and understanding:

- The purpose and uses of statistics and possible misconceptions and misuses of Statistics
- How to design a plan and collect data on the basis of the above knowledge
- The data handling cycle (Pose a question, collect data, analyse data, interpret the result and refine the original question if necessary)
- The Census at School (**CAS**) questionnaire as a means of collecting data
- Questionnaire designs
- Populations and samples
- That sampling variability influences the use of sample information to make statements about the population
- The importance of representativeness so as to avoid biased samples
- Sample selection (Simple Random Sample, stratified, cluster, quota - no formulae required, just definitions of these)
- The extent to which conclusions can be generalised
- Primary sources of data (observational (including sample surveys) and experimental studies) and secondary sources of data
    - The importance of randomisation (random assignment of subjects) and the role of the control group in studies
    - Biases, limitations and ethical issues of each type of study
- The different ways of collecting data
- How to summarise data in diagrammatic form

The students will also engage in analysing spreadsheets of data for example the spreadsheet of class data returned from the Census at School questionnaire to include:

- Recognising different types of data – category (nominal /ordinal), numerical (discrete/ continuous)
- Recognising univariate/bivariate data
- Discussing possible questions which might be answered with the data

Lesson Idea LCHL.41

Title
Analysing data graphically and numerically, interpreting and drawing inferences from data

Resources
Online Resources Leaving Certificate document

Content
These lessons will involve the students in investigating and understanding:

- The concept of a distribution of data and frequency distribution tables
- The selection and use of appropriate graphical and numerical methods to describe the sample (univariate data only)taking account of data type: bar charts, pie charts, line plots, histograms(equal class intervals), stem and leaf plots (including back to back)
- The distribution of numerical data in terms of shape (concepts of symmetry, clustering, gaps, skewness)
- The selection and use of appropriate numerical methods to describe the sample
  - The distribution of data in terms of centre (mean, median, mode and the advantages and disadvantages of each)
  - The relative positions of mean and median in symmetric and skewed data
  - The distribution of numerical data in terms of spread (range, inter-quartile range)
    - The concept of inter-quartile range as a measure of spread around the median
  - The distribution of data in terms of spread (standard deviation)
    - The concept of standard deviation as a measure of spread around the mean
    - The use of the calculator to calculator to calculate standard deviation
- Analyse plots of data to explain differences in measures of centre and spread
- How to interpret a histogram in terms of distribution of data and make decisions based on the empirical rule (based on a normal distribution)
- Outliers and their effect on measures of centre and spread

- Use percentiles to assign relative standing
- The effect on the mean of adding or subtracting a constant to each of the data points and of multiplying or dividing the data points by a constant
- Query: Should the above learning outcome be repeated for the standard deviation

Section 9: Functions (Differential Calculus and Integral Calculus)

Lesson Idea LCHL.42

Title
Revision of function concepts and relations without formulae (listed in 4.5 Junior cycle syllabus)

Resources
Online Resources Leaving Certificate document
Dynamic software package
Motion sensor

Content
These lessons will involve the students in investigating and understanding:

- Revision of function concepts
- Graphs of motion
- Quantitative graphs and drawing conclusions from them
- The connections between the shape of a graph and the story of a phenomenon
- Quantity and change of quantity on a graph

Lesson Idea LCHL.43

Title
Limits

Resources
Online Resources Leaving Certificate document
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

- The concept of a limit

- An approach to the introduction of the number *e* using $\boldsymbol{e} = \lim_{\boldsymbol{n}\to\infty}\left(1+\frac{1}{\boldsymbol{n}}\right)^{\boldsymbol{n}}$

  (Proof of the existence of this limit is not required.)
- How to apply the rules for sums, products and quotients of limits (see 3.1)
- Find by inspection the limits of sequences such as

  $\lim_{n\to\infty}\frac{n}{n+1}; \quad \lim_{n\to\infty} r^n, \ |r|<1$
- Limits and continuity of functions through informal exploration
- Derive the formula for the sum to infinity of geometric series by considering the limit of a sequence of partial sums
- Solve problems involving infinite geometric series such as recurring decimals

Lesson Idea LCHL.44

Title
Differential Calculus

Resources
Online Resources Leaving Certificate document
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

Rate of change, average rate of change, instantaneous rate of change, the derivative

This will include:

- Calculus as the study of mathematically defined change, (but not necessarily change with respect to time alone -velocity can change with height, temperature can change with energy, force can change with mass, pressure can change with depth etc.)
- How to use graphs and real life examples to analyse rates of change for:
  - Functions of the form $f(x)=k$,where *k* is a constant
  - Linear functions - links should be established to the slope of a line from coordinate geometry
  - Functions where the rate of change varies - these will include the quadratic as well as other functions where the rate of change varies
- Instantaneous rate of change (what shows on a speedometer) as opposed to average rate of change for example over the whole course of a journey
- Equality of the instantaneous and average rates of change for linear functions

- How to find the rate of change in situations where it is not constant - need to define it at every point
    - The idea of average rate of change between two points on, for example, the graph of $f(x)=x^2$ and its calculation as the slope of the line connecting the two endpoints of the interval under consideration
    - That the instantaneous rate of change is not the same as the average rate of change between two points on, for example, $f(x)=x^2$
    - That the average rate of change approaches the instantaneous rate as the interval under consideration approaches zero (the concept of a limit)
    - That the instantaneous rate of change is the slope of the tangent line at the point
- The meaning of the first derivative as the instantaneous rate of change of one quantity relative to another and the use and meaning of the terms "differentiation" and notation such as $\frac{dy}{dx}$ and $f'(x)$
- How to find the first derivatives of linear functions using the equation $y = mx + c$ and observing the slope as the first derivative
- How to differentiate linear and quadratic functions from first principles
- Differentiation by rule of the following function types
    - Polynomial
    - Exponential
    - Trigonometric
    - Rational powers
    - Inverse functions
    - Logarithms
- How to find the derivatives of sums, differences, products, quotients and composition of functions of the form of the above functions
- How to use differentiation to find the slope of a tangent to a circle
- How to apply the differentiation of the above functions to solve problems

- What it means when a function is increasing/decreasing/constant in terms of the rate of change
- How to apply an understanding of the change in $\frac{dy}{dx}$ from positive to zero to negative around a local maximum in order to identify a local maximum (concave downwards)
- How to apply an understanding of the change in $\frac{dy}{dx}$ from negative to zero to positive around a local minimum in order to identify a local minimum (concave upwards)
- Stationary points as points on a curve at which the tangent line has a slope of zero

- Turning points as points on a curve where the function changes from increasing to decreasing or vice versa. (Turning points are also stationary points but the converse may not be true.)
- The meaning of the second derivative as the rate of change of a rate of change at an instant
- The second derivative as being positive (first derivative is increasing) in a region where the graph of a function is concave upwards and negative (first derivative is decreasing) in a region where the graph of the function is concave downwards
- A point of inflection as a point on a curve at which the second derivative equals zero and changes sign (curve changes from concave upwards to concave downwards and the first derivative has a maximum or minimum point)
- Real life examples of the rate of change of a rate of change as in acceleration as a rate of change of velocity
- How to sketch a “slope- graph ”of a function given the graph of the function
- How to match a function with graphs of its first and second derivatives
- How to find second derivatives of linear, quadratic and cubic functions by rule
- The application of the second derivative to identify local maxima and local minima

  (Students might also associate the points on a normal curve which are one standard deviation away from the mean with points of inflection referred to above.)

Lesson Idea LCHL.45

Title
Integration

Resources
Online Resources Leaving Certificate document
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

- That anti-differentiation is the reverse process of differentiation.
- How to find the anti-derivative $F(x)$ of function $f(x)$
- That the indefinite anti-derivative is the set of all possible anti-derivatives of a function
- That a distinct anti-derivative is one element of this set which satisfies some initial condition
- That the process of finding the area between a curve and the x- axis (or y –axis) over an interval on the axis, is known as integration

- That the area between a function and the x-axis over an interval may be calculated using the limit of the sum of the areas of rectangles, as the number of rectangles tends to infinity and the width of each rectangle tends to zero.

