ICT & MATHS

Module 1

GeoGebra
for Synthetic Geometry

Published by

The National Centre for Technology in Education in association with
the Project Maths Development Team.

GeoGebra for Synthetic Geometry

Published by:

The National Centre for Technology in Education
Dublin City University
Glasnevin
Dublin 9
Tel: +353 1 700 8200

Email: info@ncte.ie
Web: www.ncte.ie / www.scoilnet.ie

Project Maths Development Team
Foireann Forbatha Thionscadal Mata
Drumcondra Education Centre
Drumcondra
Dublin 9
Email: grainneh@ecdrumcondra.ie
Web: www.projectmaths.ie

Please note

- Screenshots used in this manual may appear different from those on computer screens used by participants; variations in versions of the software and differing operating systems may be in use.
- The World Wide Web is constantly evolving and content and URLs (Universal Resource Locators - website addresses) change over time. It is possible that the content located at some of the URLs listed throughout this manual may change over time.
- Screenshots and software titles used throughout the manual are from a PC using Windows Vista©.
- Participants using other operating systems may encounter some differences in screen presentation and layout.

Throughout this module reference may be made to software titles and suppliers of Internet services. These references are made purely to illustrate or expound course content. Any such reference does not imply any endorsement by the NCTE of a product or company. The reader should be aware that typically there are many products and companies providing similar services in areas related to ICT. Participants should be as informed as possible before making decisions on purchases of ICT products or services.

Foreword

The introduction of the Project Maths syllabuses provides an opportunity for mathematics teachers to integrate ICT into the teaching and learning in their classrooms. With this in mind support will be provided in education centres to upskill teachers in the uses of two software packages, Excel for Strand 1 and GeoGebra for Strand 2 of the Project Maths syllabuses.

A series of professional development courses, provided by skilled ICT Tutors in the Education Centres, will consist of a series of five two and a half hour modules. Three of these modules will deal with GeoGebra and two will deal with Excel. There will also be an opportunity for participants to share resource materials developed during the course.

The NCTE and the Project Maths Development are working closely together to provide a course which will meet the needs of mathematics teachers with all levels of ICT skill. Application for a place on the course will be through the Education Centres and it is envisaged that the full course will be covered but the pace of progress will be determined by the skill level of the participants.

Duration

2.5 hours

Objectives

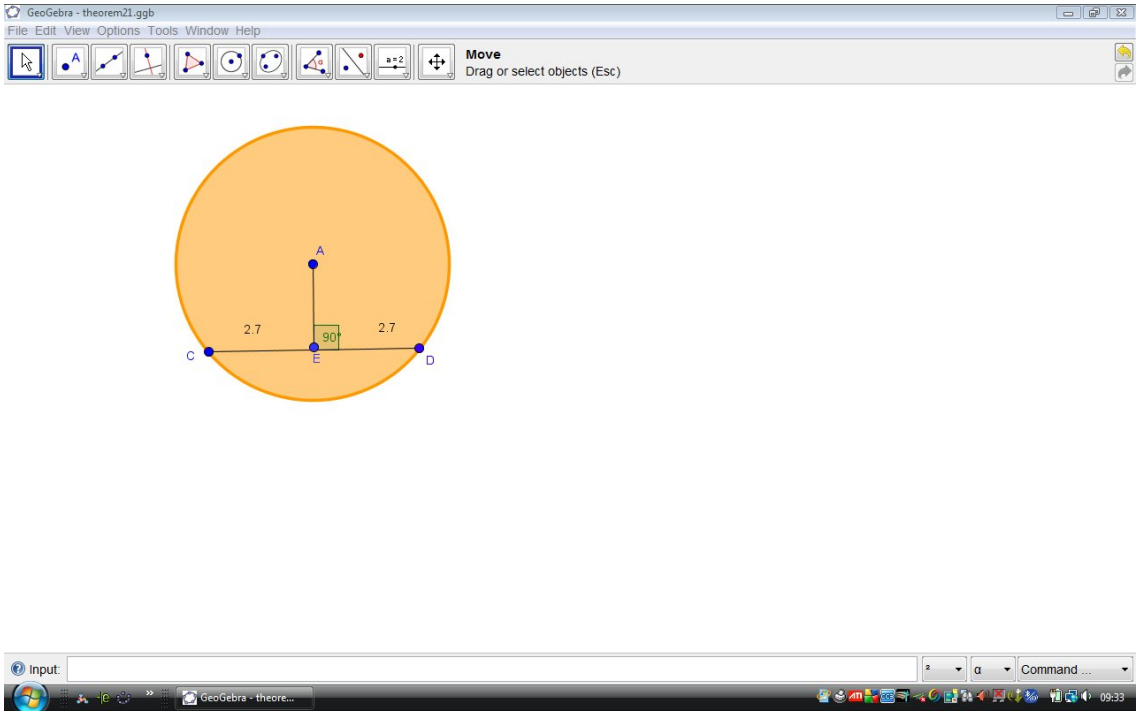
This module aims to enable the participant to:

Objectives

This module aims to enable the participant to:

- download and install GeoGebra from http://www.geogebra.org/cms/
- download and install Java from http://www.java.com/en/
- become familiar with the toolbar on the GeoGebra interface
- show or remove the Algebra View and the Input Bar
- select items on a sketch
- save GeoGebra files in various formats
- construct an interactive version of a theorem
- draw vectors
- create various types of polygons
- demonstrate reflections in a point and a line
- create a slider
- become familiar with the use of a Check box
- draw, change colour and style of a quadratic graph using the Input bar
- familiarise themselves with the Help menu
- show an enlargement with a scale factor of k, where $0 < k < 5$
- construct the centroid of a triangle
- construct the circumcircle and incircle of a triangle

