TEACHER HANDBOOK

Third Year

DRAFT

Based on the 2016 Syllabus

Contents Page

The strand structure of the syllabus should not be taken to imply that topics are to be studied in isolation. Where appropriate, connections should be made within and across the strands and with other areas of learning. (NCCA JC syllabus page 10 and LC syllabus page 8)
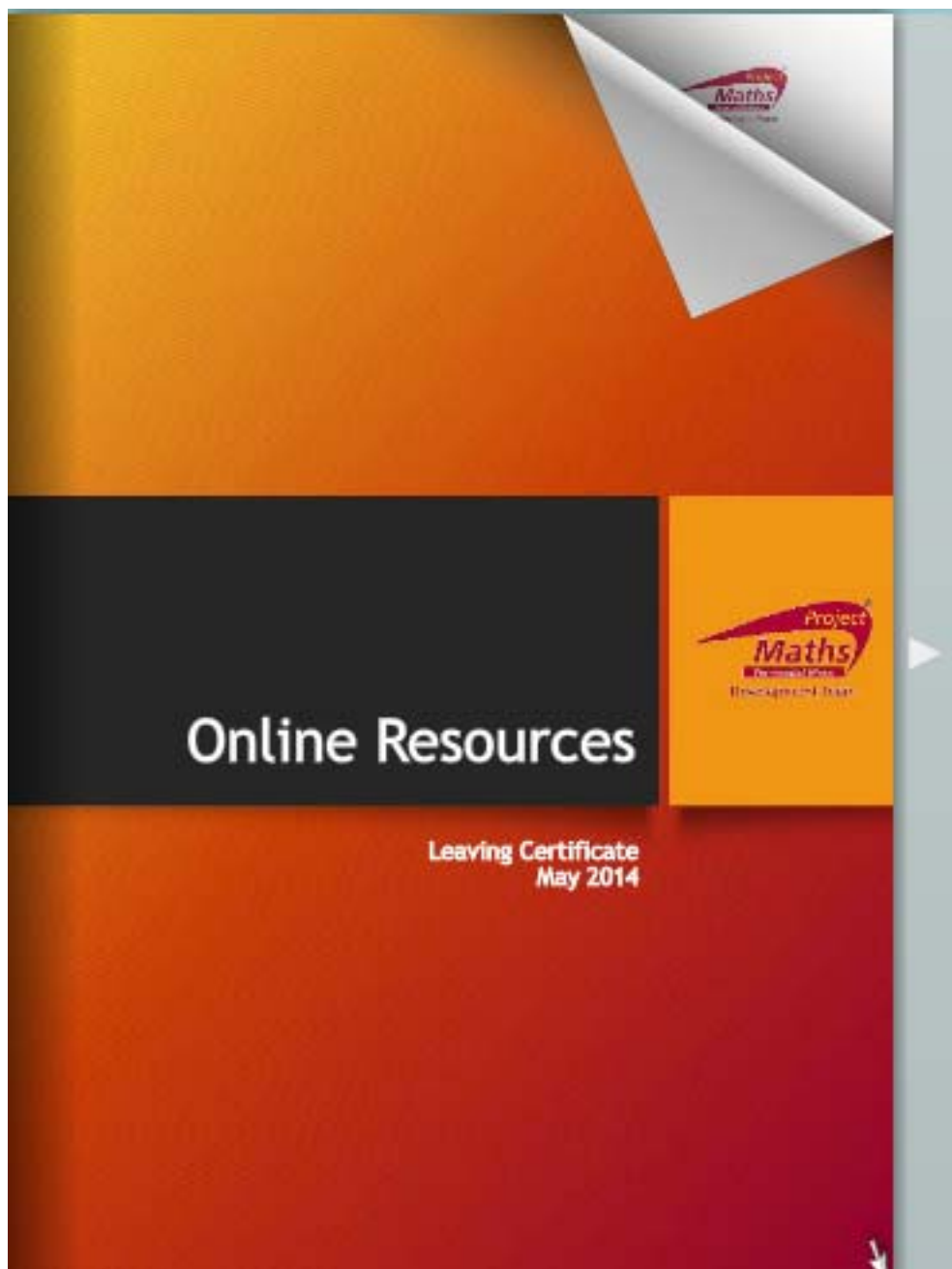
This is a comprehensive reference for resources which will allow teachers plan lessons, easily access specific learning outcomes in the syllabus and relevant support material such as "Teaching & Learning Plans" and suggested activities to support learning and teaching. Click here.

Introduction: Student Learning

While this is a handbook for teachers, it must be emphasised that <u>student learning</u> and the process of <u>mathematical thinking</u> and <u>building understanding</u> are the main focus of this document.

Information and Communications Technologies are used whenever and wherever appropriate to help to support student learning. It is also envisaged that, at all levels, learners will engage with a dynamic geometry software package.

Students with mild general learning disabilities

Teachers are reminded that the NCCA Guidelines on mathematics for students with mild general learning disabilities can be accessed at

http://www.ncca.ie/uploadedfiles/PP_Maths.pdf

This document includes

- ❖ Exemplars (from page 7)
- ❖ Approaches and Methodologies (from Page 10)

Note on the Common Introductory Course

This first year handbook contains a suggested sequence for teaching First Year students. It includes the material in the Common Introductory Course which is a minimum course for first year students and also some other sections of the Junior Certificate syllabus which teachers may wish to deal with in first year. The lesson ideas which include the **Common Introductory Course** are marked "**CIC**".

Higher level material

Material which is higher level only is shown in bold typeface.

Note: Synthesis and problem solving listed below must be incorporated into all of the Strands.
The list of skills below is taken from Strand 1of the syllabus but, an identical list is given at the end of each Strand in the syllabus.

Students learn about	Students should be able to
1.8 Synthesis and problem-solving skills	– explore patterns and formulate conjectures – explain findings – justify conclusions – communicate mathematics verbally and in written form – apply their knowledge and skills to solve problems in familiar and unfamiliar contexts – analyse information presented verbally and translate it into mathematical form – devise, select and use appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions.

Useful websites

	http://www.projectmaths.ie/ http://ncca.ie/en/Curriculum_and_Assessment/Post-Primary_Education/Project_Maths/ http://www.examinations.ie/

Literacy and Numeracy Strategy

The National Strategy to Improve Literacy and Numeracy among Children and Young People 2011-2020

Numeracy encompasses the ability to use mathematical understanding and skills to solve problems and meet the demands of day-to-day living in complex social settings. To have this ability, a young person needs to be able to think and communicate quantitatively, to make sense of data, to have a spatial awareness, to understand patterns and sequences, and to recognise situations where mathematical reasoning can be applied to solve problems.

Literacy includes the capacity to read, understand and critically appreciate various forms of communication including spoken language, printed text, broadcast media, and digital media.

Colour coding used in the suggested sequence below:

Strand 1 Statistics and Probability	Strand 2 Geometry and Trigonometry	Strand 3 Number	Strand 4 Algebra	Strand 5 Functions

Suggested sequence of topics

Section number	Strand (Syllabus section)	Corresponding Lesson Number	Title of lesson idea	Page number	
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Section number	Strand (Syllabus section)	Corresponding Lesson Number	Title of lesson idea	Page number	
			coloured objects		
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	2.1 & 2.4	3.16	Revision of second year material – a complete recap on Lessons 2.34 -2.39	22	
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	4.6	3.22	Evaluation of and operations on algebraic expressions- revision and extension of second year material	27	
	4.5	3.23	Algebraic factors with an extension to HL material	29	
	4.7	3.24	Adding algebraic fractions - revision and extension of second year material	29	
	4.7	3.25	Linear equations and linear inequalities - revision and extension of second year material	30	

Section number	Strand (Syllabus section)	Corresponding Lesson Number	Title of lesson idea	Page number	
	4.7	3.26	Solve quadratic equations - revision and extension of second year material	30	
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Section 5	5.1, 5.2	3.28	Functions - interpreting and representing linear, quadratic and exponential functions in graphical form	32	
Section 6	4.5	3.29	Relations without formulae	33	
Section 7	2.1	3.30	Theorems 14 and 15 and Proposition 9	34	
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Section 8	2.2	3.33	Co-ordinate geometry - revision and extension of second year material	36	
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Lesson Ideas