- That the area A between the graph of $f(x)$ and the $x-$ axis over the interval from $[a, b]$ is :

$$A = \lim_{n\to\infty}\sum_{i=1}^{\infty} f(x_i)\Delta x = \int_a^b f(x)dx$$

- The area beneath a function $f(x)$ from a to b is given by

$$\int_a^b f(x)dx = F(b) - F(a)$$

- The antiderivative is usually called the indefinite integral $\int f(x)dx$,

- The indefinite integral, written as, $\int f(x)dx$,
of $f(x)$ is a set of functions equal to the set of all the antiderivatives of $f(x)$. They differ by a constant C.

$$F(x) + C = \int f(x)dx$$

- The definite integral $\int_a^b f(x)dx$ is a number equal to the net signed area between the graph of $f(x)$ and the interval $[a, b]$
- That integration is the reverse process of differentiation (anti - differentiation)
$\frac{d}{dx}\int f(x)dx = f(x)$

- How to integrate sums, differences and constant multiples of functions of the form

  - x^a, where $a \in \boldsymbol{Q}$
  - a^x, where $a \in \boldsymbol{R}, a > 0$
  - Sin ax, where $a \in \boldsymbol{R}$
  - Cos ax, where $a \in \boldsymbol{R}$

- How to determine areas of plane regions bounded by polynomial and exponential functions

- How to use integration to find the average value of a function over an interval

  Average value of $f(x)$ over $[a,b] = \dfrac{\int_a^b f(x)dx}{b-a}$

Section 10: Probability and Statistics 2

Lesson Idea LCHL.46

Title
Revision of counting and probability concepts from fifth year (Section 1.1 to 1.3 JC syllabus)

Resources
Online Resources Leaving Certificate document

Content
These lessons will involve the students in investigating and understanding:

- Fundamental principle of counting, arrangements and combinations
- Concepts of probability
- Rules of probability
- Use of tree diagrams, set theory and binary/counting method in probability

Lesson Idea LCHL.47

Title
Probability Rules

Resources
Online Resources Leaving Certificate document
Report on the Trialling State Exams Commission (SEC) 2010

Content
These lessons will involve the students in investigating and understanding:

- Discuss basic rules of probability (AND/OR, mutually exclusive) through the use of Venn diagrams
- Extend their understanding of the basic rules of probability (AND/OR, mutually exclusive) through the use of formulae
- Addition Rule: $P(AUB) = P(A) + P(B) - P(A \cap B)$
- Multiplication Rule(Independent Events): $P(A \cap B) = P(A) \times P(B)$
- Multiplication Rule (General Case): $P(A \cap B) = P(A) \times P(B \mid A)$
- Solve problems involving sampling, with or without replacement

- Formal definition of independent events: A and B are independent if $P(B \mid A) = P(B)$
- Appreciate that in general $P(A \mid B) \neq P(B \mid A)$
- Examine the implications of $P(A \mid B) \neq P(B \mid A)$ in context

Lesson Idea LCHL.48

Title
Bernoulli Trials

Resources
Online Resources Leaving Certificate document

Content
These lessons will involve the students in investigating and understanding:

- Apply an understanding of Bernoulli trials
  A Bernoulli trial is an experiment whose outcome is random and can be either of two possibilities: "success" or "failure"

- Solve problems involving Bernoulli trials as follows:
  1. calculating the probability that the 1^{st} success occurs on the n^{th} Bernoulli trial where n is specified
  2. calculating the probability of k successes in n repeated Bernoulli trials (normal approximation not required)
  3. Calculating the probability that the kth success occurs on the nth Bernoulli trial

Lesson Idea LCHL.49 (should lesson 48 and 49 be switched?)

Title
Random variables and expected value

Resources
Online Resources Leaving Certificate document

Content
These lessons will involve the students in investigating and understanding:

- Random variables, discrete and continuous, which lead to discrete and continuous probability distributions

- Expected value $E(X)$ of probability distributions
- The calculation of expected value and the fact that this does not need to be one of the outcomes
- Standard deviation of probability distributions
- The role of expected value in decision making and the issue of fair games

Lesson Idea LCHL.50

Title
Consolidation of probability concepts

Resources
Online Resources Leaving Certificate document
Report on the Trialling SEC 2010

Content
These lessons will involve the students in participating in investigating and understanding:

- All probability rules and methods to date

Lesson Idea LCHL.51

Title
Revision of statistics concepts from 5th year

Resources
Online Resources Leaving Certificate document
Report on the Trialling SEC 2010

Content
These lessons will involve the students in participating in investigating and understanding:

- Revision of lessons 40 and 41

Lesson Idea LCHL.52

Title
Bivariate data, scatter plots, correlation

Resources
Online Resources Leaving Certificate document

Content
These lessons will involve the students in investigating and understanding:

- Bivariate data versus univariate data
- Different types of bivariate data
- The use of scatter plots to determine the relationship between numeric variables
- That correlation always has a value from -1 to +1 inclusive, and that it measures the extent of the **linear relationship** between two variables
- How to match correlation coefficients values to appropriate scatter plots
- That correlation does not imply causality
- How to draw the line of best fit by eye
- How to make predictions based on the line of best fit
- How to calculate the correlation coefficient by calculator

Lesson Idea LCHL.53

Title
Normal Distribution and Standard Normal

Resources
Online Resources Leaving Certificate document

Content
These lessons will involve the students in investigating and understanding:

- Continuous probability distributions – the normal distribution and the standard normal distribution
- The solution of problems involving reading probabilities from the normal distribution tables

Lesson Idea LCHL.54

Title
Drawing inferences from data, confidence intervals, margin of error, the concept of a hypothesis test,

Resources
Online Resources Leaving Certificate document Resource Document

Content
These lessons will involve the students in investigating and understanding:

- How sampling variability influences the use of sample information to make statements about the population
- How to use appropriate tools to describe variability drawing inferences about the population from the sample
- How to interpret the analysis and relate the interpretation to the original question
- How to interpret a histogram in terms of the distribution of data
- The use of the empirical rule
- The use of simulations to explore the variability of large sample proportions from a known population to construct the sampling distribution of the proportion and to draw conclusions about the sampling distribution of the proportion
- The use of the sampling distribution of the proportion as the basis for informal inference
- How to construct 95% confidence intervals for the population proportion from a large sample using z tables
- How to build on the concept of margin of error and understand that increased confidence level implies wider intervals
- How to calculate the margin of error for a population proportion ($\frac{1}{\sqrt{n}}$)
- The concept of a hypothesis test
- The distinction between a null and an alternative hypothesis
- How to conduct a hypothesis test on a population proportion using the margin of error
- How to use simulations to explore the variability of sample means from a known population to construct the sampling distribution of the mean and to draw conclusions about the sampling distribution of the mean
- The use of the sampling distribution of the mean as the basis for informal inference
- How to Construct 95% confidence intervals for the population mean from a large sample using z tables

- How to perform univariate large sample tests of the population mean (two – tailed z-test only)
- How to use and interpret p - values

Note: The margin of error referred to here is the maximum value of the 95% confidence interval.

Section 11: Synthetic Geometry 2

Lesson Idea LCHL.55

Title
Theorem *19, Corollaries 2, 3, 4 and 5

Resources
Online Resources Leaving Certificate document
A mathematical instruments set
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

Higher Level only:

- *Theorem 19: The angle at the centre of a circle standing on a given arc is twice the angle at any point of the circle standing on the same arc.
- Corollary 2: All angles at points of a circle, standing on the same arc, are equal (and the converse)
- Corollary 3: Each angle in a semi-circle is a right angle.
- Corollary 4: If the angle standing on a chord [BC] at some point of the circle is a right-angle, then [BC] is a diameter.
- Corollary 5: If ABCD is a cyclic quadrilateral, then opposite angles sum to 180°.

Lesson Idea LCHL.56

Title
Theorem 20, Corollary 6 and Construction 19

Resources
Online Resources Leaving Certificate document
A mathematical instruments set
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

- Theorem 20: (i) Each tangent to a circle is perpendicular to the radius that goes to the point of contact.