GeoGebra Overview



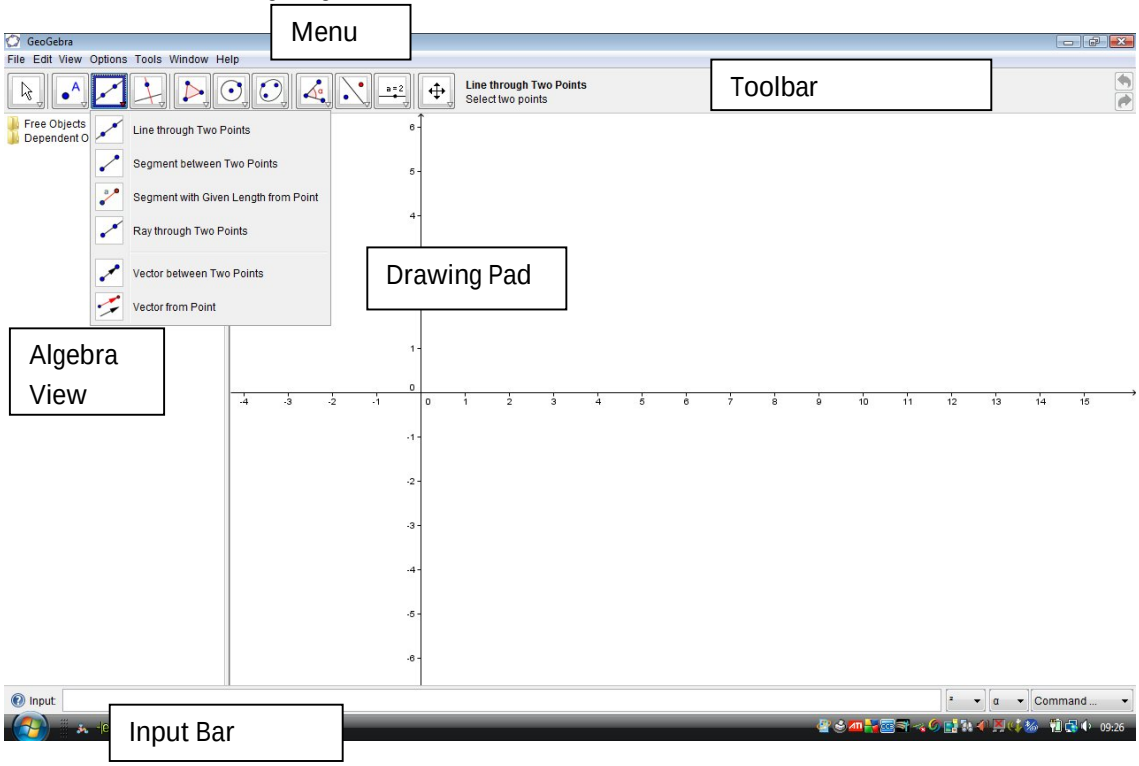
GeoGebra

The GeoGebra software package is available free at http://www.geogebra.org/cms/. It is recommended that the Download GeoGebra version be used, rather than the GeoGebra Webstart version. See Appendix A.

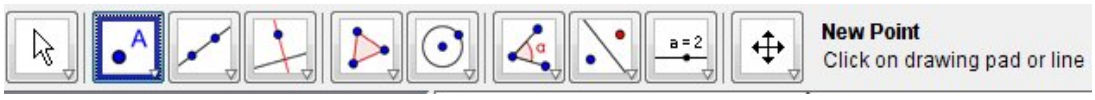
These notes and screenshots are based on Version 3.2.38.0.

Note in order for GeoGebra to work on a computer, one also needs the latest version of Java; this is available free at http://www.java.com/en/ . See Appendix B.

Interface: The following diagram demonstrates the GeoGebra interface.

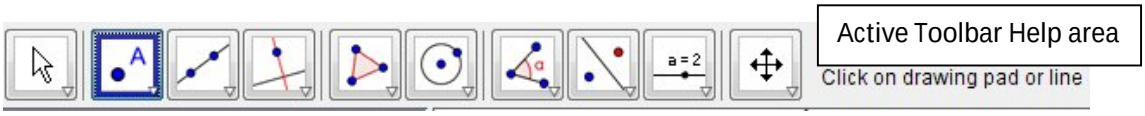


Toolbar: The Construction tools, for example, the New Point tool  are found on the Toolbar.



By following the arrows underneath each section of the toolbar, for example the arrow beside the New Point tool  , alternative tools can be found in the drop down menu. See Appendix C for a complete list of these.

The Active Toolbar Help area shows the tool that is currently open and the instructions to be followed when using that tool are also displayed in this area.



Note: once a tool has been used that tool remains as the visible one on the toolbar.

Algebra View: To show or remove the Algebra View, go to View and click Algebra View.

Input Bar: This bar enables one to input formulas and commands for constructions. To show or remove the Input Bar, go to View and click Input Bar.

Selecting items: In order to select anything in GeoGebra, click on the Move tool  and next, click on the item.

To undo the last action: Go to Edit and click Undo. (Note this can only be done until the last save) or click the Undo button  at the top right hand side of the screen.

To zoom in and out of a construction: Use either the Zoom In tool  or the Zoom Out tool  .

To change the appearance of an object: Right click on the object for example a point, choose Object Properties and make the required changes using the various options and tabs available.

To change Font Size

The default Font Size is 12, but this can be changed by going to Options, Font Size and choose the Font Size of your choice. The larger Font Sizes are often used for ease of viewing.

To change Point Style

The default Point Style is  , but this can be changed by going to Options, Point Style and choosing the Point Style of your choice.

To change the number of Decimal Places or Significant Figures displayed on the screen

Go to Options, Rounding and choose the number of Decimal Places or Significant Figures required.

To save Settings

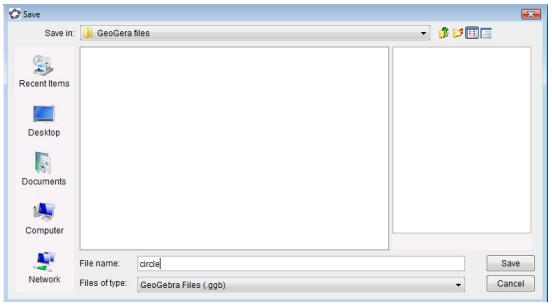
When first used Font Size, Point Style etc, are set to Default Settings, but if one wishes to apply their changes to all future usage of these settings, go to Options and click Save Settings.

To Restore Default Settings

Go to Options and click Restore Default Settings.

