TEACHER HANDBOOK

Second Year

DRAFT

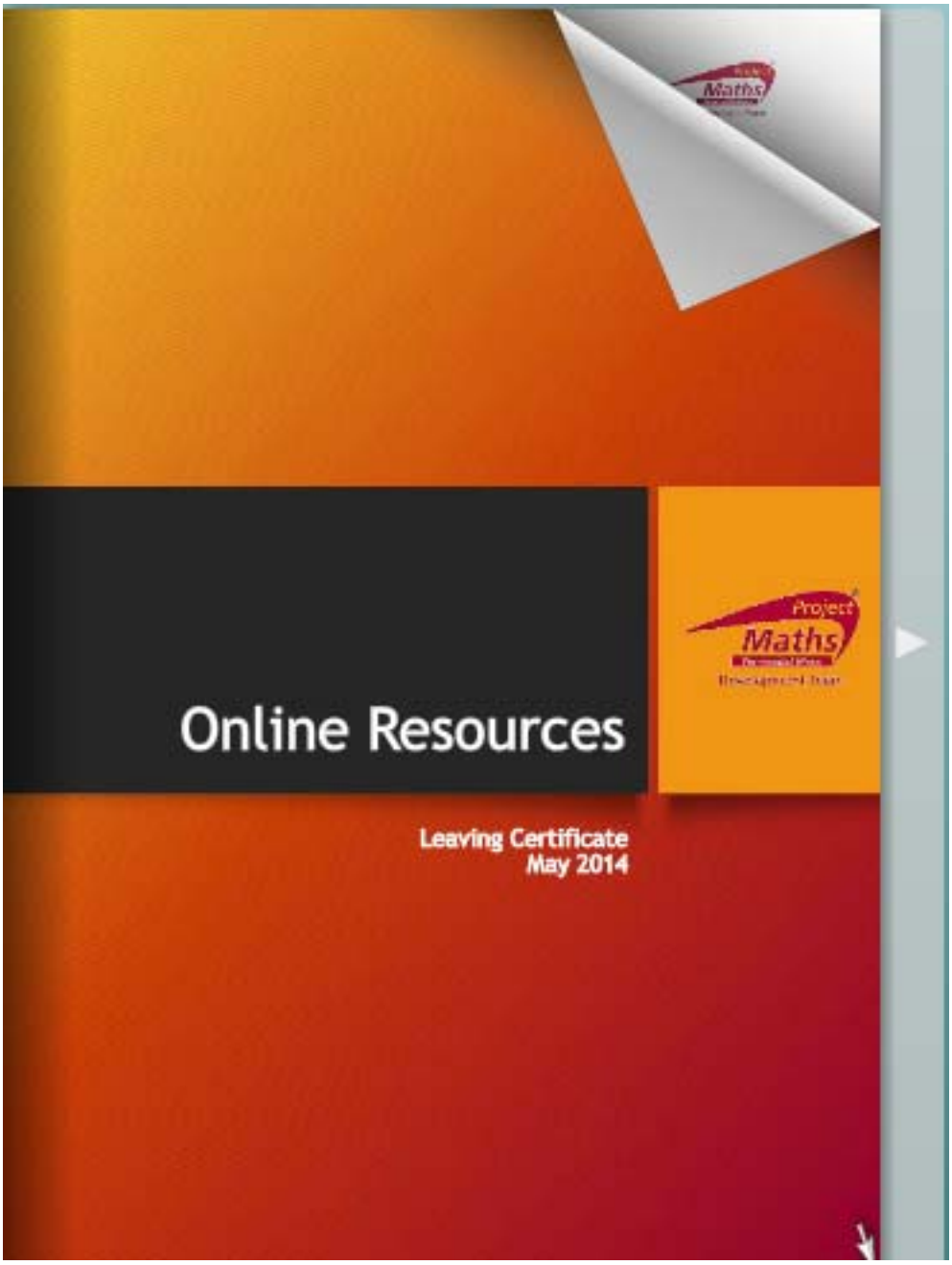
Project Maths Tionscadal Mata Development Team

Based on the 2016 Syllabus

Project Maths Tionscadal Mata Development Team

Contents

The strand structure of the syllabus should not be taken to imply that topics are to be studied in isolation. Where appropriate, connections should be made within and across the strands and with other areas of learning. (NCCA JC syllabus page 10 and LC syllabus page 8)



This is a comprehensive reference for resources which will allow teachers plan lessons, easily access specific learning outcomes in the syllabus and relevant support material such as “Teaching & Learning Plans” and suggested activities to support learning and teaching. Click here.

Introduction: Student Learning

While this is a handbook for teachers, it must be emphasised that student learning and the process of mathematical thinking and building understanding are the main focus of this document.

Information and Communications Technologies are used whenever and wherever appropriate to help to support student learning. It is also envisaged that, at all levels, learners will engage with a dynamic geometry software package.

Students with mild general learning disabilities

Teachers are reminded that the NCCA Guidelines on mathematics for students with mild general learning disabilities can be accessed at

http://www.ncca.ie/uploadedfiles/PP_Maths.pdf

This document includes

- Exemplars (from page 7)
- Approaches and Methodologies (from Page 11)

Higher level material

Material which is higher level only is shown in bold typeface.

Note: Synthesis and problem solving listed below must be incorporated into all of the Strands.
The list of skills below is taken from Strand 1of the syllabus but, an identical list is given at the end of each Strand in the syllabus.

Students learn about	Students should be able to
1.8 Synthesis and problem-solving skills	– explore patterns and formulate conjectures – explain findings – justify conclusions – communicate mathematics verbally and in written form – apply their knowledge and skills to solve problems in familiar and unfamiliar contexts – analyse information presented verbally and translate it into mathematical form – devise, select and use appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions.

Useful websites

	http://www.projectmaths.ie/ http://ncca.ie/en/Curriculum_and_Assessment/Post-Primary_Education/Project_Maths/ http://www.examinations.ie/

Literacy and Numeracy Strategy

The National Strategy to Improve Literacy and Numeracy among Children and Young People 2011-2020

Numeracy encompasses the ability to use mathematical understanding and skills to solve problems and meet the demands of day-to-day living in complex social settings. To have this ability, a young person needs to be able to think and communicate quantitatively, to make sense of data, to have a spatial awareness, to understand patterns and sequences, and to recognise situations where mathematical reasoning can be applied to solve problems.

Literacy includes the capacity to read, understand and critically appreciate various forms of communication including spoken language, printed text, broadcast media, and digital media.

Colour coding used in the suggested sequence below:

Strand 1 **Statistics and Probability**	**Strand 2** **Geometry and Trigonometry**	**Strand 3** **Number**	**Strand 4** **Algebra**	**Strand 5** **Functions**

Suggested sequence of topics

Section number	Strand (Syllabus section)	Corresponding Lesson Number	Title of lesson idea	Page number	
Section 1	3.4	2.1	Applied measure1	11	
Section 2	4.1,4.2,4.3,4.4	2.2	Linear and quadratic relationships: multi - representational approach	12	
Section 3	3.3	2.3	Applied arithmetic -financial maths	13	
	3.4	2.4	Applied measure 2 - area and volume	13	
	3.2	2.5	Investigating rules for indices	14	
Section 4	4.	2.6	Revision 1st year Algebra	15	
	4.6	2.7	Factors	16	
	4.6	2.8	Adding algebraic fractions	17	
	4.7	2.9	Linear equations, linear inequalities and simultaneous linear equations	17	
	4.7	2.10	Solve quadratic equations	18	
Section 5	2.1	2.11	Introduction to Pythagoras' Theorem (Theorem 14)	19	
Section 6	2.2	2.12	Revise first year coordinate geometry	20	
	2.2	2.13	Midpoint of a line segment	20	
	2.2	2.14	Slope of a line	21	
	2.2	2.15	Length of a line segment	21	

Section number	Strand (Syllabus section)	Corresponding Lesson Number	Title of lesson idea	Page number	
	2.2	2.16	Equation of a line	21	
	2.2	2.17	Intersection of lines	22	
Section 7	3.5	2.18	Sets	23	
Section 8	1.1, 1.2, 1.3	2.19	Probability - revision of first year material	24	
	1.1, 1.2, 1.3	2.20	2 events - tossing 2 dice (2 way tables)	24	
	1.1, 1.2, 1.3	2.21	2 events - tossing 2 coins (2 way tables + tree diagrams) Other similar scenarios	25	
	1.1, 1.2, 1.3	2.22	Probability using playing cards, spinners etc.	25	
	1.4 & 1.5	2.23	Statistics - building on first year Stats	26	
	1.4 & 1.5	2.24	Data handling cycle using categorical data	27	
	1.4 & 1.5	2.25	Data handling cycle using numeric data	27	
	1.4 & 1.5	2.26	Evaluate the effectiveness of and interpreting different graphical displays of data	28	
Section 9	2.1 Synthetic geometry1	2.27 (1.12 & 1.13)	T&L plans 6 and 7 Planes and points and Introduction to angles (Revision)	30	