Section 1: Number

Lesson Idea 3.1

Title
Applied arithmetic to include HL material

Resources
Online Resources Junior Certificate document

Content

These lessons will involve the students in investigating and understanding:

- Number systems $\mathbb{N}, \mathbb{Z}, \mathbb{Q}$ (revision)

- Reciprocals

- Factors, multiples and primes (revision)

- Further contexts with fractions, decimals, percentages, ratio and proportion involving:

    o Solving problems e.g. mobile phone tariffs, currency transactions, shopping, VAT and meter readings

    o Profit or loss, % profit or loss (on the cost price), discount and % discount, calculation of selling price, **mark up (profit as a % of cost price, margin (profit as a % of selling price)**

    o **Problems involving calculating cost price**

    o Compound interest for not more than three years (OL), **compound interest for more than three years**

    o Income tax (standard rate only for OL)

    o Net pay (including other deductions of specified amounts)

    o **Income tax and net pay including other deductions**

    o Making value- for- money calculations and judgments

Lesson Idea 3.2 Higher level only

Title
Real numbers

Resources

Online Resources Junior Certificate document

Content

These lessons will involve the students in investigating and understanding:

- **Number systems $N, Z, Q, R \backslash Q, R$**

- **Link between set R and continuous numeric data in statistics**

- **Operate on the set of irrational numbers $R \backslash Q$**

Lesson Idea 3.3

Title
Indices

Resources
Online Resources Junior Certificate document

Content
These lessons will involve the students in investigating and understanding:

- How to use and apply the rules for indices (where $a \in \mathbb{Z}$, $p, q \in \mathbb{N}$) continued from second year
  $$a^p a^q = a^{p+q}$$

  $$\frac{a^p}{a^q} = a^{p-q}$$

  $$\left(a^p\right)^q = a^{pq}$$

  Use the notation $a^{\frac{1}{2}}, a \in \mathbb{N}$

- **How to use and apply the rules for indices (where $a, b \in \mathbb{R}$, $p, q \in \mathbb{Q}$) at higher level**
  $$a^p a^q = a^{p+q}$$

  $$\frac{a^p}{a^q} = a^{p-q}$$

  $$\left(a^p\right)^q = a^{pq}$$

$$a^0 = 1$$

$$a^{\frac{1}{q}} = \sqrt[q]{a}, \quad q \in \mathbf{Z}, q \neq 0, a > 0$$

$$a^{\frac{p}{q}} = \sqrt[q]{a^p}, \quad p, q \in \mathbf{Z}, q \neq 0, a > 0$$

$$a^{-p} = \frac{1}{a^p}$$

$$(ab)^p = a^p b^p$$

$$\left(\frac{a}{b}\right)^p = \frac{a^p}{b^p}$$

Lesson Idea 3.4

Title
Numbers in standard form /scientific notation

Resources
Online Resources Junior Certificate document

Content
These lessons will involve the students in investigating and understanding:

- How to express rational numbers ≥ 1, in the approximate form $a \times 10^n$, correct to a specified number of decimal places and where $n = 0$ or $n \in N$
- **Express non-zero positive rational numbers in the approximate form $a \times 10^n$ where $n \in \mathbf{Z}$, and $1 \leq a < 10$**

Lesson Idea 3.5

Title
Applied measure

Resources
Online Resources Junior Certificate document
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

- Application and interpretation of units of measure and time

- Drawing and interpreting scaled diagrams

- Problems involving perimeter and area of the following plane figures: disc, triangle, rectangle, square and figures made from combinations of these

- The relationship between the circumference of a circle, its diameter and π

- The nets of rectangular solids

- **the nets of prisms (polygonal bases), cylinders and cones**

- The surface area of rectangular solids

- **Solution of problems involving surface area of triangular based prisms (right angle, isosceles, equilateral), cylinders and cones**

- **Solution of problems involving curved surface area of cylinders, cones and spheres**

- The volume of rectangular solids and cylinders

- **How to solve problems involving the volume of rectangular solids, cylinders, cones, triangular base prisms(right angle, isosceles, equilateral), spheres and combinations of these**

- How to model real-world situations and solve a variety of problems (including multi –step problems) involving surface areas and volumes of cylinders **and prisms.**

Lesson Idea 3.6

Title
Sets

Resources

Online Resources Junior Certificate document

Content
These lessons will involve the students in investigating and understanding:
- The concept of a set as being a collection of well defined objects, equality of sets

- Venn diagrams, universal set, null set, sub-set, cardinal number, set builder notation

- Union and intersection as operators on two sets

- Set difference

- Complement of a set

- Investigate the commutative property for intersection , union and difference

- Solve problems involving sets

- **Explore the operations of intersection, union (for three sets), set difference and complement**

- **Investigate the associative property for intersection, union and difference**

- **Investigate the distributive property of union over intersection and intersection over union**

Section 2: Statistics and Probability

Lesson Idea 3.7 (Statistics)

Title
Statistics - Revising and building on second year material using a statistical investigation

Resources
Online Resources Junior Certificate document

Content
These lessons will involve the students in investigating and understanding:

The following learning outcomes from second year:

- The use of statistics to gather information from a sample of the population with the intention of making generalisations about the whole population
- Misconceptions and misuses of statistics
- How to formulate one (or more) questions that can be answered with data
- How to plan an investigation involving statistics and conduct the investigation using the data handling cycle:
    - Pose one (or more) questions that can be answered with data
    - Collect data
    - Analyse the data
    - Interpret the results
    - Refine the question if necessary

- The different ways of collecting data, e.g. *CensusAtSchool* questionnaire
- **How to select a sample (simple random sample)**
- **The importance of representativeness so as to avoid biased samples**
- The different types of data:
    - category (nominal and ordinal)
    - numeric (discrete or continuous)
- How to summarise data in diagrammatic form including spreadsheets
- How to use bar charts, line plots, stem and leaf plots to display data
- How to select appropriate graphical methods to describe the sample
- Measures of central tendency: mean, mode, median
- Range as a measure of spread
- **How to evaluate reliability of data and data sources**

Lesson Idea 3.8 (Statistics)

Title
Data handling cycle for categorical data revising and building on second year material

Resources

Online Resources Junior Certificate document

Content
These lessons will involve the students in investigating and understanding:

- How to pose a question yielding categorical data (possibly from census at school questionnaire)
- How to collect categorical data (possibly from census at school questionnaire)
- How to use bar charts as a graphical representation of the categorical data using frequency on the y –axis

- How to use bar charts as a graphical representation of the categorical data using relative frequency(percentages) on the y -axis
- How to use a line plot as a graphical representation of categorical data
- How to represent the data in a particular category as a proportion of all the data
- How to use pie charts to compare "part to whole" using the concept of proportions
- How to select appropriate graphical displays to describe categorical data and evaluate the effectiveness of these different graphical displays
- How to use the mode as a measure of central tendency
- How to draw conclusions from graphical and numerical summaries, **recognising assumptions and limitations**
- **How sampling variability influences the use of sample information to make statements about the population**

Lesson Idea 3.9 (Statistics)

Title
Data handling cycle for continuous numeric data revising and building on second year material

Resources
Online Resources Junior Certificate document

Content
These lessons will involve the students in investigating and understanding:

- How to pose a question yielding numeric discrete or continuous data (possibly from census at school questionnaire)
- How to collect numeric discrete or continuous data (possibly from census at school questionnaire)
- How to use a line plot, if feasible, as a graphical representation of the ungrouped continuous numeric data

- How to use a stem and leaf plot as a graphical representation of the ungrouped continuous numeric data
- The effect of splitting stems in a stem and leaf plot if data is suitable
- Range as a measure of spread
- **Recognise the existence of outliers**
- **The use of stem and leaf plots to calculate quartiles and inter-quartile range**
- **How to compare range and inter-quartile range as measures of spread**
- **How to use back to back stem and leaf plots to display data**
- How to group data into equal class intervals in a grouped frequency table
- Histograms with equal class intervals for numeric continuous data and relationship with stem and leaf plots
- Histograms with relative frequency (percentages) on the y - axis
- The effect of interval width on the information conveyed by the histogram
- How to select appropriate graphical displays to describe numeric data and evaluate the effectiveness of these different graphical displays
- The use of graphical representations to describe what is typical in the data using concepts of shape and measures of central tendency: mean, mode, median
- **The mean of a grouped frequency distribution (HL only)**
- How to relate the interpretation to the original question posed
- How to draw conclusions from graphical and numerical summaries, **recognising assumptions and limitations**
- **How sampling variability influences the use of sample information to make statements about the population**