  (ii) If P lies on the circle s, and a line l through P is perpendicular to the radius to P, then l is a tangent to s.

- Corollary 6: If two circles share a common tangent line at one point, then the two centres and that point are collinear.
- Construction 19: Tangent to a given circle at a given point on it.

Lesson Idea LCHL.57

Title
Theorem 21 &Construction 18

Resources
Online Resources Leaving Certificate document
A mathematical instruments set
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

- Theorem 21: (i) The perpendicular from the centre of a circle to a chord bisects the chord.

  (ii) The perpendicular bisector of a chord passes through the centre of a circle.

- Construction 18: Angle of 60^0 , without using a protractor or set square

Lesson Idea LCHL.58

Title
Constructions 16 and 17

Resources
Online Resources Leaving Certificate document
A mathematical instruments set
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

- <u>Construction 16</u>: Circumcentre and circumcircle of a given triangle, using only straight edge and compass.
- <u>Definition 43:</u> circumcircle, circumcentre, circumradius
  (*Geometry Course for Post-Primary School Mathematics*)

- <u>Construction 17:</u> Incentre and incircle of a given triangle, using only straight edge and compass.
- <u>Definition 44:</u> incircle, incentre, inradius
  (*Geometry Course for Post-Primary School Mathematics*)

Suggested class activities
Students might engage in the following activities:

Draw the circumcentre and incentre for an acute-angled triangle, a right-angled triangle, an obtuse-angled triangle.
In which instances is the circumcentre inside the triangle?

In which instances is the incentre inside the triangle?

Lesson Idea LCHL.59

Title
Construction 21 and 22

Resources
<u>Online Resources Leaving Certificate document</u>
A mathematical instruments set
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

- Definition 45: Medians and centroid
  (*Geometry Course for Post-Primary School Mathematics*)
- Construction 21: Centroid of a triangle
- Definition 46: Orthocentre

- Construction 22: Orthocentre of a triangle

Suggested class activities
Students might engage in the following activities:

Draw the centroid and orthocentre for an acute-angled triangle, a right-angled triangle, an obtuse-angled triangle.

In which instances is the centroid inside the triangle?

In which instances is the orthocentre inside the triangle?

OPTIONAL:
Higher Level students might consider the concept of "Euler's Line".
The orthocentre, centroid and circumcentre in any triangle are always in line and this is called Euler's Line.
The incentre is on Euler's Line only in the case of an isosceles triangle.
In the case of an equilateral triangle, the orthocentre, centroid, circumcentre and incentre coincide.

Section 12: Coordinate Geometry of the Circle

Lesson Idea LCHL.60

Title
Co-ordinate geometry of the circle

Resources
Online Resources Leaving Certificate document
A mathematical instruments set, Graph paper
Dynamic geometry software package, set of board drawing instruments

Content
These lessons will involve the students in investigating and understanding:

- That $x^2 + y^2 = r^2$ represents the equation of a circle centre (0,0) and radius of length r (Link to Pythagoras' Theorem - distance from any point $p(x, y)$ on the circle to the centre of the circle is equal to the length of the radius of the circle.)
- That $(x-h)^2 + (y-k)^2 = r^2$ represents the relationship between the x and y co-ordinates of points on a circle wit centre (h, k) and radius r
  (Link to Pythagoras' Theorem - distance from any point $p(x, y)$ on the circle to the centre of the circle is equal to the length of the radius of the circle.)

- Recognise that $x^2 + y^2 + 2gx + 2fy + c = 0$ represents the relationship between the x and y coordinates of points on a circle centre (-g,-h) and radius r where $r = \sqrt{g^2 + f^2 - c}$
- How to solve problems involving a line and a circle

Section 13: Application of geometric series -Financial Maths

Lesson Idea LCHL.61

Title
Financial Maths

Resources

Online Resources Leaving Certificate document
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

- How to solve problems involving
  - Mark up (profit as a % of cost price)
  - Margin (profit as a % of selling price)
  - Income tax and net pay including other deductions
  - Compound interest investigated using multi-representations i.e. table, graph and formula (link to exponential functions and geometric sequences)
  - Compound interest rate terminology such as AER, EAR and CAR
  - How to , given an AER, covert to a different period interest rate e.g. a monthly rate and vice versa
  - Depreciation (reducing balance method) investigated using multi-representations i.e. table, graph and formula (link to exponential functions and geometric sequences)
  - Currency transactions
  - Present value
  - How to solve problems involving finite geometric series in financial applications e.g.
    - future value of regular investments over a definite period of time
    - deriving a formula for a mortgage repayment
  - How to use present value when solving problems involving loan repayments and investments

Section 14: Proof by Induction

Lesson Idea LCHL.62

Title
Proof by Induction

Resources
Online Resources Leaving Certificate document
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

Proof by induction for the following:

- Simple identities such as the sum of the first *n* natural numbers and the sum of a finite geometric series.
- Simple inequalities such as:
  $n! > 2^n$
  $2^n \geq n^2 \quad (n \geq 4)$
  $(1+x)^n \geq 1 + nx \ (x > \text{-}1)$
- Factorisation results such as 3 is a factor of $4^n - 1$

Section 15: Complex Numbers

Lesson Idea LCHL.63

Title
Complex numbers 1

Resources
Online Resources Leaving Certificate document
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

- The origin and need for complex numbers
- The use of complex numbers to model two dimensional systems as in computer games, alternating current and voltage etc.
- How to interpret multiplication by i as a rotation of 90^0 anticlockwise
- How to express complex numbers in rectangular form $a+ib$ and illustrate them on the Argand diagram
- How to investigate the operations of addition and subtraction of complex numbers in the rectangular form $(a+ib)$ using the Argand diagram
- How to interpret the modulus as distance from the origin on an Argand diagram
- How to interpret multiplication by a complex number as a "multiplication of" the modulus by a real number combined with a rotation
- How to interpret the complex conjugate as a reflection in the real axis
- Division of complex numbers in the rectangular form $(a+ib)$
- Calculate conjugates of sums and products of complex numbers
- How to solve quadratic equations having complex roots and how to interpret the solutions

Lesson Idea LCHL.64

Title
Complex numbers 2

Resources
Online Resources Leaving Certificate document
Dynamic software package

Content

These lessons will involve the students in investigating and understanding:

- How to use the *Conjugate Root Theorem* to find the roots of polynomials
- How to express complex numbers in polar form
- How to work with complex number in rectangular and polar form to solve quadratic and other equations including those in the form $z^n = a$, where $n \in \boldsymbol{Z}$ and $z = r(cos\theta + isin\theta)$
- How to use De Moivre's Theorem
- How to prove De Moivre's Theorem by induction for $n \in \mathbf{N}$
- How to use applications such as the n^{th} roots of unity, $n \in \mathbf{N}$, and identities such as $cos3\theta = 4\cos^3\theta - 3\cos\theta$

Appendix A

Geometry: Thinking at Different Levels
The Van Hiele Theory

The **Van Hiele model** describes how students learn geometry. Pierre van Hiele and Dina van Hiele-Geldof, mathematics teachers from the Netherlands, observed their geometry students in the 1950's.The following is a brief summary of the Van Hiele theory. According to this theory, students' progress through 5 levels of thinking starting from merely recognising a shape to being able to write a formal proof. The levels are as follows:

***Visualisation (Level 0)**

The objects of thought are shapes and what they look like.

Students have an overall impression of a shape. The appearance of a shape is what is important. They may think that a rotated square is a "diamond" and not a square because it is different from their visual image of a square. They will be able to distinguish shapes like triangles, squares, rectangles etc but will not be able to explain, for example, what makes a rectangle a rectangle. **Vocabulary**: Students use visual words like "pointy", "curvy", "corner" as well as correct language like angle, rectangle and parallelogram.

***Analysis (Level 1)**

The objects of thought are "classes" of shapes rather than individual shapes.