How to save a file as a Geogebra file:

1. Go to the File Menu and choose Save as. A new dialogue box appears.



2. For Save in, select the folder that you wish the folder to be saved in.

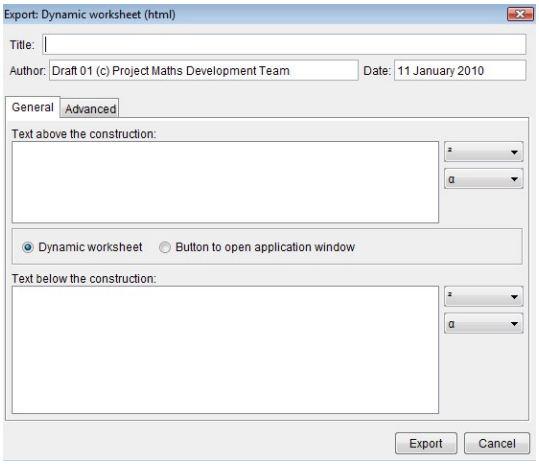
3. For File name select the name you wish to give your file.

4. Click Save. A file with the .ggb extension is created. This extension identifies the file as a GeoGebra file and it can be opened again and adjustments made using the GeoGebra application.

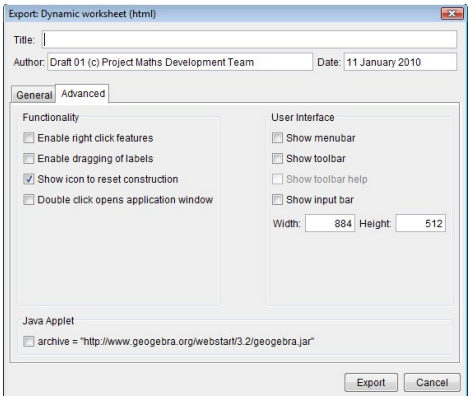
How to save a file as an interactive web page:

1. First save your file as a GeoGebra file.

2. Go to File, Export Dynamic Worksheet as Webpage (html). A new dialogue box appears.



3. Add Title, Author, etc. Click on the Advanced tab and a new dialogue box appears.



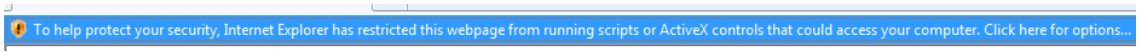
It is recommended that one clicks the Show Icon to reset construction button and sets the width and height to appropriate values for example width 1100 and height 500.

4. Click Export.

5. Select the location to Save your file and give it the name of your choice. (Note this file will have a .html extension and will be viewed using an Internet browser such as Internet Explorer 8)

6. Click Save and your default Internet browser will open.

7. When the webpage comes up you the following message will normally appear at the top of the screen.

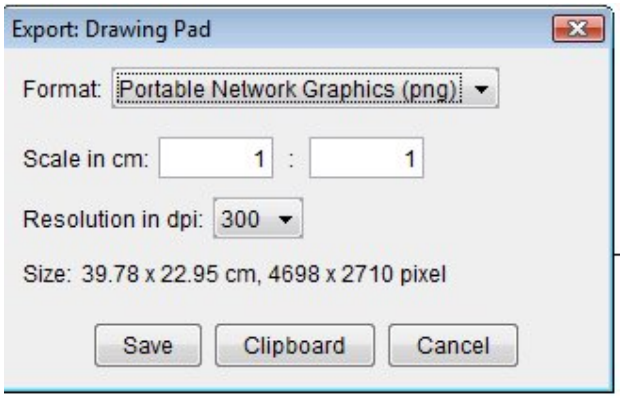


8. Click on Click here for options and choose Allow Blocked Content... and click Yes.

How to save a file as a picture:

1. First save your file as a GeoGebra file.

2. Go to File, Export and Graphics View as Picture (Png, eps). A new dialogue box appears.



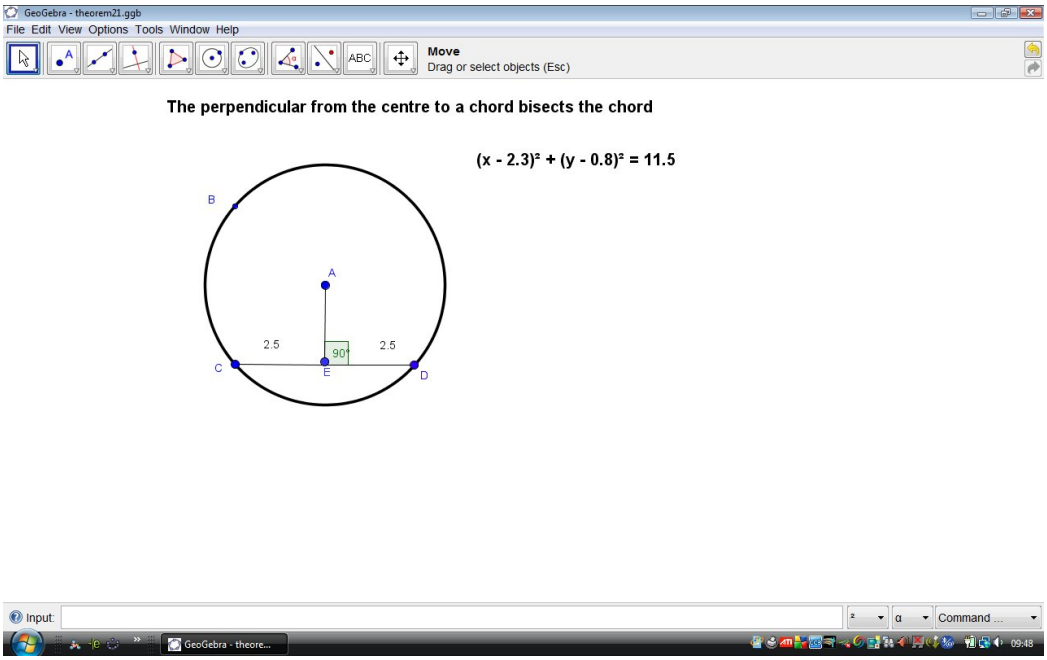
3. Choose the Format, Scale in cm and Resolution.

4. Click Save. (Note if you choose Portable Network Graphics (png) this file will have a .png extension. Other formats will give different extensions).