Section number	Strand (Syllabus section)	Corresponding Lesson Number	Title of lesson idea	Page number	
	2.1	2.28 (1.14 & 1.15)	Measuring angles, Theorem 1 (Revision)	30	
	2.1	2.29 (1.16 and 1.17)	Use of the compass, Constructions 8 & 9 (Revision)	31	
	2.1	2.30 (1.18)	Constructions 5 &6 (Revision) and **Construction 7 (H.L. only)**	32	
	2.1	2.31 (1.26)	Constructions 10 and 11 (Revision)	32	
	2.1	2.32 (1.27 & 1.28)	Construction 12 & Theorem 2 (Revision)	33	
	2.1	2.33 (1.29& 1.30)	Theorems 3,4,5,& 6 (Revision)	33	
	2.1	2.34 (1.31)	Constructions 1 ,2,3 & 4 (Revision)	34	
Section 10	2.4 Transformation geometry	2.35	Translations	35	
	2.4	2.36 (1.19)	Axial symmetry (part revision)	35	
	2.4	2.37 (1.20)	Central symmetry (part revision) and rotations	36	
Section 11	2.1 Synthetic geometry (2)	2.38	Parallelograms and Theorem 9	38	
	2.1	2.39	Parallelograms, Theorem 10	39	

Section number	Strand (Syllabus section)	Corresponding Lesson Number	Title of lesson idea	Page number	
	2.1	2.40	Investigating quadrilaterals	41	
Section 12	3.3	2.41	Ratio and proportion	41	
Section 13	2.1	2.42	Revision of Pythagoras' Theorem 14	42	
	2.3	2.43	Scaled diagrams T& L plan 8 (Introduction to trigonometry)	42	
	2.3	2.44	Trigonometric ratios T& L plan 8 (Introduction to trigonometry)	43	

(Note to self : Add in page on content from workshops and content from modular courses.)

Lesson Ideas

Section 1: Number

Lesson Idea 2.1

Title
Applied measure 1

Resources
Online Resources Junior Certificate document

Content
These lessons will involve the students in investigating and understanding:

- How to calculate, interpret and apply units of measure and time in context
- How to solve problems involving average speed, distance and time

Section 2: Algebra

Lesson Idea 2.2

Title
Representing and examining linear and quadratic relations using tables, diagrams, graphs and formulae

Resources
Online Resources Junior Certificate document
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

- The use of tables, diagrams, physical models, graphs and formulae as tools for representing and analysing linear patterns and relations in order to
    - Identify variables and constants and point out how they appear in the different representations
    - Use the representations to reason about the situation from which the relationship is derived and communicate their thinking to others
    - Discuss rate of change and the y - intercept
    - Generalise and explain patterns and relationships in words and numbers
    - Recognise that the distinguishing of a linear relationship is a constant rate of change
    - Decide if two linear relations have a common value (decide if two lines intersect and where the intersection occurs).
    - Distinguish relations of the form $y = mx$ (proportional relationships) as distinct from relationships of the form $y = mx + c$ (non - proportional relationships)
    - The concept of a linear function as a relationship between a set of inputs and a set of outputs where each input is related to only one output

- The use of tables, diagrams, graphs and formulae as tools for representing and analysing quadratic patterns and relations in order to:
    - Recognise that a distinguishing feature of quadratic relations is the way change varies i.e. the rate of change of the rate of change is constant
    - The concept of a quadratic function as a relationship between a set of inputs and a set of outputs where each input is related to only one output

Section 3: Number

Lesson Idea 2.3

Title

Applied Arithmetic

Resources
Online Resources Junior Certificate document

Content
These lessons will involve the students in investigating and understanding:

- Number systems $\mathbb{N}, \mathbb{Z}, \mathbb{Q}$ (progression from first year)
- Fractions, decimals, percentages, ratio and proportion in context involving:
    - Solving problems e.g. mobile phone tariffs, currency transactions, shopping, VAT and meter readings
    - Profit or loss, % profit or loss (on the cost price), discount and % discount, selling price
    - Compound interest for not more than three years
    - Income tax (standard rate only),
    - Net pay (including other deductions of specified amounts)
    - Making value- for- money calculations and judgments

Lesson Idea 2.4

Six class periods

Title
Applied measure 2

Resources
Online Resources Junior Certificate document
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

- Units of measure and time

- Problems involving perimeter and area of the following plane figures: disc, triangle, rectangle, square and figures made from combinations of these (revision and extension from first year)
- The relationship between the circumference of a circle, its diameter and π
- the nets of rectangular solids (revision from first year)
- The surface area of rectangular solids (revision from first year)
- The volume of rectangular solids and cylinders

Lesson Idea 2.5

Title
Rules for indices

Resources
Online Resources Junior Certificate document

Content
These lessons will involve the students in investigating and understanding:

- The rules for indices $a \in \mathbb{Z},\ a \neq 0, p, q \in \mathbb{N}$:
  $a^p a^q = a^{p+q}$
  $\frac{a^p}{a^q} = a^{p-q}$
  $\left(a^p\right)^q = a^{pq}$
  Use the notation $a^{\frac{1}{2}}, a \in N$
- How to express rational numbers ≥ 1 in the form $a \times 10^n$, where a is in decimal form correct to a specified number of places and where $n = 0$ or $n \epsilon N$
- How to compute reciprocals

Section 4: Algebra

Lesson Idea 2.6

Title
Revision and extension of algebraic expressions and simple linear equations from first year

Resources
Online Resources Junior Certificate document

Content
These lessons will involve the students in investigating and understanding:

- Indices in algebra (indices $\in \mathbb{N}$)
- Terms, coefficients and expressions
- Generating algebraic expressions from simple contexts
- Evaluating expressions of the form

$ax+by$
$a(x+y)$
x^2+bx+c
$\frac{ax+by}{cx+dy}$
axy
where $a,b,c,d,x,y \in \mathbb{Z}$ (ordinary level)

$\boldsymbol{ax^2+bx+c}$
where $a,b,c,d,x,y \in \mathbb{Q}$ (higher level)

- Adding and subtracting simple algebraic expressions of forms such as:

$(ax+by+c)\pm(dx+ey+f)$
$(ax^2+bx+c)\pm(dx^2+ex+f)$

$(\boldsymbol{ax+by+c})\pm.....\pm(\boldsymbol{dx+ey+f})$
$(\boldsymbol{ax^2+bx+c})\pm.....\pm(\boldsymbol{dx^2+ex+f})$ where $\boldsymbol{a,b,c,d,e,f} \in \mathbb{N}$

- Use of the associative and distributive property to simplify such expressions as:
(e.g. using a model such as the array model)

- $a(bx+cy+d)+e(fx+gy+h)$
- $\boldsymbol{a(bx+cy+d)+.....+e(fx+gy+h)}$
- $a(bx^2+cx+d)$
- $ax(bx^2+c)$
    - where $a,b,c,d,e,f,g,h \in Z$

      $a(bx^2+cx+d)$

      $ax(bx^2+c)$ where $a,b,c,d,e,f,g,h \in \mathbb{Z}$

      $(ax+b)(cx+d)$

- Multiply expressions of the form:
    - $(ax+b)(cx+d)$

Higher level material:

$(x+y)(x+y)$

$(x-y)(x-y)$

Lesson Idea 2.7

Title

Algebraic factors

Resources

Online Resources Junior Certificate document

Content

These lessons will involve the students in investigating and understanding:

- Use of the distributive law to multiply expressions
- Factorising expressions such as:

  ax, axy where $a \in \mathbb{Z}$

  $abxy+ay$ where $a,b \in \mathbb{Z}$

  ax^2+bx where $a,b \in \mathbb{Z}$

  $sx-ty+tx-sy$ where s,t,x,y are variable

  x^2+bx+c, where $b,c \in \mathbb{Z}$

  x^2-a^2

Higher level material:

$ax^2 + bx + c, a \in \mathbb{N}, b, c \in \mathbb{Z}$

$a^2x^2 - b^2y^2$, where $a, b \in \mathbb{N}$

Lesson Idea 2.8

Title

Adding algebraic fractions

Resources

Online Resources Junior Certificate document

Content

These lessons will involve the students in investigating and understanding:

- The addition of expressions such as:
- $\frac{ax+b}{c} \pm \frac{dx+e}{f}$ where $a, b, c, d, e, f \in \mathbb{Z}$

$\frac{ax+b}{c} \pm \ldots \pm \frac{dx+e}{f}$ where $a, b, c, d, e, f \in \mathbb{Z}$

Lesson Idea 2.9

Title

Linear equations in one and two variables and linear inequalities in one variable

Resources

Online Resources Junior Certificate document

Dynamic software package

Content

These lessons will involve the students in investigating and understanding:

- The concept of equality and what is meant by an equation
- Solve simple problems leading to linear equations
- The use of suitable strategies (graphic, numeric and algebraic, mental) in the solution of first degree equations in one variable, with coefficients elements of $\mathbb{Z}$ and solutions in $\mathbb{Z}$ including equations

of the form
$\frac{ax+b}{c} \pm \frac{dx+e}{f} = \frac{g}{h}$ where $a,b,c,e,f,g,h \in \mathbb{Z}$