- Students think about what makes a rectangle a rectangle and can separate the defining characteristics of a rectangle from irrelevant information like size and orientation. They recognize its parts (sides, diagonals and angles) and compare their properties (similar, congruent)
- They understand that if a shape belongs to a class like "rectangle", then it has all the properties of that class (2 pairs of equal sides, right angles, 2 equal diagonals, 2 axes of symmetry).
- **Vocabulary:** words like parallel, perpendicular and congruent relating to properties within a figure and the words all, always, sometimes, never, alike, different.
- A concise definition of a figure, using a sufficient rather than an exhaustive list of properties is not possible at this level.
- They do not deal with questions like "Is a square a parallelogram?" but just look at the properties of each class of shape, without comparing the classes.

*Some visualisation and analysis is covered in Primary School.

Relational/ Ordering/Informal Deduction (Level 2)

The objects of thought are the properties of shapes.

- Students are ready to understand interrelationships of properties within figures and between figures. Opposite sides of a parallelogram are parallel and opposite angles are equal.
- A rectangle is a parallelogram since it has all the properties of a parallelogram as well as all 90^0 angles.
- Students can recognise the difference between a statement and its converse. All squares are rectangles (true) is different from all rectangles are squares (not true).
- Capable of **"if –then"** thinking – if a shape is a rectangle then all the angles in it are right angles. If $|\angle A| = |\angle B|$ and $|\angle B| = |\angle C|$ then $|\angle A| = |\angle C|$
- They can select one or two properties to define a figure rather than an exhaustive list. If a quadrilateral has 4 equal sides and one right angle it must be a square.
- Students can discover new properties by simple deduction. The 2 acute angles in a right angled triangle add to 90^0 because all the angles in a triangle add up to 180^0. They can explain logically without having to measure everything.

Formal deduction (Level 3)

Students learn how to use an axiomatic system to establish geometric theory. This is the level at which proof of Theorems is learned. The sequence of theorems given in the appendix is arranged in such a manner that each theorem builds on the previous theorem(s).

Rigor (Level 4)

Comparing different axiomatic systems – not done at secondary level

Characteristics of these levels: Students cannot function at any particular level unless they are competent at all previous levels. The teacher's role is crucial in structuring activities to bring students from one level to the next.

How does the teacher bring students from any one level to the next?

5 phases of learning:

1. In an informal discussion of the topic, students are asked to give their initial observations.
2. The teacher provides structured activities such as drawing, making and measuring.
3. The students then verbalise and write down what they have learned and report back in groups to the class, which leads to a class discussion.
4. The teacher then provides an activity which will require students to apply what they have discovered
5. In the last stage students are required to summarise all they have learned and should be able to remember it as they have discovered it through guidance.

A PowerPoint presentation of the Van Hiele theory can be got at www.projectmaths.ie

2 examples are given on the PowerPoint slides

(1) Using similar triangles to show advancement between levels and

(2) Using an investigation of the rhombus to show how to progress from level 0 to level 1 with this figure using the 5 teaching phases.

A mind map of Van Hiele can be found at
http://agutie.homestead.com/files/mindmap/van_hiele_geometry_level.html

Appendix B

Guide to Theorems, Axioms and Constructions at all Levels

This is intended as a quick guide to the various axioms, theorems and constructions as set out in the ***Geometry Course for Post-Primary School Mathematics.*** You can get this from the project maths website: www.projectmaths.ie

It is not intended as a replacement for this document, merely as an aid to reading at a glance which material is required to be studied at various levels. The sequence of theorems as given must be followed.

As stated in the heading, these theorems and constructions are underpinned by 46 definitions and 20 propositions which are all set out in the ***Geometry Course for Post-Primary School Mathematics***, along with many undefined terms and definable terms used without explicit definition.

*An **axiom** is a statement accepted without proof, as a basis for argument

*A **theorem** is a statement deduced from the axioms by logical argument. Theorems can also be deduced from previously established theorems.

* A **proposition** is a useful or interesting statement that could be proved at this point, but whose proof is not stipulated as an essential part of the programme. Teachers are free to deal with them as they see fit, but they should be mentioned, at least (Appendix p. 20, footnote).

*The instruments that may be used for **constructions** are listed and described on page 38 of the Appendix and are a straight edge, compass, ruler, protractor and set-square.

Terms

Students at **Junior Certificate Higher level and Leaving Certificate Ordinary level** will be expected to understand the meanings of the following terms related to logic and deductive reasoning:
Theorem, proof, axiom, corollary, converse, implies.

In addition, students at **Leaving Certificate Higher level** will be expected to understand the meanings of the following terms related to logic and deductive reasoning:

Is equivalent to, if and only if, proof by contradiction.

Synthetic Geometry

Guide to Axioms, Theorems and Constructions for all Levels

Information Technology is used whenever and wherever appropriate to help to present mathematical concepts effectively to students. In this document the [CD] symbol appears at the corresponding position of the content to indicate that an interactive IT module is available on the Project Maths Student's CD.

	Axioms and Theorems (supported by 46 definitions, 20 propositions) *proof required for JCHL and LCHL ** proof required for LCHL only	CMN Introd. Course	JC ORD	JC HR	LC FDN	LC ORD	LC HR
	Axiom 1: There is exactly one line through any two given points	√	√	√	√	√	√
	Axiom 2: [Ruler Axiom]: The properties of the distance between points.	√	√	√	√	√	√
	[CD] **Axiom 3:** Protractor Axiom (The properties of the degree measure of an angle).	√	√	√	√	√	√
1	[CD] Vertically opposite angles are equal in measure.	√	√	√	√	√	√
	Axiom 4: Congruent triangles conditions (SSS, SAS, ASA)	√	√	√	√	√	√
2	[CD] In an isosceles triangle the angles opposite the equal sides are equal. Conversely, if two angles are equal, then the triangle is isosceles.	√	√	√	√	√	√
	Axiom 5: Given any line l and a point P, there is exactly one line through P that is parallel to l.	√	√	√	√	√	√
3	[CD] If a transversal makes equal alternate angles on two lines then the lines are parallel. Conversely**,** if two lines are parallel, then any transversal will make equal alternate angles with them.	√	√	√	√	√	√
4*	[CD] The angles in any triangle add to 180°.	√	√	√	√	√	√
5	[CD] Two lines are parallel if, and only if, for any transversal, the corresponding angles are equal.	√	√	√	√	√	√
6*	[CD] Each exterior angle of a triangle is equal to the sum of the interior opposite angles.	√	√	√	√	√	√
7	[CD] The angle opposite the greater of two sides is greater than the angles opposite the lesser. Conversely, the side opposite the greater of two angles is greater than the side opposite the lesser angle.					√	√
8	[CD] Two sides of a triangle are together greater than the third.					√	√
9*	[CD] In a parallelogram, opposite sides are equal, and opposite angles are equal. Conversely, (1) if the opposite angles of a convex quadrilateral are equal, then it is a parallelogram; (2) if the opposite sides of a convex quadrilateral are equal, then it is a parallelogram.		√	√	√	√	√
	Corollary 1. A diagonal divides a parallelogram into two congruent triangles.			√			√
10	[CD] The diagonals of a parallelogram bisect each other. Conversely, if the diagonals of a quadrilateral bisect one another,		√	√	√	√	√

