## Student Activity: To investigate polynomials

Use in connection with the interactive file, 'Polynomials', on the Student's CD.
1)

a) How many times does this graph cut the $x$ axis?
b) Where does this graph cut the x axis? What does this tell you about these points?
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c) What does this tell you about the $y$ values at these points?
$\qquad$
d) Write the equation of this function in the form $f(x)=(x-a)(x-b)$.
e) Are the y values in this function positive or negative when x is greater than 3 ? Explain.
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$\qquad$
f) Are the $y$ values in this function positive or negative between 1 and 3? Explain your answer.
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$\qquad$
g) Using the interactive file determine how $g(x)=-f(x)$ differs from $f(x)=(x-a)(x-b)$.
h) What is the equation of the polynomial represented by the graph below?

2)

a) Given the points $A(a, 0)$ and $B(b, 0)$ as shown in the diagram above, find the equation of the function represented in the diagram.
b) Where are the $y$ values increasing in this diagram?
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$\qquad$
c) Where are the $y$ values decreasing in this diagram?
$\qquad$
$\qquad$
3) Given the polynomial is $f(x)=(x-3)(x-4)(x-5)$, answer the following questions.
a) What are the roots of $f(x)=0$ ?
b)
i) For $x=5$, what is the value of $f(x)$ ?
ii) For $x=3$, what is the value of $f(x)$ ?
iii) For $x=4$, what is the value of $f(x)$ ?
iv) Plot these 3 points as part of the graph of $f(x)$.

c) F or $\mathrm{x}>5$,
i) What is the sign of $(x-3)$ ?
ii) What is the sign of $(x-4)$ ?
iii) What is the sign of $(x-5)$ ?
$\qquad$
iv) Hence what is the sign of $f(x)=(x-3)(x-4)(x-5)$ for $x>5$ ?
v) Complete the graph already started above of $f(x)=(x-3)(x-4)(x-5)$, for $x>5$.
vi) Using the same reasoning as above what is the sign of $f(x)$ for $4<x<5$ ?
d)
i) What is the sign of $f(x)$ for $3<x<4$ ?
ii) What is the sign of $f(x)$ for $x<3$ ?
iii) Complete the graph already started above of $f(x)=(x-3)(x-4)(x-5)$, for all values of $x$.
4) Given the polynomial is $g(x)=(x-2)(x-3)(x-4)$, answer the following questions.
a) What are the roots of $g(x)=0$ ?
$\qquad$
b) What is the largest root $r$ of $g(x)$ ?
$\qquad$
c) What is the value of $g(r)$ where $r$ is the largest root of $g(x)=0$ ?
$\qquad$
d) What is $\mathrm{r}+1$ and what value has $\mathrm{g}(\mathrm{r}+1)$ ?
$\qquad$
e) Is $g(r+1)$ positive or negative?
f) What is the sign of $g(x)$ when $x>r$ ?
$\qquad$
g) Hence is the graph of the polynomial $g(x)$ when $x>r$ increasing or decreasing?
h) Which of the following is the shape of the polynomial when $x>4$ ?

i) Complete a sketch of $\mathrm{y}=\mathrm{g}(\mathrm{x})=(\mathrm{x}-2)(\mathrm{x}-3)(\mathrm{x}-4)$. Note it is only a sketch and exact heights are not required.

j) Using the interactive file determine the shape of the graph $h(x)=-g(x)$ ?

5) Given the polynomial is $g(x)=(x-2)(x-1)(x-4)$, answer the following questions.
a) What are the roots of $g(x)=0$ ?
b) What is the largest root r of $\mathrm{g}(\mathrm{x})$ ?
c) What is the value of $g(r)$ where $r$ is the largest root of $g(x)=0$ ?
d) What is $\mathrm{r}+1$ and what value has $\mathrm{g}(\mathrm{r}+1)$ ?
e) Is $g(r+1)$ positive or negative?
f) What is the sign of $g(x)$ when $x>r$ ?
g) Hence is the graph of the polynomial $g(x)$ when $x>r$ increasing or decreasing?
h) What is the shape of the polynomial when $x>4$ ?
i) Complete a sketch of $\mathrm{y}=\mathrm{g}(\mathrm{x})=(\mathrm{x}-2)(\mathrm{x}-1)(\mathrm{x}-4)$. Note it is only a sketch and exact heights are not required.

j) Using the interactive file determine the shape of the graph $h(x)=-g(x)$ ?