Lesson Idea 3.10 (Statistics)

Title
Evaluate displays of statistical investigations conducted by others and use appropriate graphical displays to compare data sets

Resources
Online Resources Junior Certificate document

Content
These lessons will involve the students in investigating and understanding:

- How to evaluate the effectiveness of different displays in representing the findings of a statistical investigation conducted by others
- How to use appropriate graphical displays to compare data sets taking into account shape, centre and spread

Lesson Idea 3.11 (Probability)

Title
Revision of fundamental principle of counting and second year probability

Resources
Online Resources Junior Certificate document

Content
These lessons will involve the students in investigating and understanding:

- How to list all possible outcomes of an experiment
- How to apply the fundamental principle of counting
- How to distinguish certain from uncertain events
- How to describe events as being more or less likely from experience
- How to order events from least likely to most likely and be able to justify their choice
- How to use the scale from 0 to 1 to informally place everyday chance-related events
- How to represent and interpret probabilities as fractions, decimals and percentages
- How to represent the probability of an event as a fraction or decimal between 0 and 1 or as a percentage
- Equally likely outcomes
- How to list all possible outcomes for practical experiments such as rolling a fair die
- That the outcomes on successive throws of a die are independent of each other
- How to calculate the relative frequency for each outcome by experiment and note how it approaches the theoretical probability as the number of trials increases
- The principle that, in the case of equally likely outcomes, the probability is given by the number of outcomes of interest divided by the total number of outcomes
- The following terminology: trial, outcome, set of all possible outcomes, relative frequency, event, theoretical probability, equally likely outcomes
- How to estimate probabilities from experimental data

Lesson Idea 3.12 (Probability)

Title
Independent events 1

Resources
Online Resources Junior Certificate document

Content
These lessons will involve the students in investigating and understanding:

- How to list all the possible outcomes for throwing two dice (or similar experiments) using a two way table
- How to relate the number of outcomes to the fundamental principle of counting
- What the *concept* of *fairness* means in a game with two dice or similar games

- How to come up with rules for a game which make it fair/unfair
- How to construct a probability table
- **How to use set theory to discuss experiments, outcomes, sample spaces**
- The relationship between an event and its complement
- How to determine the probability of an event using the results of an experiment and use this to predict the result of a repetition of the experiment, for equally likely outcomes

Lesson Idea 3.13 (Probability)

Title
Independent events 2

Resources

Online Resources Junior Certificate document

Content
These lessons will involve the students in investigating and understanding:

- How to list the possible outcomes when tossing a coin (or similar experiment) and calculate the probability of getting a head or a tail
- How to list all the possible outcomes when tossing 2 coins (or similar experiment)
- How to use a tree diagram and two way table to list all the possible outcomes when tossing a coin
- How to relate the number of outcomes to the fundamental principle of counting
- How to determine the probability of an event using the results of an experiment and use this to predict the result of a repetition of the experiment, for equally likely outcomes.
- The use of the term "independent events"
- How to use tree diagrams and two way tables in other scenarios
- **How to use binary/counting methods to solve problems involving successive random events where only two possible outcomes apply to each event**
-

Lesson Idea 3.14 (Probability)

Title
Further probability with playing cards, spinners, containers with different coloured objects

Resources
Online Resources Junior Certificate document

Content
These lessons will involve the students in investigating and understanding:

- A standard deck of playing cards, knowing the names of the 4 suits, how many cards in a pack, how many in each suit, and the names of the cards in each suit
- How to calculate the theoretical probability of picking different cards from the pack
- Probabilities with spinners or containers with different coloured objects (or similar experiments)

Section 3 : Synthetic Geometry 1

While proofs are not the issue as regards informal introduction, it is important that students are kept aware that the theorems build logically.

Concepts relevant to this section in synthetic geometry:
Set, plane, point, line, ray, angle, real number, length, degree. Triangle, right-angle, congruent triangles, parallel lines, area, line segment, collinear points, distance, reflex angle, ordinary angle, straight angle, null angle, full angle, supplementary angles, vertically-opposite angles, acute angle, obtuse angle, angle bisector, perpendicular lines, perpendicular bisector of a line segment, isosceles triangle, equilateral triangle, scalene triangle, right-angled triangle, exterior angles of a triangle, interior opposite angles, alternate angles, corresponding angles, transversal line, circle.

This is a suggested sequence for teaching Third Year students. It includes the material in the Common Introductory Course and also deals with some triangle constructions and congruence of triangles. **Refer to the syllabus** for the **"Geometry for Post - primary School Mathematics"** which sets out the agreed course in geometry for both Junior Certificate Mathematics and Leaving Certificate Mathematics. Strand 2 of the relevant syllabus document specifies the learning outcomes at the different syllabus levels.

Refer to Appendix A: "Geometry - Thinking at different levels - The Van Hiele Theory"

Refer to Appendix B for the "Guide to Axioms, Theorems and Constructions for all Levels". In Appendix B, * indicates that proof of the relevant theorem is required for JCHL and LCHL and ** indicates that proof of the relevant theorem is required for LCHL only.

Teachers are also strongly recommended to use the Geometry Lesson Idea ideas in the "Junior Certificate Guidelines for Teachers" (DES 2002, Government Publications Sales Office). It is also available to download at www.projectmaths.ie.

As outlined at the workshops, the use of manipulative products such as "geostrips", "anglegs", geo-boards etc. can make the learning so much more enjoyable for students of all perceived abilities.

The first 2 lesson ideas for Geometry of year 3 are designed to give the students a chance to revisit the material met in first and second year Geometry. It is recommended that new activities and challenges be introduced during this revision so that students do not see it as too much repetition and that they can see new ways of investigating familiar situations.

Note on experimentation and experimental results:

With experimentation, when we measure, the results are only approximations and won't agree exactly. It is important for students to report faithfully what they find e.g. for a triangle they could find that the sum of the angles to be 179^0 or 181 etc. The conclusion is that the angles appear to add up to 180^0. This is a plausible working assumption. There is a distinction between what you can discover and what you can prove.

In first year we were experimenting and using words. In second year we start to problem solve in concrete situations and then in third year we prove things.

Lesson Idea 3.15

Title

Revision of second year material – Lesson Ideas 2.27 -2.34

Resources

Online Resources Junior Certificate document

A mathematical instruments set, Dynamic software package

Junior Certificate Guidelines for Teachers (DES 2002, Government Publications Sales Office). It is also available to download at www.projectmaths.ie.

Content

These lessons will involve the students in investigating and understanding:

- **The use of the term: theorem, proof, axiom, corollary and implies**
- Axioms 1, 2, 3 and 5
- Constructions 8, 9, 5, 6, 10, 11, 12, 1, 2, 3, 4 and (**7 for higher level**)
- Theorems 1 – 6
- **Proof of Theorems 4 and 6**
- Axiom 4 : Congruent triangles (SSS, ASA, SAS)
- Alternate angles and corresponding angles

Lesson Idea 3.16

Title

Revision of second year material – Lessons 2.35 -2.39

Resources

Online Resources Junior Certificate document

A mathematical instruments set, Dynamic software package

Junior Certificate Guidelines for Teachers (DES 2002, Government Publications Sales Office). It is also available to download at www.projectmaths.ie.

Content

These lessons will involve the students in investigating and understanding:

- Translations, axial symmetry, central symmetry and rotations
- Properties of parallelograms
- Square, rhombus, parallelogram, rectangle
- Theorems 9,10
- **Proof of Theorem 9**

Lesson Idea 3.17

Title
Investigating a Square

Resources

Online Resources Junior Certificate document

A mathematical instruments set
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

- The properties of a square

Suggested class activities
Students might engage in the following investigations about a square

Draw a square ABCD
Draw in the two diagonals AC and BD intersecting at E
Are the two diagonals equal in length? (Measure)
Mark in all the equal sides and angles in the triangles AED and BEC
(say why they are equal)
Can you see why triangles ADE and BEC are congruent? Give a reason
Are the triangles AEB and DEC congruent? Give a reason
Are there 4 congruent triangles in the square?
Are the diagonals perpendicular? Give a reason
Do the diagonals bisect the vertex angles of the square?
How many axes of symmetry does the square have?
Does the square have a centre of symmetry? If so, where is it?