5. Choose the location you wish to Save in and File name.

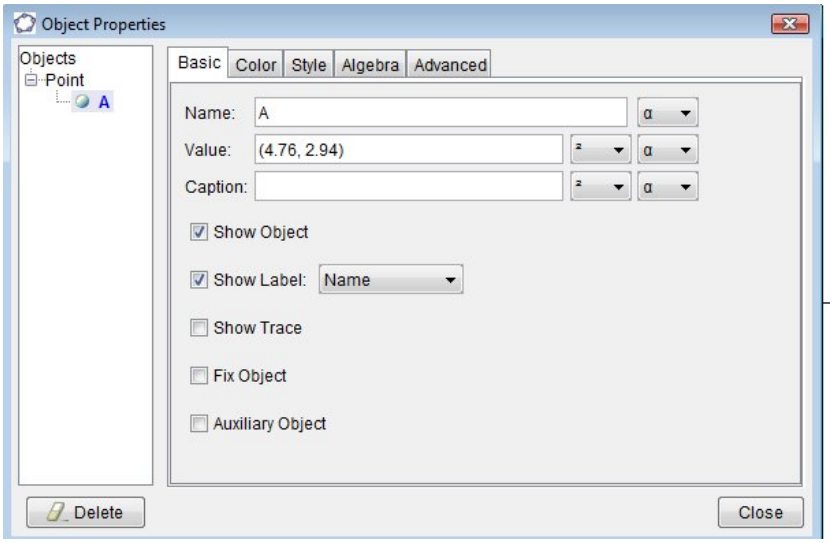
6. Click Save. This picture can now be used in other applications for example Microsoft Word.

To construct an interactive version of the theorem "The perpendicular from the centre to a chord bisects the chord".



To remove or add the axes, go to View and click Axes.

1. Construct 2 points A and B. (To construct a point, click on the New Point tool  and click on the drawing pad at the point you want to draw that particular point.)
   Note: if the label for a point is not visible, right click on the point, choose Object Properties and with the Basic tab open click the show Label box and make sure it is the Name option that is visible. In some versions of GeoGebra Object Properties is replaced by Properties.

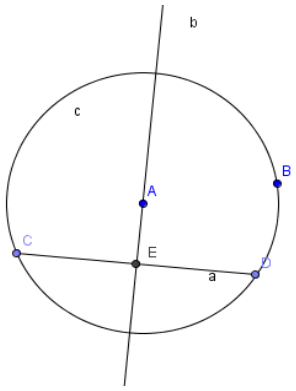


2. Construct a circle with its centre at one of your points and the other point on the circumference. (To draw a circle with centre through a point, click on the Circle with Centre through a Point tool  , click the point you wish to be centre of the circle and click the point you want to be on the circumference in that order).

3. Construct 2 points C and D on the circle that you wish to form the extremities of the chord. Note the circle is highlighted when the curser is exactly on the circle.

4. Construct the line segment that forms the chord CD. (To construct a line segment go to the Segment between Two Points tool  and click on the two points that will form the extremities of the line segment).

5. Construct a perpendicular line from the centre of the circle to the chord CD. (To construct a perpendicular line to a line segment from a point. Click on the Perpendicular Line tool  then click on the point and the line segment.

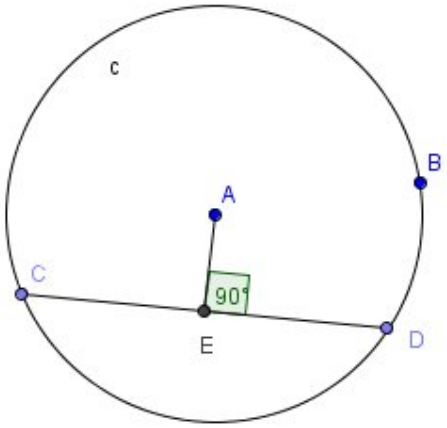


6. Mark the point of intersection of the line and the chord. (To mark the intersection of two items, click on Intersect Two Objects tool  and click the items).

7. You may wish to hide the perpendicular line and replace it with a perpendicular line segment. (To hide something, click on the item, right click and unclick Show Object).

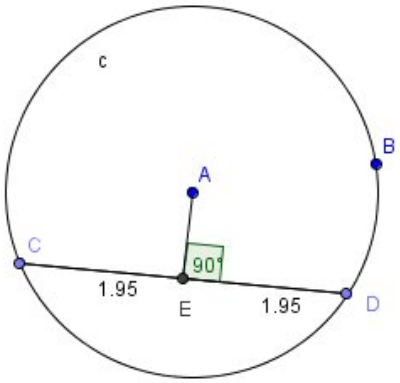
8. Draw the line segments AE, CE and ED.

9. Draw the angle AED. (To construct an angle, click on the Angle tool  and then click on the points that form the angle, making sure that if the angle is at E, then E is the second point clicked).



10. Show the angle as non reflex angles. (To show the angles as non reflex angles, right click on the angle, go to Object Properties and with the Basic tab open unclick the Allow Reflex Angle button and click Close.)

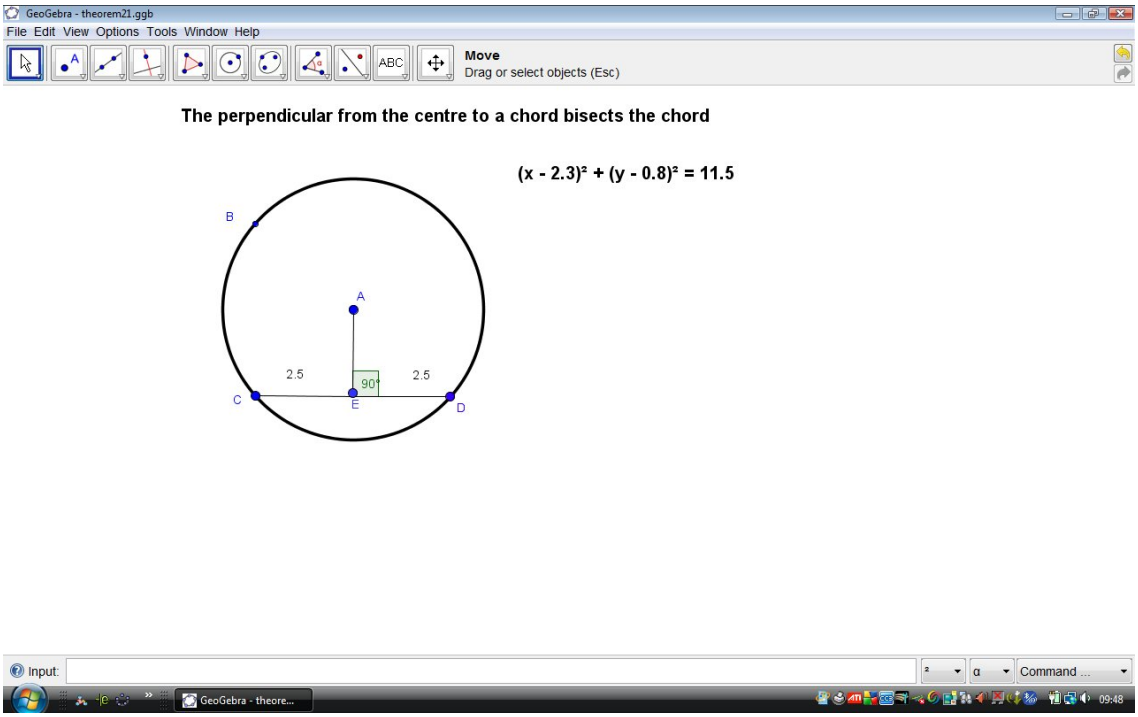
11. Show the lengths of CE and ED. (To show the length of a line segment, right click on the segment choose Object Properties and with the Basic tab open, for Show Label choose Value and click Close).