- The use of suitable strategies (graphic, numeric and algebraic, mental) in the solution of first degree equations in two variables, with coefficients elements of $\mathbb{Z}$ and solutions in $\mathbb{Z}$
- The solution of linear inequalities in one variable of the form $g(x) \leq k$ where $g(x) = ax+b, a \in \mathbb{N}$ and $b,k \in \mathbb{Z}$
- Graph solution sets on the number line for linear inequalities in one variable

Lesson Idea 2.10

Title
Solve quadratic equations

Resources
Online Resources Junior Certificate document
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

- The use of suitable strategies (graphic, numeric and algebraic, mental) in the solution of quadratic equations of the form $x^2+bx+c=0$ where $b,c \in \mathbb{Z}$ and x^2+bx+c is factorisable
- Solve simple problems leading to quadratic equations

Section 5: Geometry

Lesson Idea 2.11

Title
Introduction to Pythagoras' Theorem

Resources

Online Resources Junior Certificate document
A mathematical instruments set, graph paper
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

- Pythagoras' Theorem using models and using algebra to generalise findings
- That there are an infinite number of Pythagorean triples

Section 6: Co-ordinate Geometry

Lesson Idea 2.12

Title
Revision of first year material on the coordinate plane

Resources
Online Resources Junior Certificate document
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

- Plane, points, lines, line segments, rays, collinear points, length of a line segment
- Coordinating the plane
- Locating points on the plane using coordinates

Lesson Idea 2.13

Title
Mid-point of a line segment

Resources
A mathematical instruments set, graph paper
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

- The concept of midpoint of a line segment making connections with the concept of the mean from statistics
- How to generalise from this concept to the midpoint formula in coordinate geometry

Lesson Idea 2.14

Title
The slope of a line

Resources
Online Resources Junior Certificate document
A mathematical instruments set, graph paper
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

- Slope as a ratio of rise to run
- Links between slope and rate of growth
- Links to real life context (e.g. growth of plant with a constant growth rate)
- How to generalise to the slope of a line formula

Lesson Idea 2.15

Title
Length of a line segment

Resources
Online Resources Junior Certificate document
A mathematical instruments set, scientific graph paper,
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

- Instruments and units used to measure distance
- How Pythagoras' Theorem can be used to measure distance
- The distance formula in coordinate geometry and its connection to Pythagoras' Theorem
- The relationship between the midpoint and the length of a line segment

Lesson Idea 2.16

Title
The equation of a line

Resources
Online Resources Junior Certificate document

A mathematical instruments set, graph paper,
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

- Links to real life context (e.g. growth of plant with a constant growth rate)
- Using tables, graphs and diagrams in relation to the equation of a line
- The connection between the y-intercept, slope and the equation of a line
- The equation of line in the forms:
    - $y = mx + c$
    - $y - y_1 = m(x - x_1)$
    - $\boldsymbol{ax} + \boldsymbol{by} + \boldsymbol{c} = 0$ (HL only)

      where a, b, c are integers and m is the slope of the line
- The connection between the x and y intercepts and the equation of a line from a graphical and algebraic perspective
- Points on a line (and not on a line) from a graphical and algebraic perspective
- Properties of the equation of a line
- Equation of the $x - axis$
- Equation of the $y - axis$

Lesson Idea 2.17

Title
Intersection of lines

Resources
Online Resources Junior Certificate document
A mathematical instruments set, graph paper,
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

- How to use tables, graphs and algebra in relation to the intersection of lines
- Links to real life context (e.g. comparing plants with varying constant growth rates and starting heights)

Section 7: Number

Lesson Idea 2.18

Title
Sets -revision of first year material plus set difference and complement

Resources
Online Resources Junior Certificate document
A mathematical instruments set

Content
These lessons will involve the students in investigating and understanding:

- The concept of a set as being a collection of well-defined objects, equality of sets
- How to list the elements of a finite set
- Venn diagrams, universal set, null set, sub-set, cardinal number, set builder notation
- Union and intersection as operators on two sets
- Set difference
- Complement of a set
- Investigate the commutative property for intersection , union and difference
- Solve problems involving sets

Section 8: Probability and Statistics

Lesson Idea 2.19

Title
Revision of fundamental principle of counting and first year probability

Resources
Online Resources Junior Certificate document

Content
These lessons will involve the students in investigating and understanding:

- How to list all possible outcomes of an experiment
- How to apply the fundamental principle of counting
- How to distinguish certain from uncertain events
- How to describe events as being more or less likely from experience
- How to order events from least likely to most likely and be able to justify their choice
- How to use the scale from 0 to 1 to informally place everyday chance-related events
- How to represent and interpret probabilities as fractions, decimals and percentages
- How to represent the probability of an event as a fraction or decimal between 0 and 1 or as a percentage
- Equally likely outcomes
- How to list all possible outcomes for practical experiments such as rolling a fair die
- That the outcomes on successive throws of a die are independent of each other
- How to calculate the relative frequency for each outcome by experiment and note how it approaches the theoretical probability as the number of trials increases
- The principle that, in the case of equally likely outcomes, the probability is given by the number of outcomes of interest divided by the total number of outcomes
- The following terminology: trial, outcome, set of all possible outcomes, relative frequency, event, theoretical probability, equally likely outcomes
- How to estimate probabilities from experimental data

Lesson Idea 2.20

Title
Fair trials with two dice

Resources
Online Resources Junior Certificate document

Content
These lessons will involve the students in investigating and understanding:

- How to list all the possible outcomes for throwing 2 dice using a two way table
- How to relate the number of outcomes to the fundamental principle of counting
- What the concept of *fairness* means in a game with 2 dice
- How to come up with rules for a game which make it fair/unfair
- How to construct a probability table
- The relationship between an event and its complement
- How to determine the probability of an event using the results of an experiment and use this to predict the result of a repetition of the experiment, for equally likely outcomes

Lesson Idea 2.21

Title
Outcomes of "two coin" tosses including tree diagrams and tossing one coin and 1 die

Resources
Online Resources Junior Certificate document

Content
These lessons will involve the students in investigating and understanding:

- How to list the possible outcomes when tossing a coin and calculate the probability of getting a head or a tail
- How to list all the possible outcomes when tossing 2 coins (using a two way table)
- How to use a tree diagram and two way table to list all the possible outcomes when tossing a coin
- How to use tree diagrams and two way tables in other scenarios
- How to relate the number of outcomes to the fundamental principle of counting
- How to determine the probability of an event using the results of an experiment and use this to predict the result of a repetition of the experiment, for equally likely outcomes.
- The use of the term "independent events"

Lesson Idea 2.22

Title
Probability with coins, dice, spinners, containers with different coloured objects, playing cards, sports results, etc.

Resources
Online Resources Junior Certificate document

Content
These lessons will involve the students in investigating and understanding:

- A standard deck of playing cards, knowing the names of the 4 suits, how many cards in a pack, how many in each suit, and the names of the cards in each suit
- How to calculate the theoretical probability of picking different cards from the pack
- Probabilities with spinners or containers with different coloured objects

Lesson Idea 2.23

Title
Statistics - Revising and building on first year material using a statistical investigation

Resources
Online Resources Junior Certificate document

Content
These lessons will involve the students in investigating and understanding:

- The purpose of statistics
- The use of statistics to gather information from a sample of the population with the intention of making generalisations about the whole population
- Misconceptions and misuses of statistics
- How to formulate one (or more) questions that can be answered with data
- How to plan an investigation involving statistics and conduct the investigation using the data handling cycle:
    - Pose one (or more) questions that can be answered with data
    - Collect data
    - Analyse the data
    - Interpret the results
    - Refine the question if necessary

- The different ways of collecting data, e.g. *CensusAtSchool* questionnaire
- **How to select a sample (simple random sample)**
- **The importance of representativeness so as to avoid biased samples**
- The different types of data:
    - categorical (nominal and ordinal)
    - numerical (discrete or continuous)
- How to summarise data in diagrammatic form including spreadsheets
- How to use bar charts, line plots, stem and leaf plots to display data
- How to select appropriate graphical methods to describe the sample
- Measures of central tendency: mean, mode, median
- Range as a measure of spread

Lesson Idea 2.24

Title
Data handling cycle for categorical data

Resources
Online Resources Junior Certificate document

Content
These lessons will involve the students in investigating and understanding:

- How to pose a question yielding categorical data (possibly from census at school questionnaire)
- How to collect categorical data (possibly from census at school questionnaire)
- How to use bar charts as a graphical representation of the categorical data using frequency on the y -axis
- How to use bar charts as a graphical representation of the categorical data using relative frequency(percentages) on the y -axis
- How to use a line plot as a graphical representation of categorical data
- How to represent the data in a particular category as a proportion of all the data
- How to use pie charts to compare "part to whole" using the concept of proportionsThe effectiveness of the different graphical representations for categorical data
- How to use the mode as a measure of central tendency
- How to draw conclusions related to the original question