Lesson Idea 3.18

Title
 Investigating a Rectangle (which is not a square)

Resources

Online Resources Junior Certificate document

A mathematical instruments set

Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

- The properties of a rectangle(which is not a square)

Suggested class activities
Students might engage in the following investigations about a rectangle

Draw a rectangle ABCD (not a square)
Draw in the two diagonals AC and BD intersecting at E
Are the two diagonals equal in length? (Measure)
Mark in all the equal sides and angles in the triangles AED and BEC
(Say why they are equal.)
Can you see why triangles ADE and BEC are congruent? Give a reason
Are the triangles AEB and DEC congruent? Give a reason
Are there 4 congruent triangles in the rectangle?
Are the diagonals perpendicular? Give a reason
Do the diagonals bisect the vertex angles of the rectangle?
How many axes of symmetry does the rectangle have?
Does the Rectangle have a centre of symmetry? If so, where is it?

Possible extra activity:
Repeat the above investigations for the rhombus ABCD

Lesson Idea 3.19 (Higher level only)

Title
Theorem 11

Resources

Online Resources Junior Certificate document

A mathematical instruments set
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

Theorem 11:   **If three parallel lines cut off equal segments on some transversal line, then they will cut off equal segments on any other transversal**. **(Higher Level only)**

Lesson Idea 3.20

Title
Theorem 12 (**Higher level only**) and Theorem 13

Resources

Online Resources Junior Certificate document

A mathematical instruments set, Dynamic software package

Junior Certificate Guidelines for Teachers (DES 2002, Government Publications Sales Office).
 It is also available to download at www.projectmaths.ie.

Content

These lessons will involve the students in investigating and understanding:

- **Theorem 12: Let ABC be a triangle. If a line l is parallel to BC and cuts [AB] in the ratio s: t, then it also cuts [AC] in the same ratio.**
  (**Higher level only**)

- The meaning of similar triangles and the difference between similar and congruent triangles.

- Theorem 13: If two triangles are similar, then their sides are proportional, in order (and converse)

Suggested class activities
See Geometry Lesson Idea 12 in "Junior Certificate Guidelines for Teachers", page 70. This gives students an opportunity to discover the difference between similar and congruent triangles.

Available at www.projectmaths.ie

Section 4: Algebra

Lesson Idea 3.21

Title
Relations approach to algebra- revision and extension from second year material

Resources
Online Resources Junior Certificate document
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

- How to develop generalising strategies and ideas (considering those of others), present and interpret solutions, explaining and justifying methods, inferences and reasoning in the following:

  o The use of tables to represent a repeating pattern situation

  o Generalise and explain patterns and relationships in words and numbers

  o The use of tables, diagrams , graphs words and formulae as tools for representing and analysing <u>linear</u> patterns and relations

    - Discuss rate of change and the y - intercept. Consider how these relate to the context from which the relationship is derived and identify how they can appear in a table, in a graph and in a formula.

    - Write arithmetic expressions for particular terms in a sequence

    - Find the underlying formula written in words from which the data is derived

    - **Find the underlying formula algebraically from which the data is derived (linear, quadratic)**

    - Decide if two linear relations have a common value (decide if two lines intersect and where the intersection occurs).

    - Relations of the form $y = mx$ (proportional relationships) as distinct from relationships of the form $y = mx + c$

    - Recognise problems involving direct proportion and identify the necessary information to solve them

    - The concept of a function as a relationship between a set of inputs and a set of outputs where each input is related to only one output

    o   The use of tables, diagrams, graphs and formulae as tools for representing and analysing <u>quadratic</u> patterns and relations

- Recognise that a distinguishing feature of quadratic relations is the way change varies

- **Find the underlying formula algebraically from which the data is derived**

- The concept of a function as a relationship between a set of inputs and a set of outputs where each input is related to only one output

    o   The use of tables, diagrams, graphs and formulae as tools for representing and analysing <u>exponential</u> patterns and relations (exponential relations limited to doubling and trebling)

- Recognise that a distinguishing feature of exponential relations is the way change varies

- The concept of a function as a relationship between a set of inputs and a set of outputs where each input is related to only one output

Lesson Idea 3.22

Title

Evaluation of and operations on algebraic expressions- revision and extension of second year material

Resources

Online Resources Junior Certificate document

Content

These lessons will involve the students in investigating and understanding:

- Terms, coefficients and expressions

- Generating algebraic expressions from simple contexts

- Evaluating expressions

$$ax + by$$
$$a(x + y)$$
$$x^2 + bx + c$$
$$\frac{ax + by}{cx + dy}$$
$$axy$$
$$\text{where } a, b, c, d, x, y \in \mathbf{Z}$$

$$\boldsymbol{ax^2 + bx + c}$$
$$\boldsymbol{x^3 + bx^2 + cx + d} \textbf{ where } \boldsymbol{a, b, c, d, x, y \in Q}$$

- Adding and subtract algebraic expressions of forms such as:

$$(ax + by + c) \pm (dx + ey + f)$$
$$(ax^2 + bx + c) \pm (dx^2 + ex + f)$$

$$\boldsymbol{(ax + by + c) \pm \ldots \pm (dx + ey + f)}$$
$$\boldsymbol{(ax^2 + bx + c) \pm \ldots \pm (dx^2 + ex + f)} \text{ where } \boldsymbol{a, b, c, d, e, f \in \mathbf{Z}}$$

- Multiplying terms and expressions (using a model such as the array model) using the associative and distributive properties to simplify such expressions as:

$$a(bx + cy + d) + e(fx + gy + h)$$

$$\boldsymbol{a(bx + cy + d) + \ldots + e(fx + gy + h)}$$

$$a(bx^2 + cx + d)$$
$$ax(bx^2 + c) \text{ where } a, b, c, d, e, f, g, h \in \mathbf{Z}$$

$$\boldsymbol{(x + y)(x + y)}$$
$$\boldsymbol{(x - y)(x - y)}$$

How to multiply expressions of the form:

$$(ax + b)(cx + d)$$
$$\boldsymbol{(ax + b)(cx^2 + dx + e)} \textbf{ where } \boldsymbol{a, b, c, d, e \in \mathbf{Z}}$$

- How to divide expressions of the form:
$$ax^2 + bx + c \div dx + e, \text{ where } a, b, c, d, e \in \mathbb{Z}$$
- $$\boldsymbol{ax^3 + bx^2 + cx + d \div ex + f}, \text{ where } \boldsymbol{a, b, c, d, e \in \mathbf{Z}}$$

Lesson Idea 3.23

Title

Algebraic factors with an extension to HL material

Resources

Online Resources Junior Certificate document

Content

These lessons will involve the students in investigating and understanding:

- Use of the distributive law to multiply expressions
- Factorising expressions such as:

ax, axy where $a \in \mathbf{Z}$

$abxy + ay$ where $a, b \in \mathbf{Z}$

$sx - ty + tx - sy$ where s, t, x, y are variable

$ax^2 + bx$ where $a, b \in \mathbf{Z}$

$x^2 + bx + c$, where $b, c \in \mathbf{Z}$

$x^2 - a^2$

Higher level material:

$ax^2 + bx + c, a \in \mathbf{N}, b, c \in \mathbf{Z}$

$a^2 x^2 - b^2 y^2$, where $a, b \in \mathbf{N}$

Lesson Idea 3.24

Title

Adding algebraic fractions - revision and extension of second year material

Resources

Online Resources Junior Certificate document

Content

These lessons will involve the students in investigating and understanding:

- The addition of expressions such as:
- $\dfrac{ax+b}{c} \pm \dfrac{dx+e}{f}$ where $a, b, c, d, e, f \in \mathbf{Z}$

$\dfrac{ax+b}{c} \pm ... \pm \dfrac{dx+e}{f}$ where $a, b, c, d, e, f \in \mathbf{Z}$