12. Add a textbox containing the name of the theorem. (To add a Textbox, click on the Insert Text tool  and click on the screen at the position that you wish to add your textbox. A new dialogue box will now appear, type your text in this box and click OK).

13. Add a textbox, with the equation of the circle in it. (Click on the Insert Text tool 

and click on the position on the screen that you wish to add your textbox. A new dialogue box appears, click on the circle and click OK).



14. To change the appearance of a textbox: Right click on the textbox. Go to Object Properties. With the Text tab open choose B for bold text and change the text size or font. Click OK. With the Color tab open you can change the colour of the text. Click Close.

15. Add a colour of your choice to the drawing pad. (Go to Options, Drawing Pad and click on the box beside Background color. Pick your colour and click OK and Close).

16. To start a new file, click on File, New.

Extras

To draw a vector between two points: Create two points. Go to the Vector between Two Points tool  , click on the point you want the vector to start from and then click on the point you wish the vector to finish at.

To create a polygon shape: Draw 3 or more points. Go to the Polygon tool  and click on the points that are to form the polygon, not forgetting to re-click the first point.

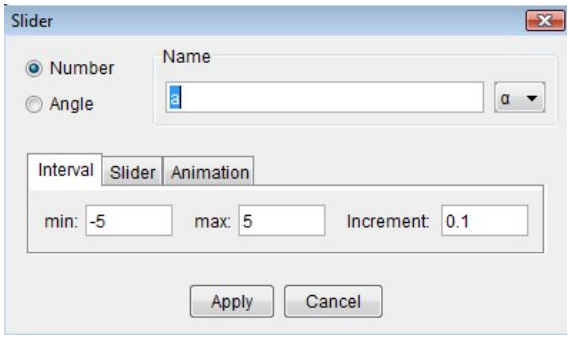
To demonstrate a Reflection in a line: Create a polygon and a line, if using a line other than the X or Y axis. Go to the Mirror Object about Line tool  and click on the polygon plus the line or the X or Y axis. Now drag the line and or the polygon to different locations and see the image change accordingly.

To demonstrate a reflection in a point: Create the polygon and the point in the usual way. Click on the Mirror Object about Point tool  and click on the polygon and the point. Now drag the point and the polygon to different locations etc. and see the image change accordingly.

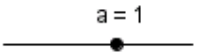
To demonstrate how to use a vector to translate a polygon: Create a polygon in the usual way and create the vector that you wish to use for the translation. Go to the Translate Object by Vector tool  and click on the polygon, followed by the vector. Note both the vector and the polygon can be changed and see how the image changes accordingly.

To create a slider:

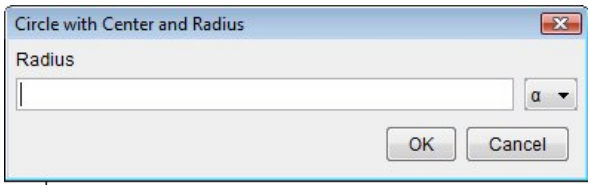
1. Go to the Slider tool  and click on the screen, where you want your slider located. A new dialogue box appears.



2. Choose if you want the slider to be an angle or a number. Give it a name. Choose the min, max values and the increment you want it to go up in. Click Apply. The slider appears as follows on the screen.



3. For example if you want to draw a circle with a varying radius, go to the Circle with Centre and Radius tool  and click on the screen where you want the centre of the circle to be. A new box appears asking you for the Radius.



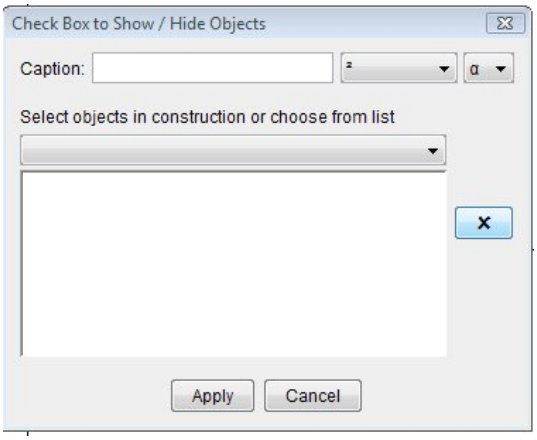
4. Type in the name of the created slider and click OK.

5. Drag the slider and see the size of the circle changing. Sliders created with angle values can be used to create angles with varying size.

To insert an image: Go to Insert Image tool and click at the location on the Drawing Pad where you want to place the image. A new box appears. Select your picture and click Open. Be careful as a lot of pictures are too big and will need to be edited to the size you require using alternative software.

Check Box to Show / Hide objects:

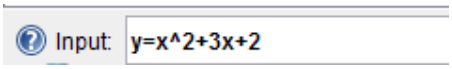
1. Click on the Check Box to Show / Hide Objects and click on the Drawing pad at the location you require the check box to appear. A new dialogue box appears.



2. In the new dialogue box that appears, type in your caption for example "Click to hide or show the circle". Follow the arrow under 'Select objects in construction or choose from list' in the dialogue box and in this case choose the circle. You can select more than one object here. Click Apply.