6) Given the polynomial is $g(x)=(x+2)(x+1)(x-2)$, answer the following questions.
a) What are the roots of $g(x)=0$ ?
b) What is the largest root r of $\mathrm{g}(\mathrm{x})$ ?
c) What is the value of $\mathrm{g}(\mathrm{r})$ where r is the largest root of $\mathrm{g}(\mathrm{x})=0$ ?
d) What is $\mathrm{r}+1$ and what value has $\mathrm{g}(\mathrm{r}+1)$ ?
e) Is $g(r+1)$ positive or negative?
f) What is the sign of $g(x)$ when $x>r$ ?
$\qquad$
$\qquad$
g) Hence is the graph of the polynomial $g(x)$ when $x>r$ increasing or decreasing?
$\qquad$
h) What is the shape of the polynomial when $x>2$ ?
$\qquad$
$\qquad$
i) Complete a sketch of $\mathrm{y}=\mathrm{g}(\mathrm{x})=(\mathrm{x}+2)(\mathrm{x}+1)(\mathrm{x}-2)$. Note it is only a sketch and exact heights are not required.

j) Using the interactive file determine the shape of the graph $h(x)=-g(x)$ ?

7) Using the interactive file determine the shape of the graph that represent $s(x)=(x-a)(x-b)$, when both $a$ and $b$ are equal to 2 . Sketch the graph.

a. When $x>2$ what is the sign of $y=s(x)$ ?
$\qquad$
b. When $\mathrm{x}<2$ what is the sign of $\mathrm{y}=\mathrm{s}(\mathrm{x})$ ?
8) With the help of the interactive file sketch the graph of the function

$$
f(x)=(x-2)(x-2)(x-3)(x-4)=(x-2)^{2}(x-3)(x-4)
$$


a. Is this graph positive or negative for $x>4$ ?
b. What happens to this graph at $x=4$ ?
c. What happens to this graph between 3 and 4?
$\qquad$
d. What happens to this graph at 3?
e. What happens to this graph between 2 and 3 ?
f. What happens to this graph at 2?
g. What happens to this graph between 1 and 2?
h. Using the interactive file determine the shape of the graph $h(x)=-g(x)$ ?

9) Sketch the graph of the polynomial $p(x)=(x-1)^{2}(x-2)(x-3)$.

10) Sketch the graph of the polynomial $p(x)=(x-1)(x-2)^{2}(x-3)$.


a. Where does this graph cut the x axis? What does this tell you about these points?
b. What is the equation of this graph?
c. For what values of x is this graph positive and increasing?
$\qquad$
d. For what values of $x$ is this graph positive and decreasing?
e. For what values of x is this graph negative and increasing?
f. For what values of x is this graph negative and decreasing?
12)

a. Where does this graph cut the x axis? What does this tell you about these points?
b. How many times does this graph cut the $x$ axis?
c. For what values of $x$ is this graph positive and increasing?
$\qquad$
$\qquad$
d. For what values of x is this graph positive and decreasing?
$\qquad$
$\qquad$
e. For what values of x is this graph negative and increasing?
$\qquad$
$\qquad$
f. For what values of x is this graph negative and decreasing?
$\qquad$
$\qquad$
g. What is the equation of this graph? Check your answer using the interactive file.
13)
a. Sketch the graph $f(x)=(x-2)(x-2)(x-2)(x-1)(x-4)=(x-2)^{3}(x-1)(x-4)$.

b. What happens to the graph at $\mathrm{x}=2$ ?
14) Sketch the graph $g(x)=(x+2)(x-2)(x-2)(x+1)(x-4)=(x+2)(x-2)^{2}(x+1)(x-4)$.

15) What is the equation of the polynomial where the highest power of $x$ is 5 , represented by the following graph?

16) What is the equation of the polynomial where the highest power of $x$ is 7 , represented by the following graph?