$\dfrac{a}{bx+c} \pm \dfrac{p}{qx+r}$ where $a, b, c, p, q, r \in \mathbf{Z}$

Lesson Idea 3.25

Title

Linear equations and linear inequalities - revision and extension of second year material

Resources

Online Resources Junior Certificate document

Dynamic software package

Content

These lessons will involve the students in investigating and understanding:

- The concept of equality and what is meant by an equation

- How to identify the necessary information, represent problems mathematically, making correct use of symbols, words, diagrams, tables and graphs

    o The use of suitable strategies (graphic, numeric, algebraic and mental) for finding solutions to equations of the form:

    ▪ First degree equations in one or two variables with coefficients elements of $\mathbf{Z}$ and solutions in $\mathbf{Z}$, including equations of the form:

    $$\frac{ax+b}{c} \pm \frac{dx+e}{f} = \frac{g}{h} \text{ where } a,b,c,e,f,g,h \in \mathbf{Z}$$

    o **The use of suitable strategies (graphic, numeric and algebraic) in the solution of first degree equations in one or two variables, with coefficients elements of Q and solutions also in Q**

    o The use of suitable strategies (graphic, numeric and algebraic) in the solution of linear inequalities in one variable of the form:

    ▪ $g(x) \leq k$ where $g(x) = ax+b, a \in \mathbf{N}$ and $b,k \in \mathbf{Z}$

    ▪ $k \leq g(x) \leq h$ where $g(x) = ax+b$ and $k,a,b,h \in \mathbf{Z}$ and $x \in \mathbf{R}$

    ▪ How to graph solution sets on the number line for linear inequalities in one variable

Lesson Idea 3.26

Title

Solve quadratic equations - revision and extension of second year material

Resources

Online Resources Junior Certificate document

Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

- How to identify the necessary information, represent problems mathematically, making correct use of symbols, words, diagrams, tables and graphs
    - The use of suitable strategies (graphic, numeric, algebraic and mental) in the solution of quadratic equations of the form $x^2 + bx + c = 0$ where $b, c \in \mathbf{Z}$ and $x^2 + bx + c$ is factorisable
    - The use of suitable strategies (graphic, numeric and algebraic) in the solution of quadratic equations of the form $\boldsymbol{ax^2 + bx + c = 0}$ **where a,** $\boldsymbol{b, c \in \mathbf{Q}}$ **and** $\boldsymbol{x \in \mathbf{R}}$
    - **Form quadratic equations give whole number roots**
    - Solve simple problems leading to quadratic equations

Lesson Idea 3.27

Title
Rearrange formulae (HL)

Resources

Online Resources Junior Certificate document

Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

- **Rearranging formulae including the use of various contexts**

Section 5: Functions

Lesson Idea 3.28

Title

Functions -interpreting and representing linear, quadratic and exponential functions in graphical form

Resources

Online Resources Junior Certificate document

Dynamic software package

Content

These lessons will involve the students in investigating and understanding:

- The concept of a function, domain, co - domain and range

- How to make use of function notation $f(x) =$, $f : x \rightarrow,$ and $y =$

- How to interpret simple graphs

- How to plot points and lines

- How to draw graphs of the following functions and interpret equations of the form $f(x) = g(x)$ as a comparison of functions

  $f(x) = ax + b$ where $a, b \in \mathbf{Z}$

  $f(x) = ax^2 + bx + c,$ where $a \in \mathbf{N}; b, c \in \mathbf{Z}; x \in \mathbf{R}$

  $\boldsymbol{f(x) = ax^2 + bx + c},$ where $\boldsymbol{a, b, c \in \mathbf{Z}; x \in \mathbf{R}}$

  $\boldsymbol{f(x) = a2^x}$ and $\boldsymbol{f(x) = a3^x}$ where $\boldsymbol{a \in \mathbf{N}, x \in \mathbf{R}}$

- How to use graphical methods to find approximate solutions where $f(x) = g(x)$ and interpret the results
- **How to find maximum and minimum values of quadratic functions from a graph**
- **How to interpret inequalities of the form $f(x) \leq g(x)$ as a comparison of functions of the above form; use graphical methods to find approximate solution sets of these inequalities and interpret the results**

Section 6: Algebra

Lesson Idea 3.29

Title

Relations without formulae

Resources

Online Resources Junior Certificate document

Dynamic software package

Content

These lessons will involve the students in investigating and understanding:

 Graphs of motion

- How to make sense of qualitative graphs and drawing conclusions from them
- The connections between the shape of a graph and the story of a phenomenon
- How to describe both quantity and change of quantity on a graph

Section 7: (Synthetic geometry Part 2)

Lesson Idea 3.30

Title
Theorems 14 and 15 and Proposition 9

Resources

Online Resources Junior Certificate document

A mathematical instruments set

Junior Certificate Guidelines for Teachers (DES 2002, Government Publications Sales Office). It is also available to download at www.projectmaths.ie.

Content
These lessons will involve the students in investigating and understanding:

- *****<u>Theorem 14</u>: [Theorem of Pythagoras] (**<u>proof required for Higher Level only</u>**)

- <u>Theorem 15</u>: [Converse to Theorem of Pythagoras] If the square of one side of a triangle is the sum of the squares of the other two sides, then the angle opposite the first side is a right angle.

- **Proposition 9**: If two right-angled triangles have hypotenuse and another side equal in length respectively, then they are congruent. (RHS)

Suggested class activities
See Geometry Lesson Idea Ideas 7, 8, 9, 10 and 11 in "Junior Certificate Guidelines for Teachers" Pp 66-69. Available at www.projectmaths.ie

Geometry Lesson Idea 3.31

Title
Constructions 13, 14 and 15

Resources

Online Resources Junior Certificate document

A mathematical instruments set

Content
These lessons will involve the students in investigating and understanding:

- <u>Construction 13</u>: Right-angled triangle, given length of hypotenuse and one other side

- <u>Construction 14</u>: Right-angled triangle, given one side and one of the acute angles (several cases)

- Construction 15: Rectangle given side lengths

Geometry Lesson Idea 3.32

Title
Theorem 19 and Corollaries 2 and 5 (higher level only) and Corollaries 3 and 4

Resources

Online Resources Junior Certificate document

A mathematical instruments set

Content
These lessons will involve the students in investigating and understanding:

*Theorem 19:   The angle at the centre of a circle standing on a given arc is twice the angle at any point of the circle standing on the same arc. (Proof required for Higher Level only)

Corollary 2:   All angles at points of a circle, standing on the same arc, are equal (and converse)

- Corollary 3: Each angle in a semi-circle is a right angle.
- Corollary 4: If the angle standing on a chord [BC] at some point of the circle is a right- angle, then [BC] is a diameter.

   Corollary 5: If ABCD is a cyclic quadrilateral, then opposite angles sum to 180°, (and converse)

Section 8: Coordinate Geometry

Lesson Idea 3.33

Title
Co-ordinate geometry - revision and extension of second year material

Resources

Online Resources Junior Certificate document

Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

- Plane, points, lines, line segments, rays, collinear points, length of a line segment
- Coordinating the plane
- Locating points on the plane using coordinates
- The relationship between the midpoint and the length of a line segment
- The midpoint formula in coordinate geometry
- The distance formula in coordinate geometry (Relate the formula to Pythagoras' theorem.)
- The concept of slope as "rise/run"
- Links between slope and rate of change (e.g. growth of plant) and other real life contexts
- Interpreting slope in context
- The formula for the slope of a line
- **Parallel and perpendicular lines and the relationship between the slopes**

Lesson Idea 3.34

Title
The equation of a line

Resources
Online Resources Junior Certificate document
Graph paper, mathematical set
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:
- Links to real life context (e.g. growth of plant)
- Using table, graphs and diagrams in relation to the equation of a line

- The connection between the y-intercept, slope and the equation of a line
- Link positive, negative and zero slope to real lie contexts.
- The equation of a line in the forms:
  - $$y - y_1 = m\left(x - x_1\right)$$
    $$y = mx + c$$
  - $ax + by + c = 0$ (HL only) where a, b, c are integers and m is the slope of the line
- The connection between the x and y intercepts and the equation of a line from a graphical and algebraic perspective
- Points on a line (and not on a line) from a graphical and algebraic perspective
- Properties of the equation of a line
- How to find the slopes of parallel and perpendicular lines from their equations
- **How to recognise parallel and perpendicular lines from their equations**
- How to find the point of intersection of two lines graphically and algebraically