	then the quadrilateral is a parallelogram.						
	Axioms and Theorems (supported by 46 definitions, 20 propositions) *proof required for JCHL and LCHL ** proof required for LCHL only	CMN Introd. Course	JC ORD	JC HR	LC FDN	LC ORD	LC HR
11**	If three parallel lines cut off equal segments on some transversal line, then they will cut off equal segments on any other transversal.			√		√	√
12**	Let ABC be a triangle. If a line l is parallel to BC and cuts [AB] in the ratio m:n, then it also cuts [AC] in the same ratio. Conversely, if the sides of two triangles are in proportion, then the two triangles are similar.			√		√	√
13**	If two triangles are similar, then their sides are proportional, in order (and converse)		√	√	√	√	√
14*	[Theorem of Pythagoras]In a right-angled triangle the square of the hypotenuse is the sum of the squares of the other two sides.		√	√	√	√	√
15	[Converse to Pythagoras]. If the square of one side of a triangle is the sum of the squares of the other two, then the angle opposite the first side is a right angle.		√	√	√	√	√
	Proposition 9: (RHS). If two right-angled triangles have hypotenuse and another side equal in length respectively, then they are congruent.		√	√	√	√	√
16	For a triangle, base x height does not depend on the choice of base.					√	√
	Definition 38: The area of a triangle is half the base by the height.					√	√
17	A diagonal of a parallelogram bisects the area.					√	√
18	The area of a parallelogram is the base x height.					√	√
19*	The angle at the centre of a circle standing on a given arc is twice the angle at any point of the circle standing on the same arc.			√			√
	Corollary 2†: All angles at points of a circle, standing on the same arc are equal (and converse).			√			√
	Corollary 3: Each angle in a semi-circle is a right angle.		√	√	√	√	√
	Corollary 4: If the angle standing on a chord [BC] at some point of the circle is a right-angle, then [BC] is a diameter.		√	√	√	√	√
	Corollary 5: If ABCD is a cyclic quadrilateral, then opposite angles sum to 180□.			√			√
20	(i) Each tangent is perpendicular to the radius that goes to the point of contact. (ii) If P lies on the circle S, and a line l is perpendicular to the radius to P, then l is a tangent to S.					√	√
	Corollary 6: If two circles intersect at one point only, then the two centres and the point of contact are collinear.					√	√
21	(i) The perpendicular from the centre to a chord bisects the chord. (ii) The perpendicular bisector of a chord passes through the centre.					√	√

† The corollaries are numbered as in the *Geometry for Post-primary School Mathematics*; corollary 2 is the first one relating to theorem 19

	Constructions (Supported by 46 definitions, 20 propositions, 5 axioms and 21 theorems)	CMN Introd. Course	JC ORD	JC HR	LC FN	LC ORD	LC HR
1	Bisector of an angle, using only compass and straight edge.	√	√	√	√	√	√
2	Perpendicular bisector of a segment, using only compass and straight edge.	√	√	√	√	√	√
3	Line perpendicular to a given line l, passing through a given point not on l.			√			√
4	Line perpendicular to a given line l, passing through a given point on l.	√	√	√	√	√	√
5	Line parallel to given line, through a given point.	√	√	√	√	√	√
6	Division of a line segment into 2 or 3 equal segments without measuring it.	√	√	√	√	√	√
7	Division of a line segment into any number of equal segments, without measuring it.			√			√
8	Line segment of a given length on a given ray.	√	√	√	√	√	√
9	Angle of a given number of degrees with a given ray as one arm.		√	√	√	√	√
10	Triangle, given lengths of 3 sides.		√	√	√	√	√
11	Triangle, given SAS data.		√	√	√	√	√
12	Triangle, given ASA data		√	√	√	√	√
13	Right-angled triangle, given length of hypotenuse and one other side		√	√	√	√	√
14	Right-angled triangle, given one side and one of the acute angles.		√	√	√	√	√
15	Rectangle given side lengths.		√	√	√	√	√
16	Circumcentre and circumcircle of a given triangle, using only straight edge and compass.					√	√
17	Incentre and incircle of a triangle of a given triangle, using only straight edge and compass.					√	√
18	Angle of 60□ without using a protractor or set square.				√	√	√
19	Tangent to a given circle at a given point on it.				√	√	√
20	Parallelogram, given the length of the sides and the measure of the angles.				√	√	√
21	Centroid of a triangle.					√	√
22	Orthocentre of a triangle.						√

Appendix C Investigations of quadrilaterals and triangles

Investigating Quadrilaterals

Quadrilaterals	Square	Rhombus (not a square)	Rectangle (not a square)	Parallelogram (not a rectangle or a rhombus)	Trapezium (not a parallelogram) and not an isosceles trapezium which has the non parallel sides equal in length)
Describe it in words.					
Draw three examples in different orientations.					
How many axes of symmetry does it have? Show on a diagram.					
Does it have a centre of symmetry? Show on a diagram.					
Which sides are equal?					
What is the sum of all the angles?					
Are all angles equal?					

Quadrilaterals	Square	Rhombus (not a square)	Rectangle (not a square)	Parallelogram (not a rectangle or a rhombus)	Trapezium (not a parallelogram) and not an isosceles trapezium which has the non parallel sides equal in length)
Which angles are equal?					
What is the sum of two adjacent angles?					
Does a diagonal bisect the angles it passes through?					
Does a diagonal divide it into two congruent triangles?					
Given the length of its sides, can you calculate the length of a diagonal?					
Are the two diagonal s equal in length?					
Do the diagonals divide it into four congruent triangles?					

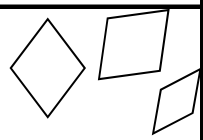
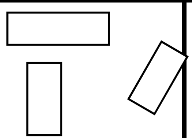
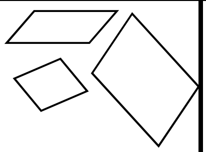
Quadrilaterals	Square	Rhombus (not a square)	Rectangle (not a square)	Parallelogram (not a rectangle or a rhombus)	Trapezium (not a parallelogram) and not an isosceles trapezium which has the non parallel sides equal in length)
Do the diagonals divide it into four triangles of equal area?					
Are the diagonals perpendicular?					
Do the two diagonals bisect each other?					
What information do you need to calculate its area? How do you calculate it?					
Does a diagonal bisect its area?					

Investigating triangles

Triangles	Equilateral	Isosceles	Right angled	Obtuse Angled
Describe it in words.				
Draw three examples in different orientations.				
How many axes of symmetry does it have? Show on a diagram.				
Does it have a centre of symmetry? Show on a diagram.				
What is the sum of the three angles?				
Are all angles equal?				
Are there any equal angles? Where?				
Can you say for certain what size the angles are?				
Apart from the isosceles triangles themselves which				

Triangles	Equilateral	Isosceles	Right angled	Obtuse Angled
of the others could also be isosceles?				
What information do you need to calculate its area? How do you calculate it?				
Draw 3 diagrams for each type of triangle showing each side as a base and the corresponding perpendicular height?				
How do you calculate the area?				
Is the centroid inside the triangle always?				
Is the circumcentre inside the triangle always?				
Is the incentre inside the triangle always?				

Suggested solutions

Quadrilaterals	**Square**	**Rhombus** (not a square)	**Rectangle** (not a square)	**Parallelogram** (not a rectangle or a rhombus)	**Trapezium** (not a parallelogram) and not an isosceles trapezium which has the non parallel sides equal in length)
Describe it in words.	A square is a quadrilateral in which all sides are equal in length and all angles are 90^0. (need only say that one angle is 90^0)	A rhombus is a quadrilateral with all sides equal and opposite angles equal. (a parallelogram with all sides equal in length.)	A rectangle is a quadrilateral with opposite sides equal and parallel and all interior angles equal to 90^0.	A parallelogram is a quadrilateral with opposite sides equal and parallel and opposite angles equal.	A trapezium is a quadrilateral which has 1 pair of parallel sides.
Draw three examples in different orientations.					
How many axes of symmetry does it have? Show on a diagram.	4	2	2	None	None if the non parallel sides are not equal in length.
Does it have a centre of symmetry? Show on a diagram.	✓	✓	✓	✓	No
Which sides are equal?	All	All	Opposite	Opposite	none
What is the sum of all the	360^0	360^0	360^0	360^0	360^0

Quadrilaterals	Square	Rhombus (not a square)	Rectangle (not a square)	Parallelogram (not a rectangle or a rhombus)	Trapezium (not a parallelogram) and not an isosceles trapezium which has the non parallel sides equal in length)
angles?					
Are all angles equal?	✓	x	✓	x	x
Which angles are equal?	All angles	Opposite angles	All angles	Opposite angles	
What is the sum of two adjacent angles?	180^0	180^0	180^0	180^0	180^0
Does a diagonal bisect the angles it passes through?	✓	✓	x	x	x
Does a diagonal divide it into two congruent triangles?	✓	✓	✓	✓	x
Given the length of its sides, can you calculate the length of a diagonal?	✓	No. Need to know an angle. Investigate using geostrips.	✓	No. Need to know an angle.	Need to know the lengths of two adjacent sides and the angle between them.
Are the two diagonal s equal in length?	✓	x	✓	x	x
Do the diagonals	✓	✓	x	x	x