Lesson Idea 2.25

Title
Data handling cycle for numerical data

Resources
Online Resources Junior Certificate document

Content
These lessons will involve the students in investigating and understanding:

- The distinction between continuous and discrete numerical data
- How to pose a question yielding numeric discrete or continuous data (possibly from census at school questionnaire)
- How to collect numeric discrete or continuous data (possibly from census at school questionnaire)
- How to use a stem and leaf plot as a graphical representation of the ungrouped continuous numeric data
- The effect of splitting stems in a stem and leaf plot if data is suitable
- Range as a measure of spread
- How to group data into equal class intervals in a grouped frequency table

- Histograms with equal class intervals for numeric continuous data and relationship with stem and leaf plots
- Histograms with relative frequency (percentages) on the y - axis
- The effect of interval width on the information conveyed by the histogram
- The effectiveness of the different graphical representations for numeric continuous data
- How to use the various graphical representations to describe what is typical in the data using concepts of shape and measures of central tendency: mean, mode, median
- **Mean of a grouped frequency distribution (HL only)**
- How to relate the interpretation to the original question posed

Lesson Idea 2.26

Title
Evaluate the effectiveness of different displays in representing the findings of a statistical investigation conducted by others

Resources
Online Resources Junior Certificate document

Content
These lessons will involve the students in investigating and understanding:

- How to evaluate the effectiveness of different displays in representing the findings of a statistical investigation conducted by others.

Section 9: Synthetic Geometry 1

A possible sequence for teaching second year students

While proofs are not the issue as regards informal introduction, it is important that students are kept aware that the theorems build logically.

Concepts relevant to this section in synthetic geometry:
Set, plane, point, line, ray, angle, real number, length, degree. Triangle, right-angle, congruent triangles, parallel lines, area, line segment, collinear points, distance, reflex angle, ordinary angle, straight angle, null angle, full angle, supplementary angles, vertically-opposite angles, acute angle, obtuse angle, angle bisector, perpendicular lines, perpendicular bisector of a line segment, isosceles triangle, equilateral triangle, scalene triangle, right-angled triangle, exterior angles of a triangle, interior opposite angles, alternate angles, corresponding angles, transversal line, circle.

Refer to the syllabus for the **"Geometry for Post - primary School Mathematics"** which sets out the agreed course in geometry for both Junior Certificate Mathematics and Leaving Certificate Mathematics. Strand 2 of the relevant syllabus document specifies the learning outcomes at the different syllabus levels.

Refer to Appendix A: "Geometry - Thinking at different levels - The Van Hiele Theory"

Refer to Appendix B for the "Guide to Axioms, Theorems and Constructions for all Levels" In this document * indicates that proof of the relevant theorem is required for JCHL and LCHL and ** indicates that proof of the relevant theorem is required for LCHL only.

Information Technology is used whenever and wherever appropriate to help to present mathematical concepts effectively to students. (Dynamic software package)

Teachers are also strongly recommended to use the Geometry Lesson Ideas in the "Junior Certificate Guidelines for Teachers" (DES 2002, Government Publications Sales Office). It is also available to download at www.projectmaths.ie.

Check: Will we refer to "Junior Certificate Guidelines for Teachers"?

As outlined at the workshops, the use of manipulative products such as "geostrips", "anglegs", geo-boards etc. can make the learning so much more enjoyable for students of all perceived abilities.

Note on experimentation and experimental results:
With experimentation, when we measure, the results are only approximations and won't agree exactly. It is important for students to report faithfully what they find e.g. for a triangle they could find that the sum of the angles to be 179^0 or 181 etc. The conclusion is that the angles appear to add up to 180^0. This is a plausible working assumption. There is a distinction between what you can discover and what you can prove.

In first year we were experimenting and using words. In second year we start to problem solve in concrete situations and then in third year we prove things.

Lesson Idea 2.27

Title
Revision of first year material – a complete recap on Lessons 1.12 and 1.13

Geometry: preliminary concepts

Resources
Online Resources Junior Certificate document
A mathematical instruments set
Dynamic software package
Syllabus: Geometry Course for Post-Primary School Mathematics
Junior Certificate Guidelines for Teachers (DES 2002, Government Publications Sales Office). It is also available to download at www.projectmaths.ie.

Content
These lessons will involve the students in investigating and understanding:

- Plane, points, lines, line segments, rays, collinear points, length of a line segment
- **Axiom 1:** There is exactly one line through any two given points.
- **Axiom 2:** [Ruler Axiom]: The properties of the distance between points (***Syllabus: Geometry Course for Post-Primary School Mathematics*** page 8)
- Angle as a rotation, angles in different orientations.
- How to estimate angles in degrees, naming angles
- Terms: Perpendicular, parallel, vertical, horizontal
- **Axiom 3:** Protractor Axiom (The properties of the degree measure of an angle).

Lesson Idea 2.28
One class period

Title
Revision of first year material – a complete recap on Lessons 1.14 and 1.15
Measuring angles, Theorem 1

Resources
Online Resources Junior Certificate document
A mathematical instruments set, angle estimator
Syllabus: Geometry Course for Post-Primary School Mathematics
NCCA Student Resources
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

- Measuring of angles using the protractor
- Possible misconceptions: Students thinking that size of an angle varies with arm or arc-length; failure to recognise equal angles in different orientations
  Common error: Reading from the incorrect scale on the protractor
- A straight angle has 180°
- Supplementary angles
- **Axiom 3 (Protractor axiom)**: see Syllabus: ***Syllabus: Geometry Course for Post-Primary School Mathematics***
- Vertically opposite angles
- Theorem 1: Vertically opposite angles are equal in measure.

Suggested class activities
For students with mild general learning disabilities see the activities on pages 128 – 134 of the NCCA *Guidelines for Teachers of Students with Mild General Learning Disabilities* at http://www.ncca.ie/uploadedfiles/PP_Maths.pdf

Lesson Idea 2.29

Title

Revision of first year material – a complete recap on Lessons 1.16 and 1.17

Resources

Online Resources Junior Certificate document

A mathematical instruments set

Dynamic software package
Syllabus: Geometry Course for Post-Primary School Mathematics

Content
These lessons will involve the students in investigating and understanding:

- The use of the compass
- Construction 8: Line segment of a given length on a given ray
- Construction 9: Angle of a given number of degrees with a given ray as one arm

Lesson Idea 2.30

Title
Revision of first year material – a complete recap on Lessons 1.18 and **Construction 7 (higher level only)**

Resources
Online Resources Junior Certificate document
A mathematical instruments set
Dynamic software package
Syllabus: Geometry Course for Post-Primary School Mathematics

Content
These lessons will involve the students in investigating and understanding:

- Construction 5: Line parallel to a given line, through a given point
- Axiom 5: Given any line l and a point P, there is exactly one line through P that is parallel to l.
- Construction 6: Division of a line segment into 2 or 3 equal segments without measuring it
- **Construction 7: Division of a line segment into any number of equal segments without measuring it (Higher Level only)**

Lesson Idea 2.31

Title
Revision of first year material – a complete recap on **Lesson Idea 1.26**

Resources
A mathematical instruments set
Dynamic software package
Syllabus: Geometry Course for Post-Primary School Mathematics
Junior Certificate Guidelines for Teachers (DES 2002, Government Publications Sales Office €3.81). It is also available to download at www.projectmaths.ie.

Content
These lessons will involve the students in investigating and understanding:

- Triangles: scalene, isosceles, equilateral, right-angled
- Construction 10: Triangle given SSS – Congruent triangles (Axiom 4)
- Construction 11: Triangle given SAS - Congruent triangles (Axiom 4)

Suggested class activities
Geometry Lesson Idea Ideas 1 and 14 in *Junior Certificate Guidelines for Teachers* **Check**
Lesson Idea 1 (pp. 58 - 59) aims to help students to recognise various types of triangles and provide them with concrete experience of dealing with triangles.
Lesson Idea 14 (pp. 72 – 73) introduces students to the idea of congruency with concrete materials

Lesson Idea 2.32

Title
Revision of first year material – a complete recap on Lesson Ideas 1.27 and 1.28

Resources
Online Resources Junior Certificate document
A mathematical instruments set
Dynamic software package
Syllabus: Geometry Course for Post-Primary School Mathematics

Content
These lessons will involve the students in investigating and understanding:

- Construction 12: Triangle given ASA - Congruent triangles (Axiom 4)
- More constructions of triangles with SSS, SAS and ASA
- By construction, show that AAA and AAS are not sufficient conditions for congruence.
- Theorem 2: (i) In an isosceles triangle the angles opposite the equal sides are equal.
  (ii) Conversely, if two angles are equal, then the triangle is isosceles

Lesson Idea 2.33

Title
Revision of first year material – a complete recap on Lesson Ideas 1.29 and 1.30

Resources
Online Resources Junior Certificate document
A mathematical instruments set
Syllabus: Geometry Course for Post-Primary School Mathematics
***Junior Certificate Guidelines for Teachers*(DES 2002, Government Publications Sales Office €3.81).** It is also available to download at www.projectmaths.ie.
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

- Alternate angles by examples and measuring
- Theorem 3: (i) If a transversal makes equal alternate angles on two lines, then the lines are parallel.