Section 9: Trigonometry

Lesson Idea 3.35

Title
Revision of Pythagoras' Theorem

Resources
Online Resources Junior Certificate document
Graph paper, calculator,
A mathematical instruments set
Dynamic software package
NCCA Mathematics Resources for Students – Junior Certificate Strand 2
Mathematics Junior Certificate Guidelines for Teachers Pages 66-69

Content
These lessons will involve the students in investigating and understanding:
- Pythagoras' Theorem using a visual geometric model and using algebra
- The properties of a right-angled triangle generated by Pythagoras' Theorem (differentiate between the hypotenuse and any other side, finding length of unknown side, verify if a triangle is (or is not) right-angled
- Apply the theorem of Pythagoras to solve right – angled triangle problems of a simple nature involving heights and distances
- **Proof of Theorem 14 (Pythagoras' Theorem)**

Lesson Idea 3.36

Title
Revision of scaled diagrams from second year

Resources
Online Resources Junior Certificate document
Graph paper, calculator,
A mathematical instruments set
Dynamic software package
NCCA Mathematics Resources for Students – Junior Certificate Strand 2

Content
These lessons will involve the students in investigating and understanding:

- The use of a scaled diagram to measure angles of elevation and depression and unknown lengths (Trigonometric ratios are not being used yet. See pages 12 to 14 for ordinary-level, **pages 12 to 17 for higher-level**, of Teaching and Learning Plan 8 : *Introduction to Trigonometry*)

Lesson Idea 3.37

Title
Trigonometric ratios in right-angled triangles (sine, cosine, tangent)

Resources

Online Resources Junior Certificate document

Graph paper, calculator,
A mathematical instruments set
Dynamic software package
NCCA Mathematics Resources for Students – Junior Certificate Strand 2

Content
These lessons will involve the students in investigating and understanding:

- The labelling of the sides of a right-angled triangle (opposite, adjacent and hypotenuse)
- The meaning of complementary angles
- Similar right – angled triangles and practical uses
- The trigonometric ratios:
    - Understanding tan (tangent) as the <u>ratio</u> of opposite to adjacent in a right-angled triangle and link to slope of a line (rise/run)
    - Understanding sine and cosine as ratios between the sides of the right-angled triangle
- The use of these trigonometric ratios to solve problems involving angles (integer values) between 0° and 90°
- **The use of these trigonometric ratios to solve problems involving right – angled triangles**
- The use of a clinometer in conjunction with trigonometry to solve practical problems
- **How to work with trigonometric ratios in surd form for right –angled triangles with angles of 30^0, 45^0 and 60^0**
- **The solution of problems involving surds**
- **The manipulation and measure of angles in both decimal and DMS forms**

Appendix A

Geometry: Thinking at Different Levels

The Van Hiele Theory

The **Van Hiele model** describes how students learn geometry. Pierre van Hiele and Dina van Hiele-Geldof, mathematics teachers from the Netherlands, observed their geometry students in the 1950's.The following is a brief summary of the Van Hiele theory. According to this theory, students progress through 5 levels of thinking starting from merely recognising a shape to being able to write a formal proof. The levels are as follows:

*Visualisation (Level 0)

The objects of thought are shapes and what they look like.

Students have an overall impression of a shape. The appearance of a shape is what is important. They may think that a rotated square is a "diamond" and not a square because it is different from their visual image of a square. They will be able to distinguish shapes like triangles, squares, rectangles etc but will not be able to explain, for example, what makes a rectangle a rectangle. **Vocabulary**: Students use visual words like "pointy", "curvy", "corner" as well as correct language like angle, rectangle and parallelogram.

*Analysis (Level 1)

 The objects of thought are "classes" of shapes rather than individual shapes.

- Students think about what makes a rectangle a rectangle and can separate the defining characteristics of a rectangle from irrelevant information like size and orientation. They recognize its parts (sides, diagonals and angles) and compare their properties (similar, congruent)
- They understand that if a shape belongs to a class like "rectangle", then it has all the properties of that class (2 pairs of equal sides, right angles, 2 equal diagonals, 2 axes of symmetry).
- **Vocabulary:** words like parallel, perpendicular and congruent relating to properties within a figure and the words all, always, sometimes, never, alike, different.
- A concise definition of a figure, using a sufficient rather than an exhaustive list of properties is not possible at this level.
- They do not deal with questions like "Is a square a parallelogram?" but just look at the properties of each class of shape, without comparing the classes.

*Some visualisation and analysis is covered in Primary School.

Relational/ Ordering/Informal Deduction (Level 2)

The objects of thought are the properties of shapes.

- Students are ready to understand interrelationships of properties within figures and between figures. Opposite sides of a parallelogram are parallel and opposite angles are equal.
- A rectangle is a parallelogram since it has all the properties of a parallelogram as well as all 90^0 angles.
- Students can recognise the difference between a statement and its converse. All squares are rectangles (true) is different from all rectangles are squares (not true).
- Capable of **"if –then"** thinking – if a shape is a rectangle then all the angles in it are right angles. If $|<A| = |<B|$ and $|<B| = |<C|$ then $|<A| = |<C|$
- They can select one or two properties to define a figure rather than an exhaustive list. If a quadrilateral has 4 equal sides and one right angle it must be a square.
- Students can discover new properties by simple deduction. The 2 acute angles in a right angled triangle add to 90^0 because all the angles in a triangle add up to 180^0. They can explain logically without having to measure everything.

Formal deduction (Level 3)

Students learn how to use an axiomatic system to establish geometric theory. This is the level at which proof of Theorems is learned. The sequence of theorems given in the appendix is arranged in such a manner that each theorem builds on the previous theorem(s).

Rigor (Level 4)

Comparing different axiomatic systems – not done at secondary level

Characteristics of these levels: Students cannot function at any particular level unless they are competent at all previous levels. The teacher's role is crucial in structuring activities to bring students from one level to the next.

How does the teacher bring students from any one level to the next?

5 phases of learning:

1. In an informal discussion of the topic, students are asked to give their initial observations.
2. The teacher provides structured activities such as drawing, making and measuring.
3. The students then verbalise and write down what they have learned and report back in groups to the class, which leads to a class discussion.
4. The teacher then provides an activity which will require students to apply what they have discovered
5. In the last stage students are required to summarise all they have learned and should be able to remember it as they have discovered it through guidance.

A PowerPoint presentation of the Van Hiele theory can be got at www.projectmaths.ie (Workshop 2 Show 2) http://www.projectmaths.ie/workshops/WS2_NR/WS2_NR_PPTS.zip

A mind map of the Van Hiele theory can be found at
http://agutie.homestead.com/files/mindmap/van_hiele_geometry_level.html

Appendix B

Guide to Theorems, Axioms and Constructions at all Levels*

This is intended as a quick guide to the various axioms, theorems and constructions as set out in the *Geometry Course for Post-Primary School Mathematics.* You can get this from the project maths website: www.projectmaths.ie

It is not intended as a replacement for this document, merely as an aid to reading at a glance which material is required to be studied at various levels. The sequence of theorems as given must be followed.

As stated in the heading, these theorems and constructions are underpinned by 46 definitions and 20 propositions which are all set out in the *Geometry for Post-Primary School Mathematics*, along with many undefined terms and definable terms used without explicit definition.

*An **axiom** is a statement accepted without proof, as a basis for argument

*A **theorem** is a statement deduced from the axioms by logical argument. Theorems can also be deduced from previously established theorems.

* A **proposition** is a useful or interesting statement that could be proved at this point, but whose proof is not stipulated as an essential part of the programme. Teachers are free to deal with them as they see fit, but they should be mentioned, at least (Appendix p. 20, footnote).

*The instruments that may be used for **constructions** are listed and described on page 38 of the Appendix and are a straight edge, compass, ruler, protractor and set-square.

Terms   Students at **Junior Certificate Higher level and Leaving Certificate Ordinary level** will be expected to understand the meanings of the following terms related to logic and deductive reasoning:
Theorem, proof, axiom, corollary, converse, implies.