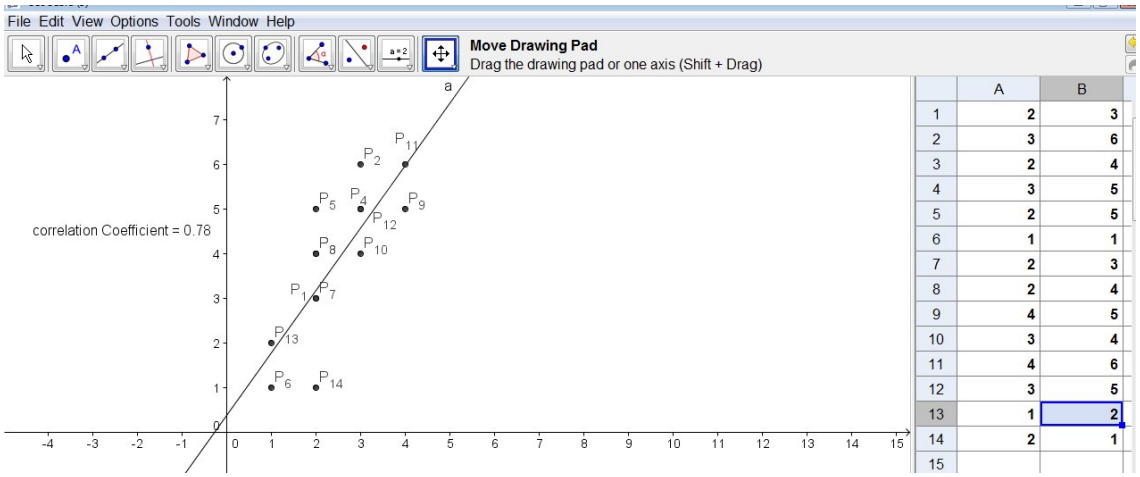
3. With the Selection arrow clicked, click the check box and the circle disappears.

To draw a graph: Go to where it says Input at the bottom of the screen, type in your equation say y=x^2 +3x+2 and press Enter. Note x^2, means x^2.



To change the colour, style etc. of the graph: Click on the graph, right click and choose Object Properties and choose the Colour, Style, etc.

To draw a Scatter Plot and show Correlation



	A	B
1	2	3
2	3	6
3	2	4
4	3	5
5	2	5
6	1	1
7	2	3
8	2	4
9	4	5
10	3	4
11	4	6
12	3	5
13	1	2
14	2	1
15		

1. If the Spreadsheet View is not visible go to View and click Spreadsheet View.

2. Type the following table into the Spreadsheet View.

	A	B
1	2	3
2	3	6
3	2	4
4	3	5
5	2	5
6	1	1
7	2	3
8	2	4
9	4	5
10	3	4
11	4	6
12	3	5
13	1	2
14	2	1
15		

3. Select the cells A2:B14. Right click and select Create List of Points.

4. The name of the list of points, *list1*, or whatever name it has been allocated will now appear in the Algebra View.

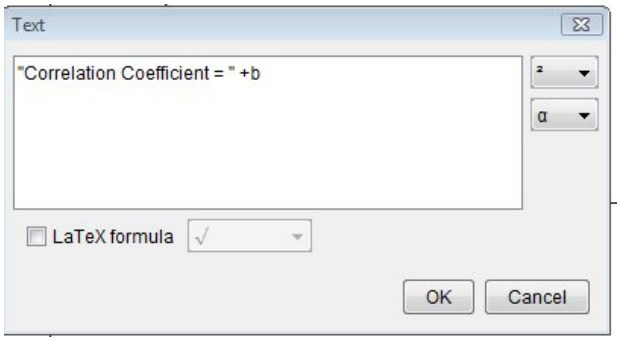
5. To draw the line of Best it, go to the Line of Best Fit Tool  . Drag the cursor to create a rectangle over all the points in the Drawing Pad.

6. Calculate the correlation. (To calculate the correlation, go to the drop down menu at the far right hand side of the Input Bar). Select Correlation Coefficient.

7. Inside the square brackets after Correlation Coefficient type list1 or the name of your list of points. Press Enter. The value for the correlation coefficient will now appear in the Algebra View.

8. To show the correlation coefficient in a textbox. Go to the Insert Text tool 

and click on the Drawing Pad at the location you require the textbox to appear.



9. In the dialogue box that appeared type *"Correlation Coefficient* = "+*b* or replace the b by whatever your correlation coefficient is known as. With the LaTex formula button in the dialogue box unclicked, click OK.

10. Change the values in the Spreadsheet View and see the Scatter Plot, Line of Best Fit and Correlation change.

To Find the Mean, Median and Mode of a Data set

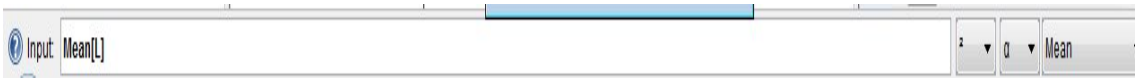
1. Open the Spreadsheet View and insert the data in column A.

	A
1	12
2	13
3	14
4	15
5	16
6	12
7	13
8	14
9	15
10	15

2. Highlight the data in column A. Right click and select Create List.

3. As the list will automatically be called L_1 in the Algebra View, it is easier if one right clicks it in the Algebra View and selects Rename and give it the name L.

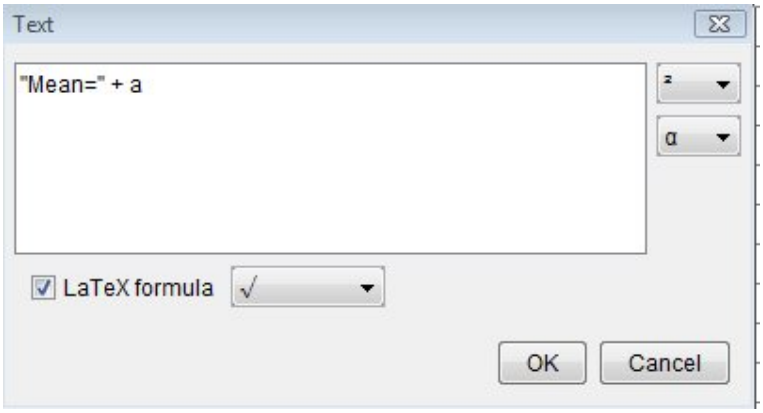
4. To calculate the mean, go to the drop down menu at the far right hand side of the Input Bar and select Mean.



5. Inside the square brackets after Mean type L or the name of your list of points. The value for the mean will now appear in the Algebra View.

6. Go to the Insert Text Tool 

and a new dialogue box appears.



7. Type the text “Mean=”+a in this dialogue box and with LaTeX formula button in the dialogue box unclicked click OK. The following text Mean=13.9 should now appear on the Drawing Pad.