Quadrilaterals	**Square**	**Rhombus** (not a square)	**Rectangle** (not a square)	**Parallelogram** (not a rectangle or a rhombus)	**Trapezium** (not a parallelogram) and not an isosceles trapezium which has the non parallel sides equal in length)
divide it into four congruent triangles?					
Do the diagonals divide it into four triangles of equal area?	✓	✓	✓	✓	x
Are the diagonals perpendicular?	✓	✓	x	x	x
Do the two diagonals bisect each other?	✓	✓	✓	✓	x
What information do you need to calculate its area? **How do you calculate it?**	One side length x. Area = x^2 (Base (*b*) and perpendicular height (*h*)from a vertex to that base Area = *b* x *h*)	Base (*b*) and perpendicular height (*h*)from a vertex to that base Area = *b* x *h* If you know the lengths of the diagonals *x* and *y* Area = ½ *x y*.	Lengths of 2 adjacent sides *l* and *b*. Area = *l* x *b*. (Base (*b*) and perpendicular height (*h*)from a vertex to that base Area = *b* x *h*)	Base (*b*) and perpendicular height (*h*)from a vertex to that base Area = *b* x *h*	The lengths of its parallel sides (*a* and *b*)and the perpendicular distance between them. Area = ½ (*a*+*b*) *h*
Does a diagonal bisect its area?	✓	✓	✓	✓	

Appendix D

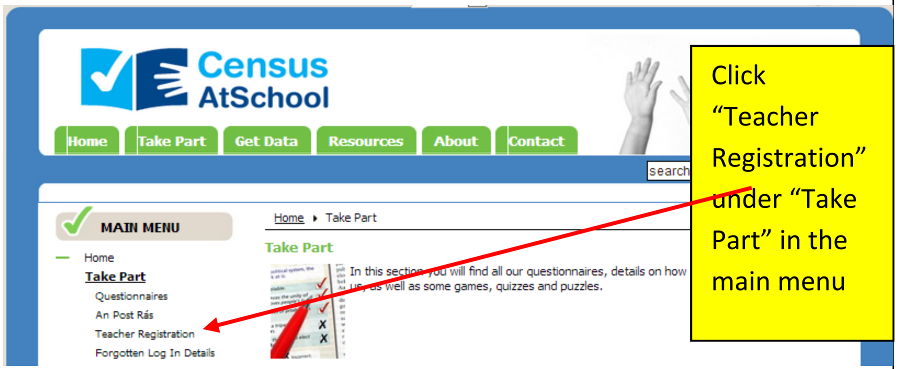
Part 1: Registration

www.CensusAtSchool.ie

This is the Irish *CensusAtSchool* homepage.

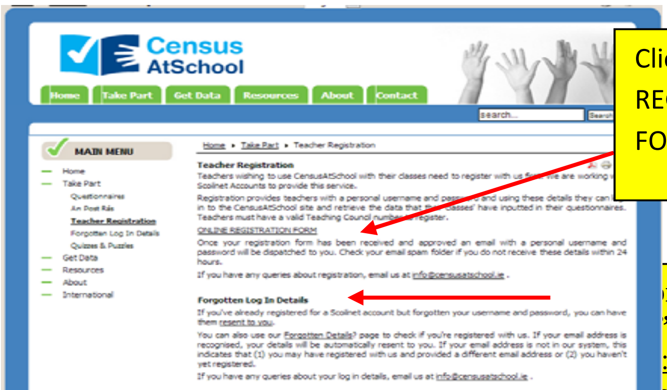
Click “Take Part” under the main menu.

1



Click “Teacher Registration” under “Take Part” in the main menu

2

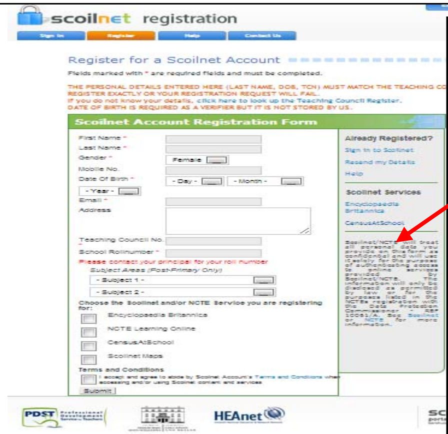


Click “ONLINE REGISTRATION FORM”

rgotten Log In

nfo@censusatschool.ie

3

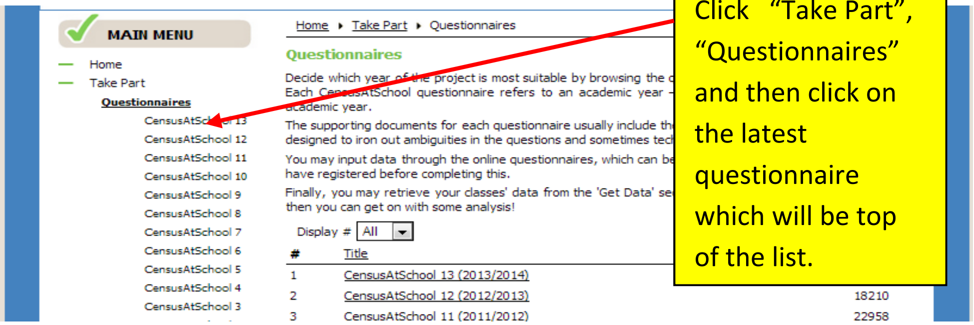


Fill out and click on “Submit”

Once your registration form has been received and approved an email with a personal username and password will be dispatched to you. Check your email spam folder if you do not receive these details within 48 hours.

4

Part 2: Accessing the questionnaires -use the highest phase available



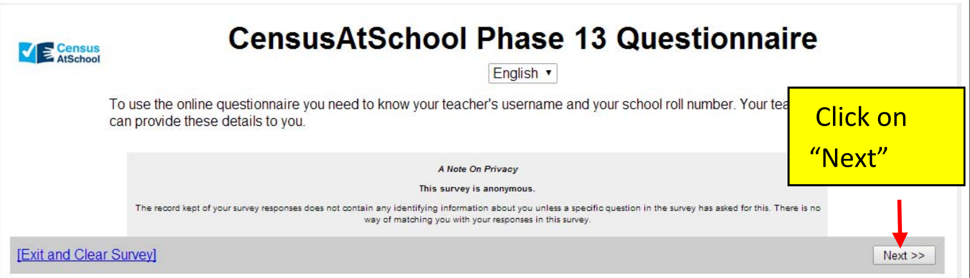
Click "Take Part", "Questionnaires" and then click on the latest questionnaire which will be top of the list.