Synthetic Geometry

Guide to Axioms, Theorems and Constructions for all Levels

Information Technology is used whenever and wherever appropriate to help to present mathematical concepts effectively to students.

In this document the 💿 **symbol appears at the corresponding position of the content to indicate that an interactive IT module is available on the Project Maths Student's CD.**

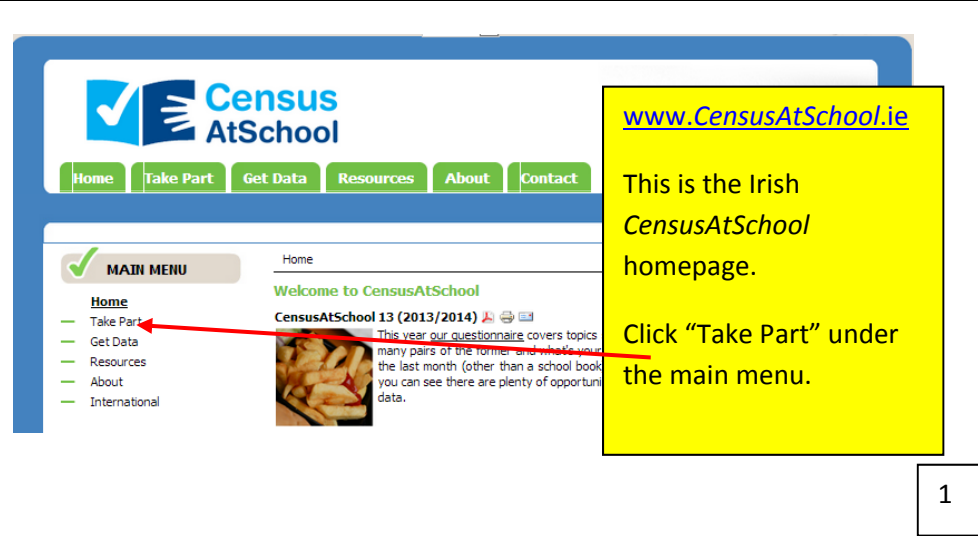|	Axioms and Theorems (supported by 46 definitions, 20 propositions) *proof required for JCHL and LCHL ** proof required for LCHL only	CMN Introd. Course	JC ORD	JC HR	LC FDN	LC ORD	LC HR
	Axiom 1: There is exactly one line through any two given points	√	√	√	√	√	√
	Axiom 2: [Ruler Axiom]: The properties of the distance between points.	√	√	√	√	√	√
	💿 **Axiom 3:** Protractor Axiom (The properties of the degree measure of an angle).	√	√	√	√	√	√
1	💿 Vertically opposite angles are equal in measure.	√	√	√	√	√	√
	Axiom 4: Congruent triangles conditions (SSS, SAS, ASA)	√	√	√	√	√	√
2	💿 In an isosceles triangle the angles opposite the equal sides are equal. Conversely, if two angles are equal, then the triangle is isosceles.	√	√	√	√	√	√
	Axiom 5: Given any line l and a point P, there is exactly one line through P that is parallel to l.	√	√	√	√	√	√
3	💿 If a transversal makes equal alternate angles on two lines then the lines are parallel. Conversely, if two lines are parallel, then any transversal will make equal alternate angles with them.	√	√	√	√	√	√
4*	💿 The angles in any triangle add to 180 .	√	√	√	√	√	√
5	💿 Two lines are parallel if, and only if, for any transversal, the corresponding angles are equal.	√	√	√	√	√	√
6*	💿 Each exterior angle of a triangle is equal to the sum of the interior opposite angles.	√	√	√	√	√	√
7	💿 The angle opposite the greater of two sides is greater than the angles opposite the lesser. Conversely, the side opposite the greater of two angles is greater than the side opposite the lesser angle.					√	√
8	💿 Two sides of a triangle are together greater than the third.					√	√
9*	💿 In a parallelogram, opposite sides are equal, and opposite angles are equal. Conversely, (1) if the opposite angles of a convex quadrilateral are equal, then it is a parallelogram; (2) if the opposite sides of a convex quadrilateral are equal, then it is a parallelogram.		√	√	√	√	√
	Corollary 1. A diagonal divides a parallelogram into two congruent triangles.			√			√
10	💿 The diagonals of a parallelogram bisect each other. Conversely, if the diagonals of a quadrilateral bisect one another, then the quadrilateral is a parallelogram.		√	√	√	√	√

	Axioms and Theorems (supported by 46 definitions, 20 propositions) *proof required for JCHL and LCHL ** proof required for LCHL only	CMN Introd. Course	JC ORD	JC HR	LC FDN	LC ORD	LC HR
11**	If three parallel lines cut off equal segments on some transversal line, then they will cut off equal segments on any other transversal.			√		√	√
12**	Let ABC be a triangle. If a line l is parallel to BC and cuts [AB] in the ratio m:n, then it also cuts [AC] in the same ratio. Conversely, if the sides of two triangles are in proportion, then the two triangles are similar.			√		√	√
13**	If two triangles are similar, then their sides are proportional, in order (and converse)		√	√	√	√	√
14*	[Theorem of Pythagoras]In a right-angled triangle the square of the hypotenuse is the sum of the squares of the other two sides.		√	√	√	√	√
15	[Converse to Pythagoras]. If the square of one side of a triangle is the sum of the squares of the other two, then the angle opposite the first side is a right angle.		√	√	√	√	√
	Proposition 9: (RHS). If two right-angled triangles have hypotenuse and another side equal in length respectively, then they are congruent.		√	√	√	√	√
16	For a triangle, base x height does not depend on the choice of base.					√	√
	Definition 38: The area of a triangle is half the base by the height.					√	√
17	A diagonal of a parallelogram bisects the area.					√	√
18	The area of a parallelogram is the base x height.					√	√
19*	The angle at the centre of a circle standing on a given arc is twice the angle at any point of the circle standing on the same arc.			√			√
	Corollary 2†: All angles at points of a circle, standing on the same arc are equal (and converse).			√			√
	Corollary 3: Each angle in a semi-circle is a right angle.		√	√	√	√	√
	Corollary 4: If the angle standing on a chord [BC] at some point of the circle is a right-angle, then [BC] is a diameter.		√	√	√	√	√
	Corollary 5: If ABCD is a cyclic quadrilateral, then opposite angles sum to 180 .			√			√
20	(i)　　Each tangent is perpendicular to the radius that goes to the point of contact. (ii)　　If P lies on the circle S, and a line l is perpendicular to the radius to P, then l is a tangent to S.					√	√
	Corollary 6: If two circles intersect at one point only, then the two centres and the point of contact are collinear.					√	√
21	(i)　　The perpendicular from the centre to a chord bisects the chord. (ii)　　The perpendicular bisector of a chord passes through the centre.					√	√

† The corollaries are numbered as in the *Geometry for Post-primary School Mathematics*; corollary 2 is the first one relating to theorem 19

	Constructions (Supported by 46 definitions, 20 propositions, 5 axioms and 21 theorems)	CMN Introd. Course	JC ORD	JC HR	LC FN	LC ORD	LC HR
1	Bisector of an angle, using only compass and straight edge.	√	√	√	√	√	√
2	Perpendicular bisector of a segment, using only compass and straight edge.	√	√	√	√	√	√
3	Line perpendicular to a given line l, passing through a given point not on l.			√			√
4	Line perpendicular to a given line l, passing through a given point on l.	√	√	√	√	√	√
5	Line parallel to given line, through a given point.	√	√	√	√	√	√
6	Division of a line segment into 2 or 3 equal segments without measuring it.	√	√	√	√	√	√
7	Division of a line segment into any number of equal segments, without measuring it.			√			√
8	Line segment of a given length on a given ray.	√	√	√	√	√	√
9	Angle of a given number of degrees with a given ray as one arm.		√	√	√	√	√
10	Triangle, given lengths of 3 sides.		√	√	√	√	√
11	Triangle, given SAS data.		√	√	√	√	√
12	Triangle, given ASA data		√	√	√	√	√
13	Right-angled triangle, given length of hypotenuse and one other side		√	√	√	√	√
14	Right-angled triangle, given one side and one of the acute angles.		√	√	√	√	√
15	Rectangle given side lengths.		√	√	√	√	√
16	Circumcentre and circumcircle of a given triangle, using only straight edge and compass.					√	√
17	Incentre and incircle of a triangle of a given triangle, using only straight edge and compass.					√	√
18	Angle of 60 without using a protractor or set square.				√	√	√
19	Tangent to a given circle at a given point on it.				√	√	√
20	Parallelogram, given the length of the sides and the measure of the angles.				√	√	√
21	Centroid of a triangle.					√	√
22	Orthocentre of a triangle.						√