  (ii) Conversely, if two lines are parallel, then any transversal will make equal alternate angles with them.

- ***Theorem 4**: The angles in any triangle add to 180°.
  (**Proof required for Higher Level only**).

- Corresponding angles explained by examples and measuring
- Theorem 5: Two lines are parallel if and only if for any transversal, corresponding angles are equal.

- *Theorem 6: Each exterior angle of a triangle is equal to the sum of the interior opposite angles.
  (**Proof required for Higher Level only**).

Suggested class activities
See Geometry Lesson Idea 4 in "Junior Certificate Guidelines for Teachers", page 62. Available at www.projectmaths.ie

Lesson Idea 2.34

Title
Revision of first year material – a complete recap on Lesson Ideas 1.31 and 1.32

Resources
Online Resources Junior Certificate document
A mathematical instruments set
Syllabus: Geometry Course for Post-Primary School Mathematics
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

- Construction 1: Use of compass to bisect an angle
- Construction 2: Use of compass to draw the perpendicular bisector of a line segment
- **Construction 3: Line perpendicular to a given line *l*, passing through a given point not on *l***
- Construction 4: Line perpendicular to a given line *l*, passing through a given point on *l*

Section 10: Transformation geometry

Lesson Idea 2.35

Title
Introduction to translations

Resources
Online Resources Junior Certificate document
A mathematical instruments set
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

- Translations
- Recognise images of points and objects under translation

Suggested class activities
Students might engage in the following investigations:

Does a translation preserve length?
Does a translation preserve angle size?
Does a translation maps a line onto a parallel line?
Does a translation map a triangle onto a congruent triangle?

Lesson Idea 2.36

Title
Axial symmetry, axis of symmetry - revision of and building on Lesson Idea 1.19 from first year

Resources
Online Resources Junior Certificate document
A mathematical instruments set
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

- Axis of symmetry and axial symmetry
- Recognise images of points and objects under axial symmetry

Suggested class activities
Students might engage in the following investigations:

Does an axial symmetry preserve length?
Does an axial symmetry preserve angle size?
Does an axial symmetry maps a line onto a parallel line?
Does an axial symmetry map a triangle onto a congruent triangle?
How many axes of symmetry does an isosceles triangle have?
How many axes of symmetry does an equilateral triangle have?
How many axes of symmetry does a circle have?
(Draw examples of the above)

Lesson Idea 2.37

Title
Introduction to centre of symmetry, central symmetry, centre of rotation and rotations - revision of and building on Lesson Idea 1.20 from first year

Resources
Online Resources Junior Certificate document
A mathematical instruments set
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

- Centre of symmetry and central symmetry
- Recognise images of points and objects under central symmetry

- Rotation of points: centre of rotation, angle of rotation, direction of rotation
- Recognise images of points and objects under rotations

Suggested class activities
Students might engage in the following investigations:

Does a central symmetry preserve length?
Does a central symmetry preserve angle size?
Does a central symmetry map a line onto a parallel line?
Does a central symmetry map a triangle onto a congruent triangle?
Does an isosceles triangle have a centre of symmetry?
Does an equilateral triangle have a centre of symmetry?
Which types of triangle have a centre of symmetry?
Does a circle have a centre of symmetry?

Does rotation preserve length?

Does rotation preserve angle size?

Does rotation map a line onto a parallel line?

Does rotation map a triangle onto a congruent triangle?

Does an isosceles triangle have a centre of rotation?
Does an equilateral triangle have a centre of rotation?
Which types of triangle have a centre of rotation?
Does a circle have a centre of rotation?

Section 11: Synthetic Geometry 2

Lesson Idea 2.38

Title
Parallelograms and Theorem 9

Resources
Online Resources Junior Certificate document
A mathematical instruments set
Dynamic software package
Syllabus: Geometry Course for Post-Primary School Mathematics

Content
These lessons will involve the students in investigating and understanding:

- Properties of parallelograms
- ***Theorem 9**: In a parallelogram, opposite sides are equal, and opposite angles are equal. (**Proof required for Higher Level only**).
- Conversely, (1) if the opposite angles of a convex quadrilateral are equal, then it is a parallelogram; (2) if the opposite sides of a convex quadrilateral are equal, then it is a parallelogram.
- **Corollary 1**:**A diagonal divides a parallelogram into two congruent triangles.**

  (Higher Level Only)

- Remark 1 of ***Syllabus: Geometry Course for Post-Primary School Mathematics***: Sometimes it happens that the converse of a true statement is false. For example, it is true that if a quadrilateral is a rhombus, then its diagonals are perpendicular. But it is not true that a quadrilateral whose diagonals are perpendicular is always a rhombus.
- Remark 2 of ***Syllabus: Geometry Course for Post-Primary School Mathematics***: The converse of Corollary 1 is false: it may happen that a diagonal divides a convex quadrilateral into two congruent triangles, even though the quadrilateral is not a parallelogram.

Suggested class activities
Students might engage in the following investigations which lead to an informal proof of theorem 9:
Draw a parallelogram ABCD which is not a rectangle or a rhombus
Draw in one diagonal BD
Mark in all the alternate angles –you should have 2 pairs
Can you see why triangles ABD and BCD are congruent? (Give a reason)
What does this tell you about the opposite sides of ABCD?
What can you deduce about the opposite angles of ABCD?

Possible further investigations:

Does the diagonal bisect the angles at the vertex?
What is the sum of the four angles of parallelogram ABCD?
What would you get if you add two adjacent angles of the parallelogram? Why?

Lesson Idea 2.39

Title
More about parallelograms and Theorem 10

Resources
Online Resources Junior Certificate document
A mathematical instruments set
Dynamic software package
Syllabus: Geometry Course for Post-Primary School Mathematics

Content
These lessons will involve the students in investigating and understanding:

- Theorem 10: The diagonals of a parallelogram bisect each other.
- Conversely, if the diagonals of a quadrilateral bisect one another, then the quadrilateral is a parallelogram.

Suggested class activities
Students might engage in the following investigations which lead to an informal proof of the theorem:
Draw a parallelogram ABCD which is not a rectangle or a rhombus
Draw in the two diagonals AC and BD intersecting at E
Are the two diagonals equal in length? (Measure)
Mark in all the equal sides and angles in the triangles AED and BEC
Can you see why triangles ADE and BEC are congruent? (Give a reason)

Possible further investigations:

Are the triangles AEB and DEC congruent? Give a reason.
Are the diagonals perpendicular? Can you give a reason?
Are there 4 congruent triangles in the parallelogram?
Do the diagonals bisect the vertex angles of the parallelogram?
How many axes of symmetry does the parallelogram have?
Does the parallelogram have a centre of symmetry? If so, where is it located?

Lesson Idea 2.40

Title
Investigations of quadrilaterals: square, rhombus, parallelogram, rectangle.

Resources
Online Resources Junior Certificate document
A mathematical instruments set
Dynamic software package
Syllabus: Geometry Course for Post-Primary School Mathematics

Content
These lessons will involve the students in investigating and understanding:

- The properties of different quadrilaterals

Suggested class activities
Students might engage in the following investigations about a square, rhombus, parallelogram and rectangle:

Describe each of these in words.
Draw 3 examples of each in different orientations.
Which sides are equal?
What is the sum of all the angles?
Are all angles equal? Which angles are equal?
What is the sum of 2 adjacent angles?
Does a diagonal bisect the angles it passes through?
Are the diagonals perpendicular?
Does a diagonal divide it into two congruent triangles?
Given the length of its sides, can you calculate the length of a diagonal?
Are the two diagonals equal in length?
Do the diagonals divide it into 4 congruent triangles?
Do the diagonals bisect each other?

How many axes of symmetry does a square have?
How many axes of symmetry does a rectangle (not a square) have?
How many axes of symmetry does a parallelogram (not a rectangle) have?
How many axes of symmetry does a rhombus have?
(Draw them in on your diagrams, cut out and fold).
In the case of each shape, does it have a centre of symmetry? If so, indicate them on your diagrams.

Option: do the above investigations on a KITE.