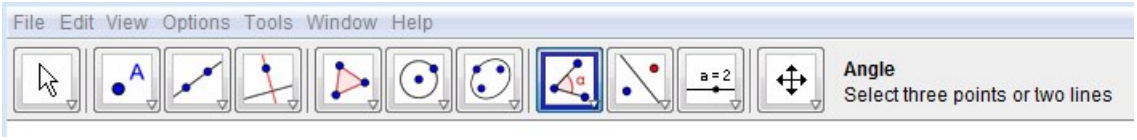
To calculate the Median or Mode of a data set replace Mean in the previous 7 instructions by Median or Mode, whichever is required.

Examples from the Internet:

Go to
http://www.geogebra.org/cms/index.php?option=com_content&task=blogcategory&id=69&Itemid=56 (in particular the Ladder against the Wall example).

http://math247.pbwiki.com/Learn-and-Use-GeoGebra Learn and Use GeoGebra - Step-by-Step help on how to use GeoGebra.

GeoGebra Help Files:

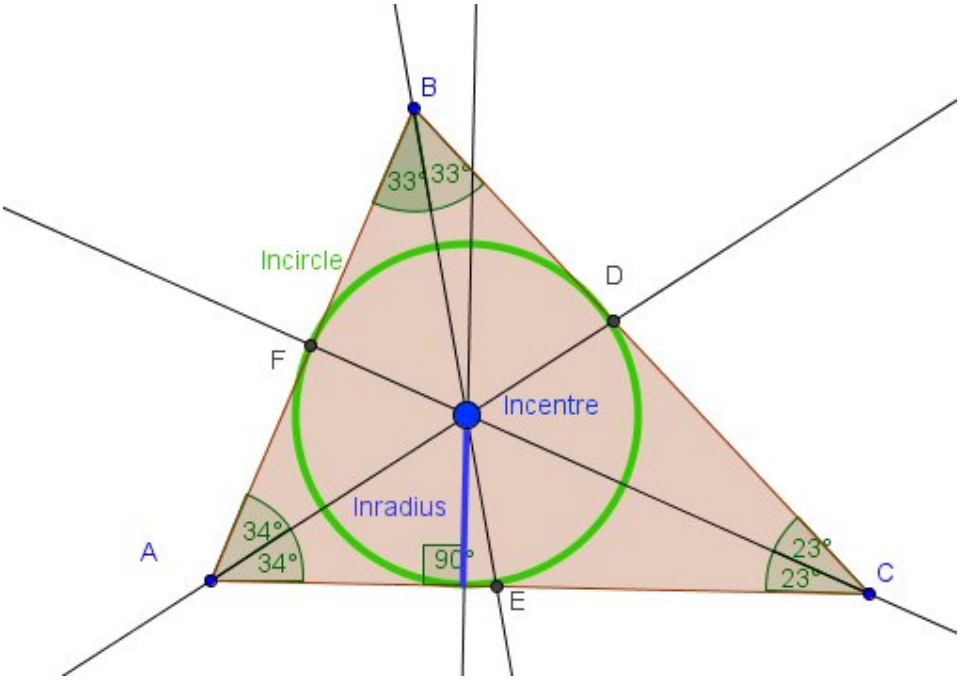


Clicking on Help in the Help menu in GeoGebra will automatically launch your browser and open up the GeoGebra online help documentation. This can be searched by either going through the Contents section or by using the Search function and entering a keyword.

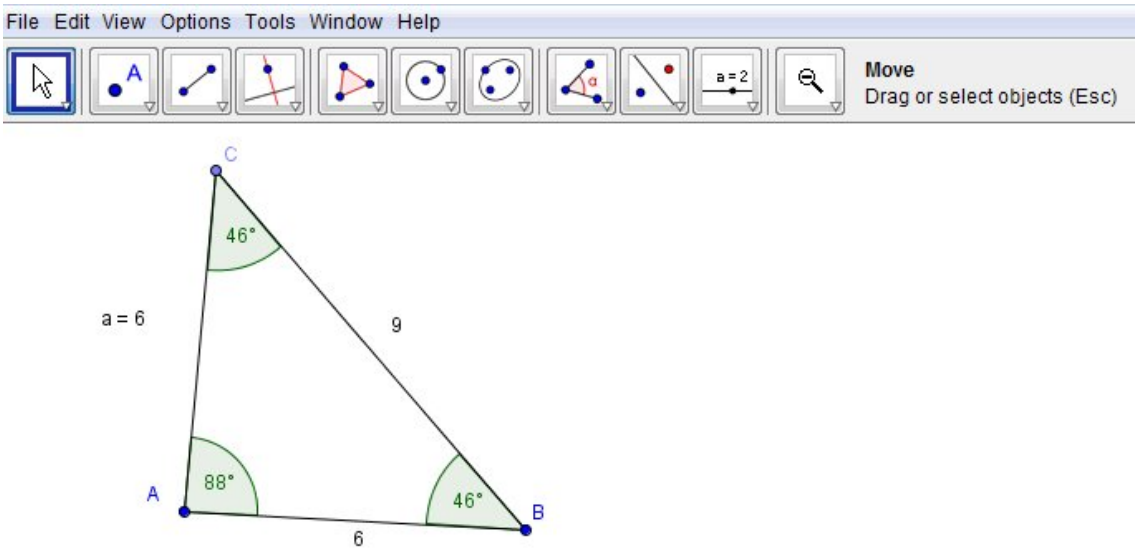
Shortcut keys: See http://www.geogebra.org/help/docuen/index.html?n=161.html

Book: Introduction to GeoGebra by Markus Hohenwarter, Judith Hohenwarter

GeoGebra Sample Constructions



To construct an isosceles triangle

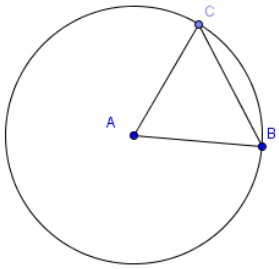


1. Select the Circle with Centre through Point tool  and construct a circle, centre A through point B. If the labels are not showing, right click, select Object Properties and with the Basic tab open, click on Show Label.

2. Select the New Point tool  and construct any point C on the circumference of circle c.

3. Select the Segment between Two Points tool  and construct [AC]

4. Construct [BC] and [AB]



5. Right click on one side of the triangle, select Object Properties, and with the Basic tab open, click on the drop down arrow beside the Show Label box. Select Value to show the name and length of this side of the triangle. Click close. Repeat for the other triangle sides.