Appendix C

How to register for *CensusAtSchool*, complete online questionnaires and retrieve class data
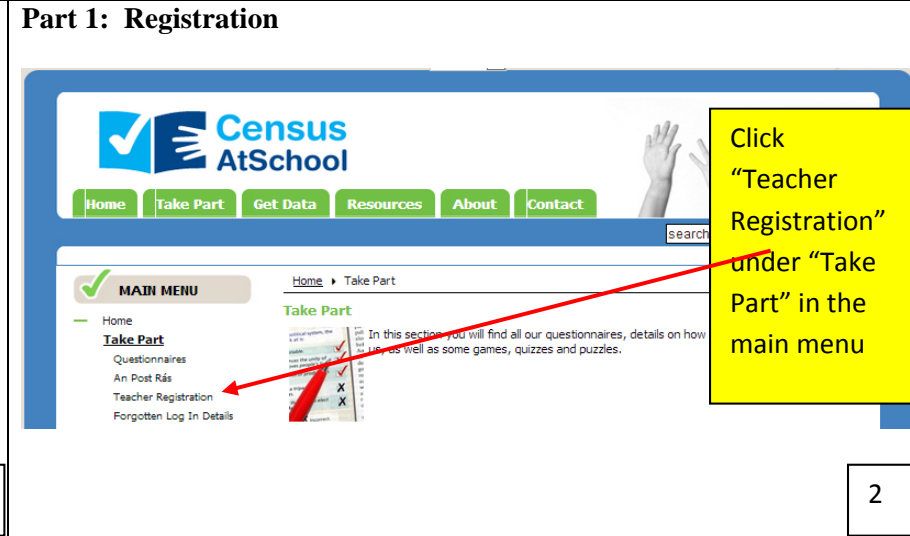

www.*CensusAtSchool*.ie

This is the Irish *CensusAtSchool* homepage.

Click "Take Part" under the main menu.

1

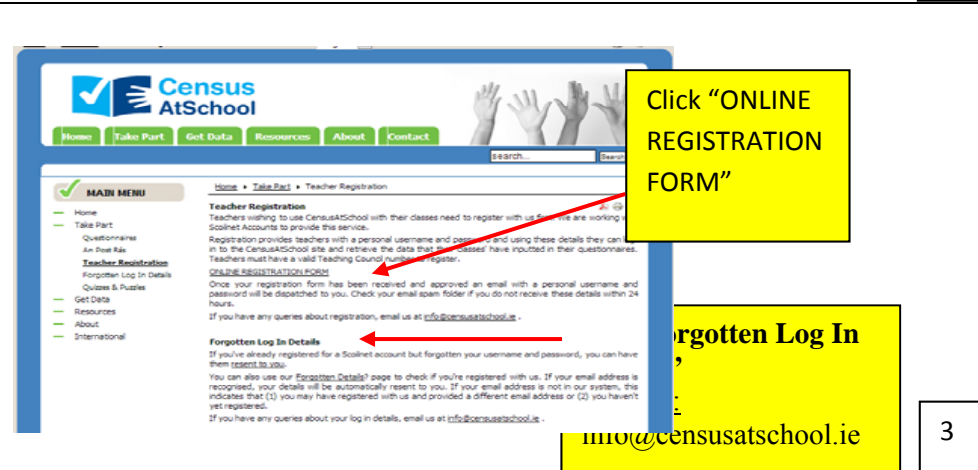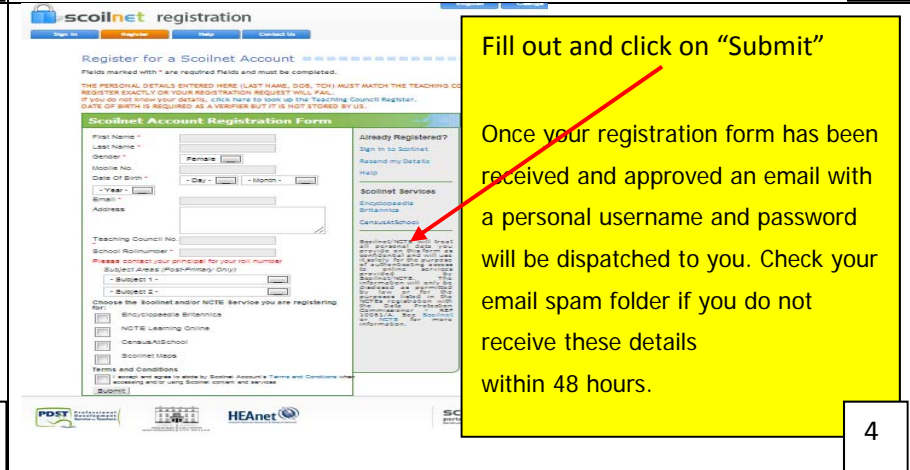**Part 1: Registration**

Click "Teacher Registration" under "Take Part" in the main menu

2

Click "ONLINE REGISTRATION FORM"

...rgotten Log In ...,

info@censusatschool.ie

3

Fill out and click on "Submit"

Once your registration form has been received and approved an email with a personal username and password will be dispatched to you. Check your email spam folder if you do not receive these details within 48 hours.

4

Part 2: Accessing the questionnaires -use the highest phase available

Click "Take Part", "Questionnaires" and then click on the latest questionnaire which will be top of the list.

5

Click "Take the Phase* questionnaire"

6

Part 3: Filling out the questionnaire

Click on "Next"

7

Each student must fill in accurately, the school roll number and the teacher's username (sent by email) and continue until the questionnaire is completed. The student then clicks "Submit".

8

Part 4 Now that my students have filled in the data – What next?

Under "Get Data" click on "Request your class data" .

9

Logging in to retrieve data

Fill in your username and password. If you forget them "click on the "forgot your password or username" link.

Click "Login".

10

Once logged in you will be asked for your school roll number and username.

Select whichever phase your students have completed. Click on "Submit" and your data will be returned to you in spreadsheet form.

11

Your data will look similar to the spreadsheet below.

Sex	BrthMnth	BrthYear	BrthPlce	BedRms	PeoplHme	InetAxs	NumPts	PtsNone	PtsDog	PtsCat	PtsMouse	PtsRabt
Female	July	1992	New Sout	4	5	Yes - broa	4	No	Yes	Yes	No	No
Female	February	1996	Australian	4	5	Yes - broa	4	No	Yes	Yes	No	No
Male	September	1994	Victoria	4	5	Yes - dial-	5	No	Yes	No	No	No
Female	September	1991	Fiji	4	4	No - Inter	0	Yes	No	No	No	No
Male	August	1993	Queenslar	3	4	No - Inter	0	No	No	No	No	No
Female	December	1993	Tasmania	3	5	Yes - dial-	3	No	No	Yes	No	No
Female	August	1990	Queenslar	4	5	Yes - broa	3	No	Yes	No	No	No
Female	November	1991	New Sout	11	5	Yes - dial-	4	No	Yes	No	No	Yes
Male	June	1995	Victoria	3	5	Yes - othe	6	No	Yes	No	No	No
Female	March	1993	Western A	30	122	No - Inter	2	No	Yes	No	No	No
Female	December	1989	Queenslar	3	3	Yes - broa	2	No	No	No	No	No
Female	April	1993	South Aus	3	5	No - Inter	4	No	Yes	Yes	No	No
Male	March	1996	New Sout	3	2	No - Inter	0	Yes	No	No	No	No
Male	September	1993	Queenslar	4	4	Yes - dial-	18	No	Yes	No	No	No
Male	May	1995	Tasmania	3	4	Yes - broa	2	No	No	No	No	Yes
Female	March	1994	Western A	4	4	Yes - dial-	4	No	Yes	Yes	No	No

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