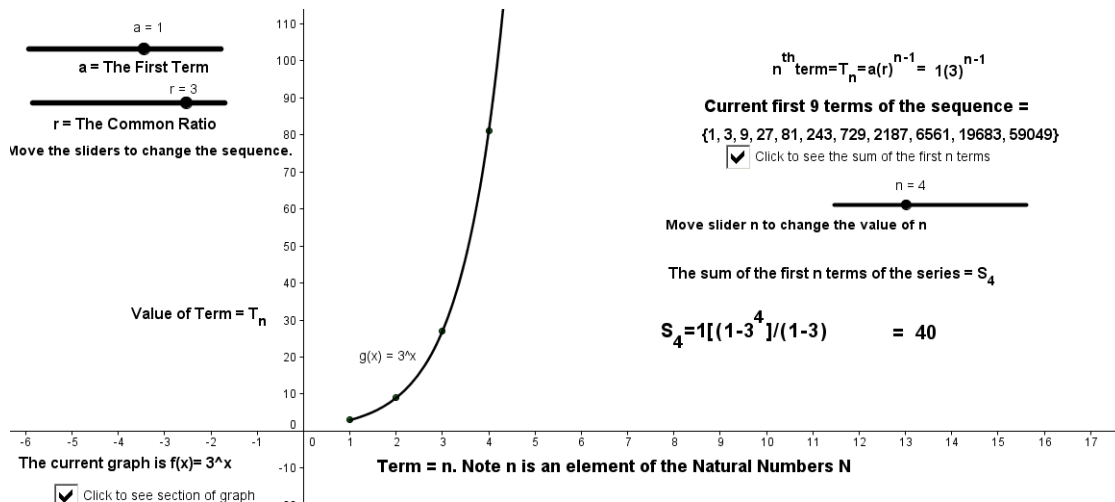# Student Activity: To investigate geometric sequences and series

Use in connection with the interactive file, 'Geometric Sequence and Series', on the Student's CD.

a = 1

a = The First Term

r = 3

r = The Common Ratio

Move the sliders to change the sequence.

Value of Term = T_n

g(x) = 3^x

The current graph is f(x)= 3^x

☑ Click to see section of graph

$n^{th} \text{term} = T_n = a(r)^{n-1} = 1(3)^{n-1}$

Current first 9 terms of the sequence =

{1, 3, 9, 27, 81, 243, 729, 2187, 6561, 19683, 59049}

☑ Click to see the sum of the first n terms

n = 4

Move slider n to change the value of n

The sum of the first n terms of the series = S_4

$S_4 = 1[(1-3^4)]/(1-3)$ = 40

Term = n. Note n is an element of the Natural Numbers N

1. Consider the sequence 1, 2, 4, 8, 16.......

   What is the first term of this sequence? How is each term related to the one before it? What type of sequence is this?

   _____

   _____

2. The first term, "a", of a geometric sequence is equal to 3 and the common ratio, "r", of the sequence is 2.

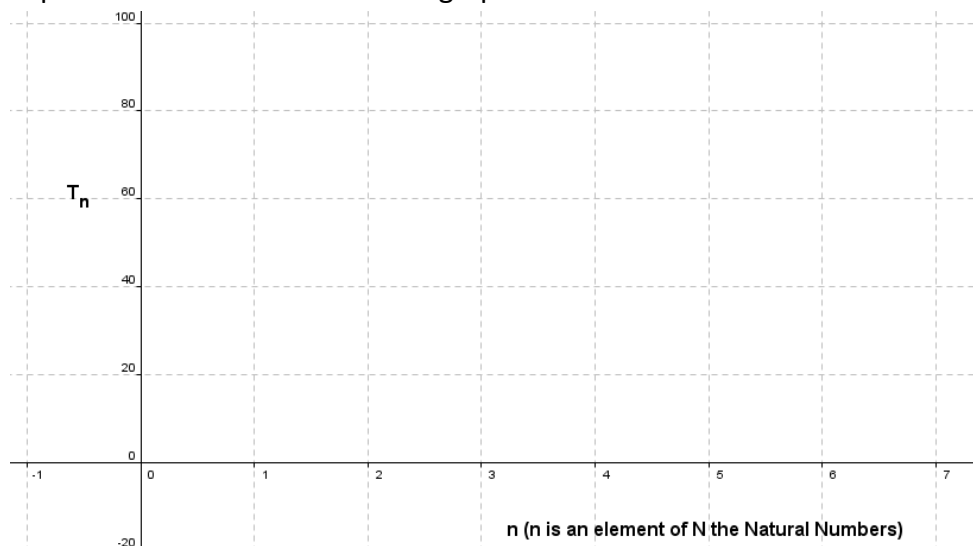   a. Write down the first 6 terms. Show your calculations.

   _____

   _____

   _____

   b. Represent all these terms on the graph below.

T_n

n (n is an element of N the Natural Numbers)

c. Complete the following table using the geometric sequence in the graph and from your data and the table above determine if the pattern formed is linear, quadratic, exponential or none of these.

n	n^{th} term	Change	Change of the change	Ratio of $(n+1)^{th}$ term to the previous term
1				
2				
3				
4				
5				

The pattern formed is_____

3. The first term of a geometric sequence is equal to a and the common ratio of the sequence is r.

　　a. Write down the first 6 terms in terms of a and r.

　　_____

　　_____

　　b. From the pattern of the terms in a above, determine a formula for T_n, the nth term of a geometric sequence.

　　_____

　　_____

4. What is the ratio between T_n and T_{n+1} of any geometric sequence where T_n is its nth term and T_{n+1} is its (n+1)th term?

_____

_____

5. When, if ever, is T_n less than T_{n+1}, in a geometric sequence?

_____

6. What is the difference between a geometric sequence and geometric series?

_____

_____

$$S_n = a\left(\frac{1-r^n}{1-r}\right)$$

7. Using the formula ，find the sum of the first 10 terms of a geometric series given a =2 and r=3. Check this using your calculator.

_____

_____

_____

_____

8. Given that a geometric series is a+ar+ar^2+ar^3+.........+ar^{n-1}, prove that S$_n$, the sum of the

first n terms of a geometric series, is equal to $S_n = a\left(\dfrac{1-r^n}{1-r}\right)$ If necessary check the file

'Proof of the Sum of Geometric Series'.

_____

_____

_____

_____

_____

_____

_____

_____

9. If you toss a coin once you get 2 possible outcomes, toss it twice you get 4 possible outcomes and toss it three times you get 8 possible outcomes. If this pattern continues, how many outcomes would one get for 20 tosses? Show your calculations.

_____

_____

_____

_____

_____

10. An antique costing €4,000 appreciates each year by 6% of the value it was at the

beginning of that year. Find its value after 11 years. Show your calculations.

_____

_____

_____

_____

_____

11. The birth rate in an area is increasing by 6% per annum. If 20,000 children were born in this area in 2009
   a) How many children will be born in 2018 in this area?

   _____

   b) How many children in total will be born in this area between the years 2009 to 2018 inclusive?

_____