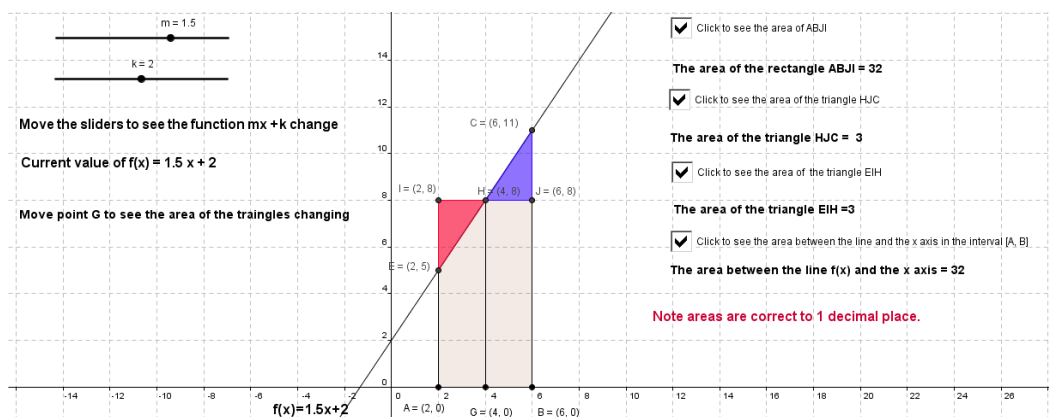## Student Activity: To investigate the Average Value of a Linear Function

Use in connection with the interactive file, 'Average Value 2', on the Student's CD.

1. In the interactive file, move the sliders *m* and *k* so that f(x) = 2x + 2 and move the points G to (3, 0), A to (2,0) and B to (6,0).

   a. What is the area of the rectangle ABJI?

   _____

   _____

   b. What is the area of the triangle HJC?

   _____

   _____

   c. What is the area of the triangle EIH?

   _____

   _____

   d. Is the area between the line f(x) = 2x + 2 and the x-axis in the interval [2, 6] equal to the area of the rectangle ABJI? Explain why this is so.

   _____

   _____

2. Keeping f(x) = 2x + 2, point A at (2, 0) and point B at (6, 0), move the point G to (4, 0).

   a. What is the area of the rectangle ABJI?

   _____

   b. What is the area of the triangle HJC?

   _____

c. What is the current area of the triangle EIH?

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d. Is the area between the line f(x) = 2x + 2 and the x-axis in the interval [2, 6] equal to the current area of the rectangle ABJI? Explain why this is so?

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e. Find the average value of the function f(x)= 2x+2 in the interval [2,6] using f(2) and f(6) only.

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f. Find the average value of the function f(x)= 2x+2 in the interval [2,6] using f(2), f(3), f(4), f(5) and f(6).

_____

g. How many values can x ∈ R have in the interval [2, 6]? What would be the average value using all these values?

_____

h. Express the area between the line f(x) = 2x + 2 and the x-axis in the interval [2, 6] using integration.

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i. Find $\int_{2}^{6}(2x+2)dx$

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_____

j. Hence, find the area between the line f(x) = 2x + 2 and the x-axis in the interval [2, 6].

_____

k. Did the answer to part j equal the answer to part a? Why does this tell us?

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l. Find the area of the rectangle ABJI in terms of the x co-ordinate of A, the x co-ordinate of B and the y co-ordinate of H, when the area of the triangles HJC and EIH are equal.

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_____

m. Did your answer in part l equal the answer to part j?

_____

n. Do you agree that the average value of the function in the interval [2, 6] is the y co-ordinate of the point H when the area of the triangles HJC and EIH are equal?

_____

o. Using part l of this question write the average value of the function f(x) = 2x+2 in the interval [2, 6] in terms of $\int_{2}^{6}(2x+2)dx$.

_____

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3.

a. For any linear function f(x)=mx+c, the area between the graph of the function and the x-axis in the interval [A, B] is equal to the area of the rectangle ABJI when the height of the rectangle is the **average value of the function f(x)** in the interval [A, B]. Write the area between the line and the x-axis in terms of a = x(A), the x co-ordinate of A, b = x(B), the x co-ordinate of B and the average value of the function in that interval.

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_____

b. Give an expression for the area between graph of the function and the x-axis in terms of the integral of f(x), in the interval [A, B] equal to. [Use a = x(A), the x co-ordinate of A and b = x(B), the x co-ordinate of B.]

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_____

c. At the point where the two answers above are equal, what is the average value of the function in the interval [A, B] in terms of a, b and the integral of f(x)?

_____

4. Use the formula for the average value of a function $= \dfrac{1}{b-a}\displaystyle\int_a^b f(x)dx$ to find the average value of the function f(x) = x + 1 in the interval [0, 3]?
Check this using the interactive file.

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5. Find the average value of the function f(x) = x+2 in the interval [0, 3].

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_____

6. Given that the average value of the function f(x) = ax is equal to 15 in the interval [2, 8], find the value of a.

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7. Given that the average value of a linear function f(x) = 2x+ 2 is 12 in the interval

[0, b], find the value of b.

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