Section 12: Ratio and proportion

Lesson Idea 2.41

Title
Ratio and proportion

Resources
Online Resources Junior Certificate document

Content
These lessons will involve the students in investigating and understanding:

- Distinguish between absolute comparison and relative comparison
- See ratios as comparing part to part and fractions as comparing part to whole, where the quantities being compared have the same units.
- See rates as the ratio of two quantities having different units
- Appreciate the importance of order when dealing with ratios
- Find equivalent ratios
- Divide a number into a given ratio
- Recognise a proportion as a statement of equivalent ratios 5:2 = 10:4 or set up a proportion to find x as in 5:2 = 8: x
- Distinguish between proportional and non proportional situations recognising the multiplicative relationship that exists between the quantities in proportional situations as seen in tables, graphs and algebraic expressions
- Use a variety of techniques including the unitary method, factor of scale and tables, to solve proportional tasks and to recognise that these techniques are all related
- Solve problems involving proportional reasoning in different contexts
- How to draw and interpret scaled diagrams

Section 13: Trigonometry

Lesson Idea 2.42

Title
Revision of Pythagoras' Theorem (see Lesson Idea 2.10)

Resources
Online Resources Junior Certificate document
Mathematical set, graph paper,
Dynamic software package
NCCA Mathematics Resources for Students - Junior Certificate Strand 2

Content

These lessons will involve the students in investigating and understanding:

- Pythagoras' Theorem using models and using algebra to generalise
- That there are an infinite number of Pythagorean triples
- How to apply the result of the theorem of Pythagoras to solve right - angled triangle problems of a simple nature involving heights and distances

Lesson Idea 2.43

Title

The application of scaled diagrams to trigonometry

Resources
Online Resources Junior Certificate document
Graph paper, mathematical set
Dynamic software package
Calculator
NCCA Mathematics Resources for Students – Junior Certificate Strand 2

Content
These lessons will involve the students in investigating and understanding:

- The use of diagrams drawn to scale to measure angles of elevation and depression and unknown lengths

Lesson Idea 2.44

Title
Trigonometric ratios in right-angled triangles (sine, cosine, tangent)

Resources
Online Resources Junior Certificate document
Graph paper,
A mathematical instruments set
Dynamic software package
Calculator
NCCA Mathematics Resources for Students – Junior Certificate Strand 2

Content
These lessons will involve the students in investigating and understanding:

- The labelling of the sides of a right-angled triangle (opposite, adjacent and hypotenuse)
- The meaning of complementary angles
- Understanding sine, cosine and tangent as the ratios between the sides of the triangle
- The connection between slope as a ratio of rise to run and the tan of an angle in a right angled triangle as the ratio of the length of the opposite side to the length of the adjacent side
- The use of these ratios in a practical way to solve unknowns in right-angled triangles
- The use of trigonometric ratios to solve problems involving angles (integer values) between 0° and 90°
- The use of a clinometer in conjunction with trigonometry to solve practical problems

Appendix A

Geometry: Thinking at Different Levels

The Van Hiele Theory

The **Van Hiele model** describes how students learn geometry. Pierre van Hiele and Dina van Hiele-Geldof, mathematics teachers from the Netherlands, observed their geometry students in the 1950's.The following is a brief summary of the Van Hiele theory. According to this theory, students progress through 5 levels of thinking starting from merely recognising a shape to being able to write a formal proof. The levels are as follows:

***Visualisation (Level 0)**

The objects of thought are shapes and what they look like.

Students have an overall impression of a shape. The appearance of a shape is what is important. They may think that a rotated square is a "diamond" and not a square because it is different from their visual image of a square. They will be able to distinguish shapes like triangles, squares, rectangles etc but will not be able to explain, for example, what makes a rectangle a rectangle. **Vocabulary**: Students use visual words like "pointy", "curvy", "corner" as well as correct language like angle, rectangle and parallelogram.

***Analysis (Level 1)**

The objects of thought are "classes" of shapes rather than individual shapes.

- Students think about what makes a rectangle a rectangle and can separate the defining characteristics of a rectangle from irrelevant information like size and orientation. They recognize its parts (sides, diagonals and angles) and compare their properties (similar, congruent)
- They understand that if a shape belongs to a class like "rectangle", then it has all the properties of that class (2 pairs of equal sides, right angles, 2 equal diagonals, 2 axes of symmetry).
- **Vocabulary:** words like parallel, perpendicular and congruent relating to properties within a figure and the words all, always, sometimes, never, alike, different.
- A concise definition of a figure, using a sufficient rather than an exhaustive list of properties is not possible at this level.
- They do not deal with questions like "Is a square a parallelogram?" but just look at the properties of each class of shape, without comparing the classes.

*Some visualisation and analysis is covered in Primary School.

Relational/ Ordering/Informal Deduction (Level 2)

The objects of thought are the properties of shapes.

- Students are ready to understand interrelationships of properties within figures and between figures. Opposite sides of a parallelogram are parallel and opposite angles are equal.
- A rectangle is a parallelogram since it has all the properties of a parallelogram as well as all 90^0 angles.
- Students can recognise the difference between a statement and its converse. All squares are rectangles (true) is different from all rectangles are squares (not true).
- Capable of **"if –then"** thinking – if a shape is a rectangle then all the angles in it are right angles. If |<A |= |<B |and |<B| = |<C| then |< A| =|<C|
- They can select one or two properties to define a figure rather than an exhaustive list. If a quadrilateral has 4 equal sides and one right angle it must be a square.
- Students can discover new properties by simple deduction. The 2 acute angles in a right angled triangle add to 90^0 because all the angles in a triangle add up to 180^0. They can explain logically without having to measure everything.

Formal deduction (Level 3)

Students learn how to use an axiomatic system to establish geometric theory. This is the level at which proof of Theorems is learned. The sequence of theorems given in the appendix is arranged in such a manner that each theorem builds on the previous theorem(s).

Rigor (Level 4)

Comparing different axiomatic systems – not done at secondary level

Characteristics of these levels: Students cannot function at any particular level unless they are competent at all previous levels. The teacher's role is crucial in structuring activities to bring students from one level to the next.

How does the teacher bring students from any one level to the next?

5 phases of learning:

1. In an informal discussion of the topic, students are asked to give their initial observations.
2. The teacher provides structured activities such as drawing, making and measuring.
3. The students then verbalise and write down what they have learned and report back in groups to the class, which leads to a class discussion.
4. The teacher then provides an activity which will require students to apply what they have discovered
5. In the last stage students are required to summarise all they have learned and should be able to remember it as they have discovered it through guidance.

A PowerPoint presentation of the Van Hiele theory can be got at www.projectmaths.ie(Workshop 2 Show 2) http://www.projectmaths.ie/workshops/WS2_NR/WS2_NR_PPTS.zip

A mind map of Van Hiele can be found at http://agutie.homestead.com/files/mindmap/van_hiele_geometry_level.html

Appendix B

Guide to Theorems, Axioms and Constructions at all Levels*

This is intended as a quick guide to the various axioms, theorems and constructions as set out in the ***Syllabus: Geometry Course for Post-Primary School Mathematics.*** You can get this from the project maths website: www.projectmaths.ie

It is not intended as a replacement for this document, merely as an aid to reading at a glance which material is required to be studied at various levels. The sequence of theorems as given must be followed.

As stated in the heading, these theorems and constructions are underpinned by 46 definitions and 20 propositions which are all set out in the ***Syllabus: Geometry Course for Post-Primary School Mathematics***, along with many undefined terms and definable terms used without explicit definition.

*An **axiom** is a statement accepted without proof, as a basis for argument

*A **theorem** is a statement deduced from the axioms by logical argument. Theorems can also be deduced from previously established theorems.

* A **proposition** is a useful or interesting statement that could be proved at this point, but whose proof is not stipulated as an essential part of the programme. Teachers are free to deal with them as they see fit, but they should be mentioned, at least (Appendix p. 20, footnote).

*The instruments that may be used for **constructions** are listed and described on page 38 of the Appendix and are a straight edge, compass, ruler, protractor and set-square.

Terms Students at **Junior Certificate Higher level and Leaving Certificate Ordinary level** will be expected to understand the meanings of the following terms related to logic and deductive reasoning:
Theorem, proof, axiom, corollary, converse, implies.

Synthetic Geometry

Guide to Axioms, Theorems and Constructions for all Levels

Information Technology is used whenever and wherever appropriate to help to present mathematical concepts effectively to students.

In this document the  symbol appears at the corresponding position of the content to indicate that an interactive IT module is available on the Project Maths Student's CD.