5

Click "Take the Phase* questionnaire"

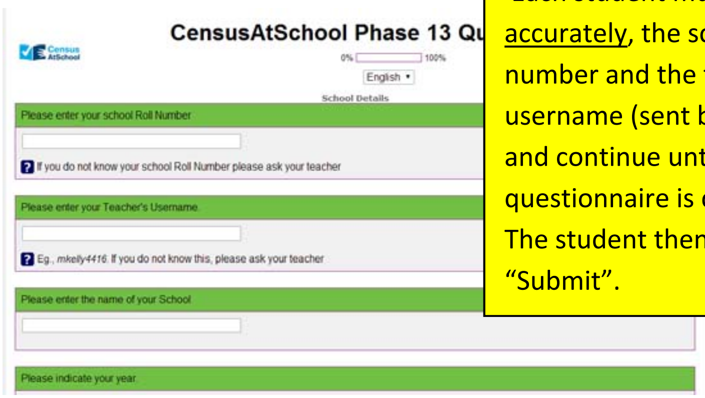
6

Part 3: Filling out the questionnaire



Click on "Next"

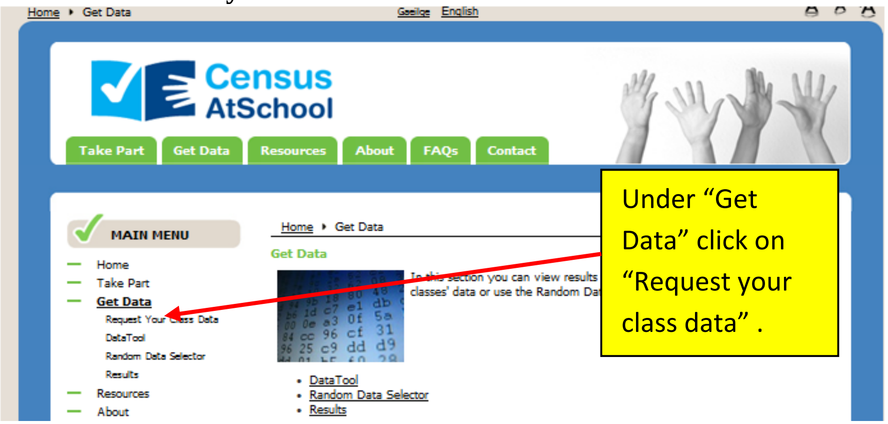
7



Each student must fill in accurately, the school roll number and the teacher's username (sent by email) and continue until the questionnaire is completed. The student then clicks "Submit".

8

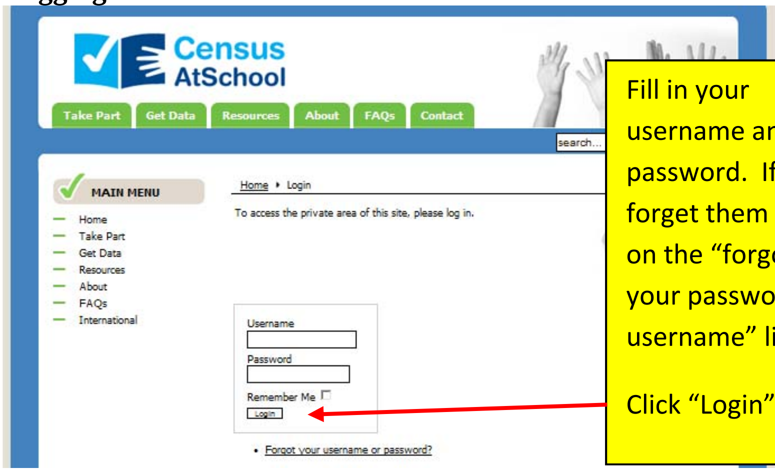
Part 4 Now that my students have filled in the data – What next?



Under "Get Data" click on "Request your class data".

9

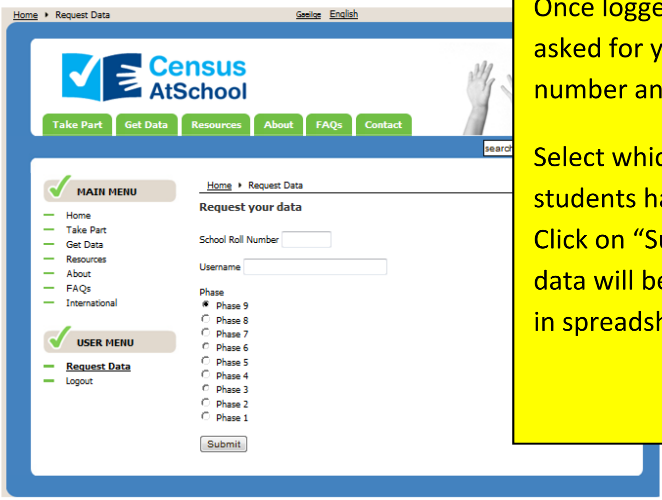
Logging in to retrieve data



Fill in your username and password. If you forget them "click on the "forgot your password or username" link.

Click "Login".

10



Once logged in you will be asked for your school roll number and username.

Select whichever phase your students have completed. Click on "Submit" and your data will be returned to you in spreadsheet form.

11

Your data will look similar to the spreadsheet below.

Sex	BrthMnth	BrthYear	BrthPlce	BedRms	PeoplHme	InetAxs	NumPts	PtsNone	PtsDog	PtsCat	PtsMouse	PtsRabt
Female	July	1992	New Sout	4	5	Yes - broa	4	No	Yes	Yes	No	No
Female	February	1996	Australian	4	5	Yes - broa	4	No	Yes	Yes	No	No
Male	Septembe	1994	Victoria	4	5	Yes - dial-	5	No	Yes	No	No	No
Female	Septembe	1991	Fiji	4	4	No - Interi	0	Yes	No	No	No	No
Male	August	1993	Queenslai	3	4	No - Interi	0	No	No	No	No	No
Female	Decembei	1993	Tasmania	3	5	Yes - dial-	3	No	No	Yes	No	No
Female	August	1990	Queenslai	4	5	Yes - broa	3	No	Yes	No	No	No
Female	Novembe	1991	New Sout	11	5	Yes - dial-	4	No	Yes	No	No	Yes
Male	June	1995	Victoria	3	5	Yes - othe	6	No	Yes	No	No	No
Female	March	1993	Western A	30	122	No - Interi	2	No	Yes	No	No	No
Female	Decembei	1989	Queenslai	3	3	Yes - broa	2	No	No	No	No	No
Female	April	1993	South Aus	3	5	No - Interi	4	No	Yes	Yes	No	No
Male	March	1996	New Sout	3	2	No - Interi	0	Yes	No	No	No	No
Male	Septembe	1993	Queenslai	4	4	Yes - dial-	18	No	Yes	No	No	No
Male	May	1995	Tasmania	3	4	Yes - broa	2	No	No	No	No	Yes
Female	March	1994	Western A	4	4	Yes - dial-	4	No	Yes	Yes	No	No

12

Appendix E: Trigonometric Formulae

* Proof required for higher level

1.* $\cos^2 A + \sin^2 A = 1$

2.* sine rule: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

3.* cosine rule: $a^2 = b^2 + c^2 - 2bc\cos A$

4.* $\cos(A - B) = \cos A\cos B + \sin A\sin B$

5.* $\cos(A + B) = \cos A\cos B - \sin A\sin B$

6.* $\cos 2A = \cos^2 A - \sin^2 A$

7.* $\sin(A + B) = \sin A\cos B + \cos A\sin B$

8. $\sin(A - B) = \sin A\cos B - \cos A\sin B$

9.* $\tan(A + B) = \dfrac{\tan A + \tan B}{1 - \tan A\tan B}$

10. $\tan(A - B) = \dfrac{\tan A - \tan B}{1 + \tan A\tan B}$

11. $\sin 2A = 2\sin A\cos A$

12. $\sin 2A = \dfrac{2\tan A}{1 + \tan^2 A}$

13. $\cos 2A = \dfrac{1 - \tan^2 A}{1 + \tan^2 A}$

14. $\tan 2A = \dfrac{2\tan A}{1 - \tan^2 A}$

15. $\cos^2 A = \dfrac{1}{2}(1 + \cos 2A)$

16. $\sin^2 A = \dfrac{1}{2}(1 - \cos 2A)$

17. $2\cos A\cos B = \cos(A + B) + \cos(A - B)$

18. $2\sin A\cos B = \sin(A + B) + \sin(A - B)$

19. $2\sin A\sin B = \cos(A - B) - \cos(A + B)$

20. $2\cos A\cos B = \sin(A + B) + \sin(A - B)$

21. $\cos A + \cos B = 2\cos\dfrac{A + B}{2}\cos\dfrac{A - B}{2}$

22. $\cos A - \cos B = -2\sin\dfrac{A + B}{2}\sin\dfrac{A - B}{2}$

23. $\sin A + \sin B = 2\sin\dfrac{A + B}{2}\cos\dfrac{A - B}{2}$

24. $\sin A - \sin B = 2\cos\dfrac{A + B}{2}\sin\dfrac{A - B}{2}$

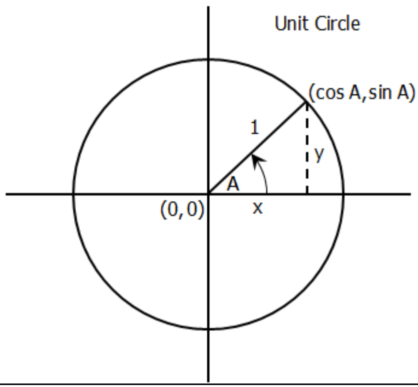
Appendix F – Sample Derivations of the formulae 1, 2, 3, 4, 5, 6, 7, 9

