Student Activity: To investigate arithmetic sequences and series

Use in connection with the interactive file, ‘Arithmetic Sequence and Series’, on the Student’s CD.

1. The first term “a” of an arithmetic sequence is equal to 3 and the common difference “d” of the sequence is 2.
   a. Write down the first 6 terms. Show your calculations.

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   ____________________________________________________________

   b. Represent all these terms on a graph.

   ![Graph](image)

   c. Use any 2 points on the graph to find the slope of the line joining these two points.

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   d. Repeat step c. for 3 more sets of points.

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e. What do you notice about the relationship between the slope and the common difference?

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f. Change the first term in the interactive file and keep the common difference as 2. What do you notice about this graph?

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g. The first term “a” of an arithmetic sequence is equal to 2 and the common difference “d” of the sequence is 2. Draw the graph to represent this situation. How does this graph differ from the graph in section b?

h. What quantity do you need to change in the sequence to change the slope of its graph?

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2. The first term “a” of an arithmetic sequence is equal to 5 and the common difference “d” is 2.
   a. Write down the first 6 terms. Show your calculations.

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b. Represent all these terms on a graph.

![Graph of arithmetic sequence terms](image)

3. The first term “a” of an arithmetic sequence is equal to 6 and the common difference “d” is −1.
   a. Write down the first 6 terms. Show your calculations.

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   b. Represent all these terms on a graph.

   ![Graph of arithmetic sequence terms](image)
c. What do you think the slope of the line joining these points will be? Check your answer using any 2 of the points on the graph. Explain the reason for this.

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4. The first term of an arithmetic sequence is 12 and the common difference is 5.
   a. Calculate the first 6 terms of this sequence. Show calculations.

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b. Calculate the sum of the first 6 terms of this sequence. Show calculations.

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c. Calculate the sum of the first 10 terms of this sequence. Show calculations.

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d. Complete the following table:

<table>
<thead>
<tr>
<th>T₁</th>
<th>T₂</th>
<th>T₃</th>
<th>T₄</th>
<th>T₅</th>
<th>T₆</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

e. Calculate T₁ + T₆.

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f. Calculate T₂ + T₅.

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g. Calculate T₃ + T₄.

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h. Use the pattern you have seen in the last 3 sections of this problem to find the sum of the first 6 terms

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i. Compare your results with that found in the interactive file.
5. 

Given that the sum of the first \(n\) terms of an arithmetic sequence \(S_n = \frac{n}{2}[2a + (n-1)d]\) and the \(n^{th}\) term of an arithmetic sequence \(T_n = a + (n-1)d\).

Show that \(S_n = \frac{n}{2} \left[ \text{first term} + \text{n}^{th} \text{term} \right].\)

6. After knee surgery, your trainer tells you to return to your jogging program slowly. He suggests jogging for 10 minutes the day after your treatment. Each day thereafter, he suggests that you increase that time by 2 minutes.
   
   a. Is this an arithmetic sequence? Explain your answer.

   b. What is the first term?

   c. What is the common difference?

   d. What are your jogging times for the first 6 days after your visit to the trainer?

   e. For how many minutes will you jog on the 9th day after your treatment? Show calculations.

   f. What is the total time you will jog in the first 9 days after your treatment? Show calculations.

   g. Check your results for the previous 3 sections of this question using the interactive file.
h. If you started jogging on the first day of the month, how long will you be jogging per day after 1 month (30 days)? Show your calculations.

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i. How much time will you spend jogging in the first month (30 days)?

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7. A water tank contains 200 litres of water when full. The tank develops a drip and water leaks out at a constant rate of 5 litres per day.
   a. Does the amount of water left in the tank at the end of each day form an arithmetic sequence? Why?

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b. What is the first term?

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c. What is the common difference?

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d. How much water does the tank contain for each of the first 6 days?

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e. What is the total amount of water lost by the tank in the first 6 days?

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f. How many litres of water does the tank contain at the end of the 12th day?

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g. On which day will the tank be half empty?

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h. On which day will the tank be completely empty?

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