	Axioms and Theorems (supported by 46 definitions, 20 propositions) *proof required for JCHL and LCHL ** proof required for LCHL only	CMN Introd. Course	JC ORD	JC HR	LC FDN	LC ORD	LC HR
	Axiom 1: There is exactly one line through any two given points	√	√	√	√	√	√
	Axiom 2: [Ruler Axiom]: The properties of the distance between points.	√	√	√	√	√	√
	Axiom 3: Protractor Axiom (The properties of the degree measure of an angle).	√	√	√	√	√	√
1	Vertically opposite angles are equal in measure.	√	√	√	√	√	√
	Axiom 4: Congruent triangles conditions (SSS, SAS, ASA)	√	√	√	√	√	√
2	In an isosceles triangle the angles opposite the equal sides are equal. Conversely, if two angles are equal, then the triangle is isosceles.	√	√	√	√	√	√
	Axiom 5: Given any line l and a point P, there is exactly one line through P that is parallel to l.	√	√	√	√	√	√
3	If a transversal makes equal alternate angles on two lines then the lines are parallel. Conversely, if two lines are parallel, then any transversal will make equal alternate angles with them.	√	√	√	√	√	√
4*	The angles in any triangle add to 180°.	√	√	√	√	√	√
5	Two lines are parallel if, and only if, for any transversal, the corresponding angles are equal.	√	√	√	√	√	√
6*	Each exterior angle of a triangle is equal to the sum of the interior opposite angles.	√	√	√	√	√	√
7	The angle opposite the greater of two sides is greater than the angles opposite the lesser. Conversely, the side opposite the greater of two angles is greater than the side opposite the lesser angle.					√	√
8	Two sides of a triangle are together greater than the third.					√	√
9*	In a parallelogram, opposite sides are equal, and opposite angles are equal. Conversely, (1) if the opposite angles of a convex quadrilateral are equal, then it is a parallelogram; (2) if the opposite sides of a convex quadrilateral are equal, then it is a parallelogram.		√	√	√	√	√
	Corollary 1. A diagonal divides a parallelogram into two congruent triangles.			√			√
10	The diagonals of a parallelogram bisect each other. Conversely, if the diagonals of a quadrilateral bisect one another, then the quadrilateral is a parallelogram.		√	√	√	√	√

	Axioms and Theorems (supported by 46 definitions, 20 propositions) *proof required for JCHL and LCHL ** proof required for LCHL only	CMN Introd. Course	JC ORD	JC HR	LC FDN	LC ORD	LC HR
11**	 If three parallel lines cut off equal segments on some transversal line, then they will cut off equal segments on any other transversal.			√		√	√
12**	 Let ABC be a triangle. If a line l is parallel to BC and cuts [AB] in the ratio m:n, then it also cuts [AC] in the same ratio. Conversely, if the sides of two triangles are in proportion, then the two triangles are similar.			√		√	√
13**	 If two triangles are similar, then their sides are proportional, in order (and converse)		√	√	√	√	√
14*	 [Theorem of Pythagoras]In a right-angled triangle the square of the hypotenuse is the sum of the squares of the other two sides.		√	√	√	√	√
15	 [Converse to Pythagoras]. If the square of one side of a triangle is the sum of the squares of the other two, then the angle opposite the first side is a right angle.		√	√	√	√	√
	Proposition 9: (RHS). If two right-angled triangles have hypotenuse and another side equal in length respectively, then they are congruent.		√	√	√	√	√
16	 For a triangle, base x height does not depend on the choice of base.					√	√
	Definition 38: The area of a triangle is half the base by the height.					√	√
17	 A diagonal of a parallelogram bisects the area.					√	√
18	 The area of a parallelogram is the base x height.					√	√
19*	 The angle at the centre of a circle standing on a given arc is twice the angle at any point of the circle standing on the same arc.			√			√
	Corollary 2†: All angles at points of a circle, standing on the same arc are equal (and converse).			√			√
	Corollary 3: Each angle in a semi-circle is a right angle.		√	√	√	√	√
	Corollary 4: If the angle standing on a chord [BC] at some point of the circle is a right-angle, then [BC] is a diameter.		√	√	√	√	√
	Corollary 5: If ABCD is a cyclic quadrilateral, then opposite angles sum to 180°.			√			√
20	 (i) Each tangent is perpendicular to the radius that goes to the point of contact. (ii) If P lies on the circle S, and a line l is perpendicular to the radius to P, then l is a tangent to S.					√	√
	Corollary 6: If two circles intersect at one point only, then the two centres and the point of contact are collinear.					√	√
21	 (i) The perpendicular from the centre to a chord bisects the chord. (ii) The perpendicular bisector of a chord passes through the centre.					√	√

† The corollaries are numbered as in the *Geometry for Post-primary School Mathematics*; corollary 2 is the first one relating to theorem 19

	Constructions (Supported by 46 definitions, 20 propositions, 5 axioms and 21 theorems)	CMN Introd. Course	JC ORD	JC HR	LC FN	LC ORD	LC HR
1	Bisector of an angle, using only compass and straight edge.	√	√	√	√	√	√
2	Perpendicular bisector of a segment, using only compass and straight edge.	√	√	√	√	√	√
3	Line perpendicular to a given line l, passing through a given point not on l.			√			√
4	Line perpendicular to a given line l, passing through a given point on l.	√	√	√	√	√	√
5	Line parallel to given line, through a given point.	√	√	√	√	√	√
6	Division of a line segment into 2 or 3 equal segments without measuring it.	√	√	√	√	√	√
7	Division of a line segment into any number of equal segments, without measuring it.			√			√
8	Line segment of a given length on a given ray.	√	√	√	√	√	√
9	Angle of a given number of degrees with a given ray as one arm.		√	√	√	√	√
10	Triangle, given lengths of 3 sides.		√	√	√	√	√
11	Triangle, given SAS data.		√	√	√	√	√
12	Triangle, given ASA data		√	√	√	√	√
13	Right-angled triangle, given length of hypotenuse and one other side		√	√	√	√	√
14	Right-angled triangle, given one side and one of the acute angles.		√	√	√	√	√
15	Rectangle given side lengths.		√	√	√	√	√
16	Circumcentre and circumcircle of a given triangle, using only straight edge and compass.					√	√
17	Incentre and incircle of a triangle of a given triangle, using only straight edge and compass.					√	√
18	Angle of 60° without using a protractor or set square.				√	√	√
19	Tangent to a given circle at a given point on it.				√	√	√
20	Parallelogram, given the length of the sides and the measure of the angles.				√	√	√
21	Centroid of a triangle.					√	√
22	Orthocentre of a triangle.						√

Appendix C

How to register for *CensusAtSchool*, complete online questionnaires and retrieve class data

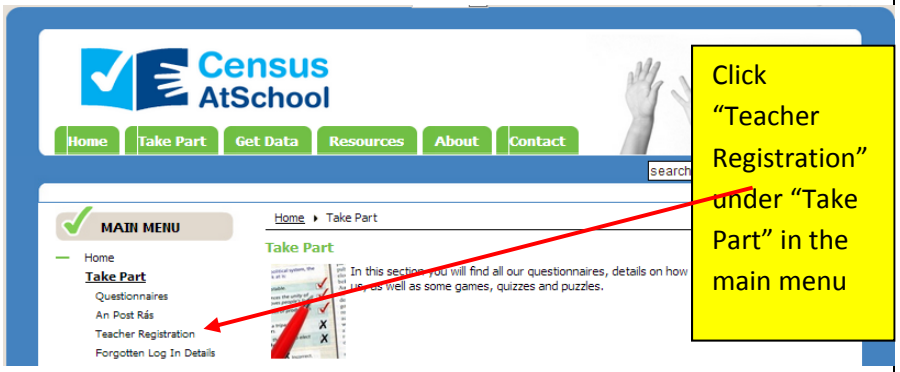
Part 1: Registration

www.CensusAtSchool.ie

This is the Irish *CensusAtSchool* homepage.

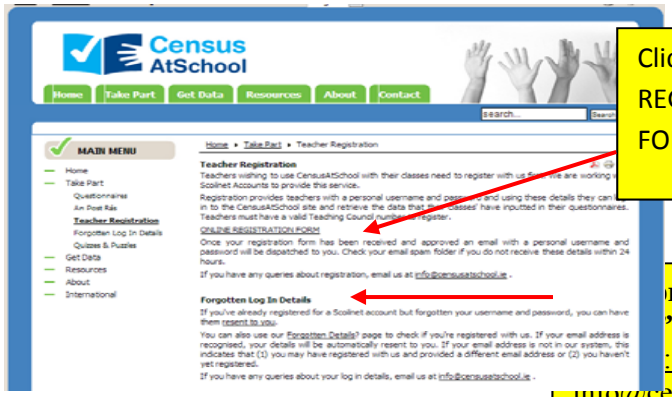
Click "Take Part" under the main menu.

1



Click "Teacher Registration" under "Take Part" in the main menu

2

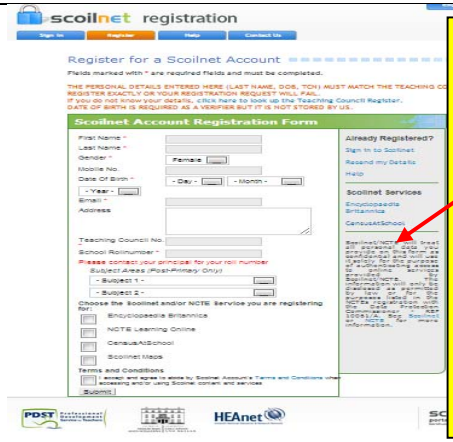


Click "ONLINE REGISTRATION FORM"

rgotten Log In

nfo@censusatschool.ie

3

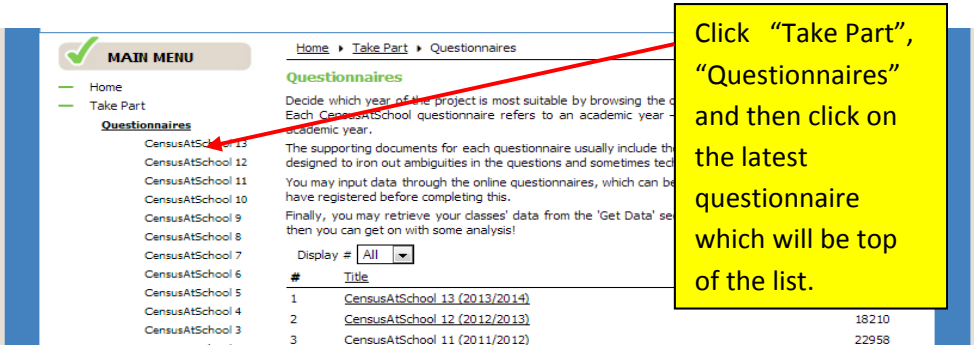


Fill out and click on "Submit"

Once your registration form has been received and approved an email with a personal username and password will be dispatched to you. Check your email spam folder if you do not receive these details within 48 hours.