1.* $\cos^2 A + \sin^2 A = 1$ [Pg. 13, Trigonometry]

Distance from (0, 0) to $(\cos A, \sin A)$ is 1

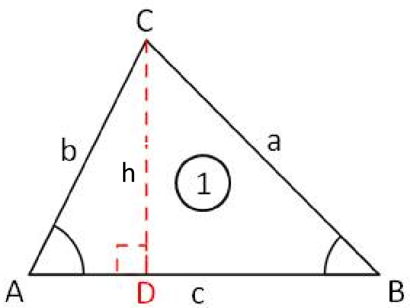
$\Rightarrow \sqrt{(\cos A - 0)^2 + \sin(A-0)^2} = 1$

$\Rightarrow \cos^2 A + \sin^2 A = 1$



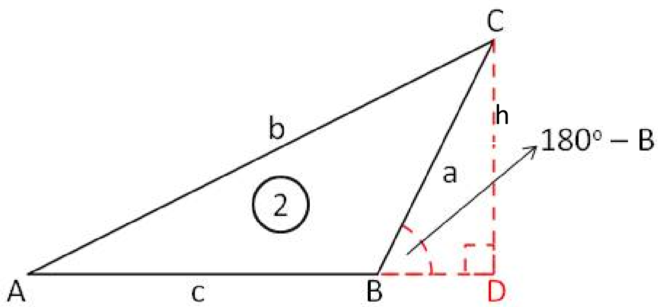
2.* sine formula: $\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$ [*Pg*. 16, Trigonometry of the triangl

Need to examine two cases – acute angled triangles such as ΔACB and obtuse angled triangles such as ΔBCD



Case 1: ΔACB (acute angled)

$\sin A = \dfrac{h}{b} \Rightarrow h = b \sin A$



Case 2: ΔBCD (obtuse angled)

$\sin A = \dfrac{h}{b} \Rightarrow h = b \sin A$

$\sin B = \frac{h}{a} \Rightarrow \text{h} = a \sin B$

$\sin(180 - B) = \frac{h}{a}$

$\sin B = \frac{h}{a}$ [as $\sin(180 - B) = \sin B$]

$\Rightarrow h = a \sin B$

In both cases: h= bsinA and h = asinB

Equating h's

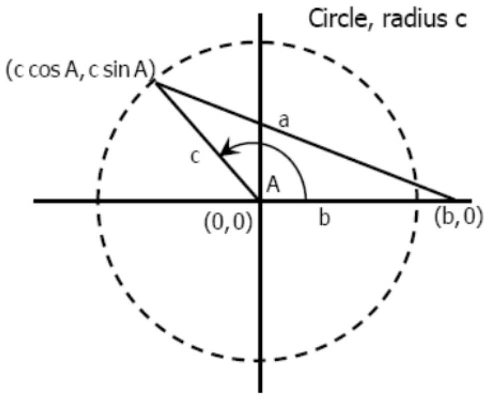
$a \sin B = b \sin A \Rightarrow \frac{a}{\sin A} = \frac{b}{\sin B}$

Similarly if the perpendicular height was dropped from A it would yield:

$\frac{b}{\sin B} = \frac{c}{\sin C}$

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

3.* cosine formula: $a^2 = b^2 + c^2 - 2bc \cos A$ [Pg. 16, Trigonometry of the triangle



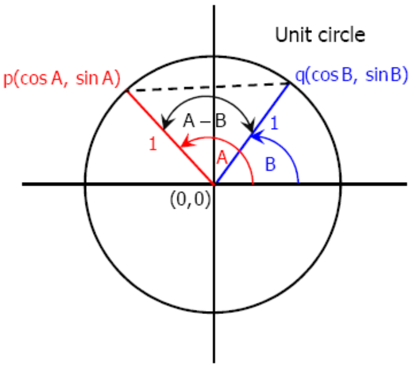
$a = \sqrt{(c \cos A - b)^2 + (c \sin A - 0)^2}$ using the distance formula

$a^2 = c^2 \cos^2 A - 2bc \cos A + b^2 + c^2 \sin^2 A$

$a^2 = b^2 + c^2(\cos^2 A + \sin^2 A) - 2bc \cos A$

$a^2 = b^2 + c^2 - 2bc \cos A$ as $\cos^2 A + \sin^2 A = 1$

4.*	$\cos(A-B)=\cos A\cos B+\sin A\sin B$	[Pg. 14, Compound angle formulae]



Find the distance between p and q in two different ways and equate the answers

$|pq|^2 = 1^2 + 1^2 - 2(1)(1)\cos(A-B)$ using cosine formula, $a^2 = b^2 + c^2 - 2bc\cos A$

$|pq|^2 = 2 - 2\cos(A-B)$

$|pq| = \sqrt{(\cos A - \cos B)^2 + (\sin A - \sin B)^2}$ using distance formula

$|pq|^2 = \cos^2 A - 2\cos A\cos B + \cos^2 B + \sin^2 A - 2\sin A\sin B + \sin^2 B$

$|pq|^2 = 2 - 2\cos A\cos B - 2\sin A\sin B$

Equating both:

$2 - 2\cos(A-B) = 2 - 2\cos A\cos B - 2\sin A\sin B$

$-2\cos(A-B) = -2\cos A\cos B - 2\sin A\sin B$

$\cos(A-B) = \cos A\cos B + \sin A\sin B$

5.*	$\cos(A+B)=\cos A\cos B-\sin A\sin B$	[Pg. 14, Compound angle formulae]
	$\cos(A-B)=\cos A\cos B+\sin A\sin B$	using formula 4
	$\cos(A-(-B))=\cos A\cos(-B)+\sin A\sin(-B)$	changing B to $-B$
	$\cos(A+B)=\cos A\cos B-\sin A\sin B$	as $\cos(-B)=\cos B$ and $\sin(-B)=-\sin B$

6.*	$\cos 2A=\cos^2 A-\sin^2 A$	[Pg. 14, Double angle formulae]

$\cos(A+B)=\cos A\cos B-\sin A\sin B$	using formula 5
$\cos(A+A)=\cos A\cos A-\sin A\sin A$	changing B to A
$\cos 2A \ =\cos^2 A-\sin^2 A$	

7.* $\sin(A+B)=\sin A\cos B+\cos A\sin B$ [Pg. 14, Compound angle formulae]

$\sin(A+B) \ =\cos[90^o-(A+B)]$ using complementary angles, $\sin\theta=\cos(90^\circ-\theta)$

$=\cos[90^o-A-B]$

$=\cos[(90^o-A)-B]$

$=\cos(90^o-A)\cos B+\sin(90^o-A)\sin B$ using formula 4, $\cos(A-B)=\cos A\cos B +$

$=\sin A\cos B+\cos A\sin B$

9.* $\tan(A+B)=\dfrac{\tan A+\tan B}{1+\tan A\tan B}$ [*Pg*. 14, Compound angle formulae]

$\tan(A+B)=\dfrac{\sin(A+B)}{\cos(A+B)}$ $\tan\theta=\dfrac{\sin\theta}{\cos\theta}$

$=\dfrac{\sin A\cos B+\cos A\sin B}{\cos A\cos B-\sin A\sin B}$

$=\dfrac{\dfrac{\sin A\cos B}{\cos A\cos B}+\dfrac{\cos A\sin B}{\cos A\cos B}}{1-\dfrac{\sin A\sin B}{\cos A\cos B}}$ dividing everywhere by $\cos A\cos B$

$=\dfrac{\tan A+\tan B}{1-\tan A\tan B}$