4

Part 2: Accessing the questionnaires -use the highest phase available



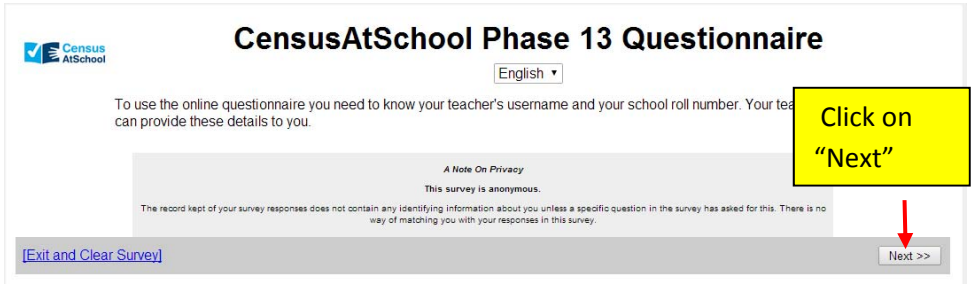
Click "Take Part", "Questionnaires" and then click on the latest questionnaire which will be top of the list.

5

Click "Take the Phase* questionnaire"

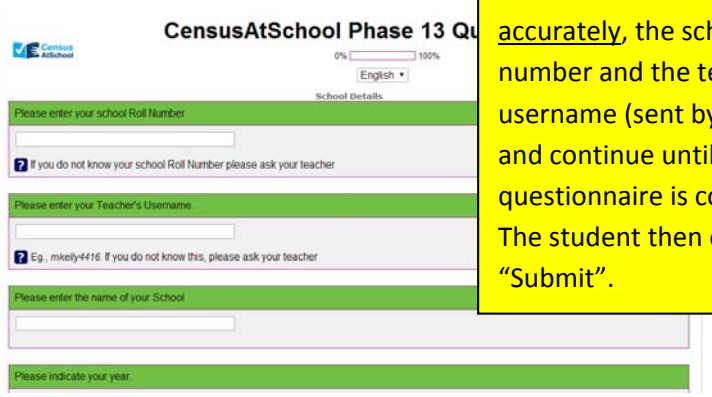
6

Part 3: Filling out the questionnaire



Click on "Next"

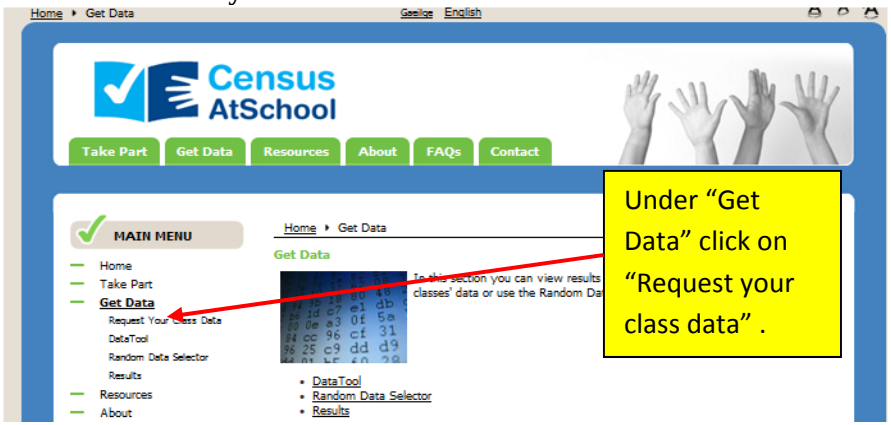
7



Each student must fill in accurately, the school roll number and the teacher's username (sent by email) and continue until the questionnaire is completed. The student then clicks "Submit".

8

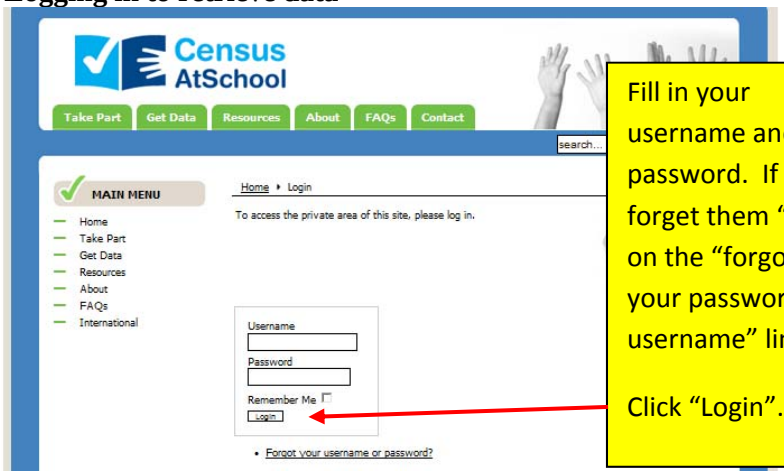
Part 4 Now that my students have filled in the data – What next?



Under "Get Data" click on "Request your class data" .

9

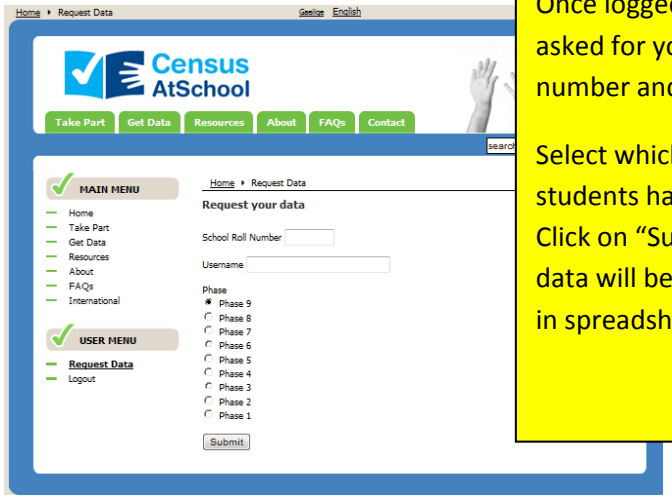
Logging in to retrieve data



Fill in your username and password. If you forget them "click on the "forgot your password or username" link.

Click "Login".

10



Once logged in you will be asked for your school roll number and username.

Select whichever phase your students have completed. Click on "Submit" and your data will be returned to you in spreadsheet form.

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Your data will look similar to the spreadsheet below.

Sex	BrthMnth	BrthYear	BrthPlce	BedRms	PeoplHme	InetAxs	NumPts	PtsNone	PtsDog	PtsCat	PtsMouse	PtsRabt
Female	July	1992	New Sout	4	5	Yes - broa	4	No	Yes	Yes	No	No
Female	February	1996	Australian	4	5	Yes - broa	4	No	Yes	Yes	No	No
Male	Septembe	1994	Victoria	4	5	Yes - dial-	5	No	Yes	No	No	No
Female	Septembe	1991	Fiji	4	4	No - Interi	0	Yes	No	No	No	No
Male	August	1993	Queensla	3	4	No - Interi	0	No	No	No	No	No
Female	Decembe	1993	Tasmania	3	5	Yes - dial-	3	No	No	Yes	No	No
Female	August	1990	Queensla	4	5	Yes - broa	3	No	Yes	No	No	No
Female	Novembe	1991	New Sout	11	5	Yes - dial-	4	No	Yes	No	No	Yes
Male	June	1995	Victoria	3	5	Yes - othe	6	No	Yes	No	No	No
Female	March	1993	Western A	30	122	No - Interi	2	No	Yes	No	No	No
Female	Decembe	1989	Queensla	3	3	Yes - broa	2	No	No	No	No	No
Female	April	1993	South Aus	3	5	No - Interi	4	No	Yes	Yes	No	No
Male	March	1996	New Sout	3	2	No - Interi	0	Yes	No	No	No	No
Male	Septembe	1993	Queensla	4	4	Yes - dial-	18	No	Yes	No	No	No
Male	May	1995	Tasmania	3	4	Yes - broa	2	No	No	No	No	Yes
Female	March	1994	Western A	4	4	Yes - dial-	4	No	Yes	Yes	No	No

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