6. Drag each vertex of triangle ABC and note the length of its sides.

7. Hide the circle, by right clicking on it and clicking on Show Object.

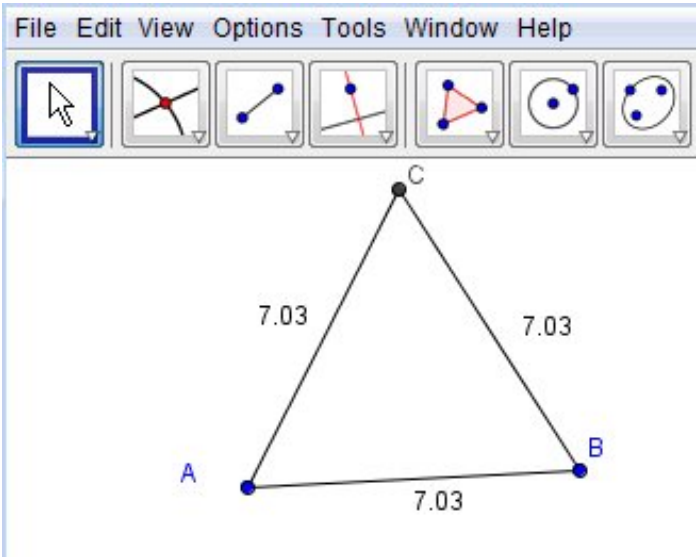
8. Measure the 3 angles in the triangle using the Angle tool  .

9. Drag any of the vertices of the triangle ABC and observe how the angle measures change.

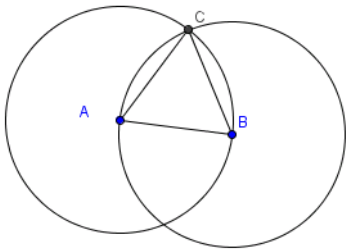
10. Click on File, Save as and choose the location you wish to save your GeoGebra file to. Enter a filename that describes the file content and click on Save.

11. To start a new file, click on File, New.

To construct an Equilateral Triangle

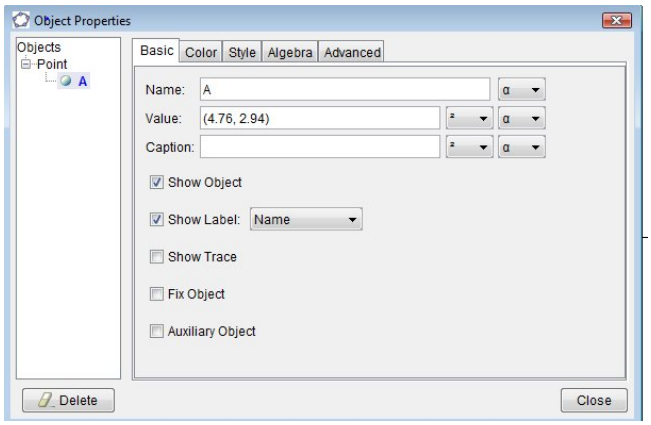


1. Draw two points A and B.
2. Draw the line segment AB.
3. From A draw a circle through B.
4. From B draw a circle through A.
5. Find one intersection C of the two circles.



6. Draw the line segments AC and BC.
7. Hide the circles.

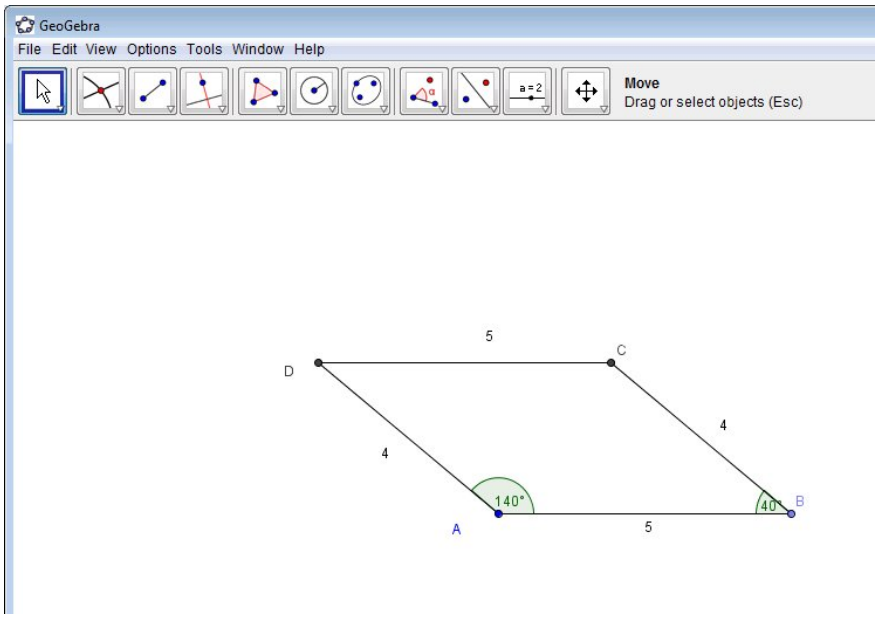
8. Show the lengths of each of the sides of the triangle. (To show the length of a line segment right click on the line segment and choose Object Properties. With the Basic tab open for Show Label choose Value. Click close.)



9. Click on File, Save as and choose the location you wish to save your GeoGebra file to. Enter a filename that describes the file content and click on Save.

10. To start a new file, click on File, New.

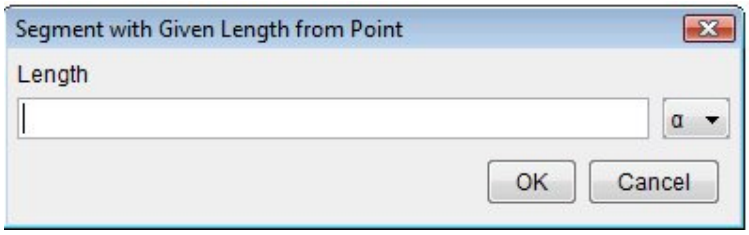
To construct a parallelogram ABCD, where the length of AB = 5 cm, the length of BC=4 cm and the angle ABC is 40^0



Remove the axes, by clicking on View and click Axes.

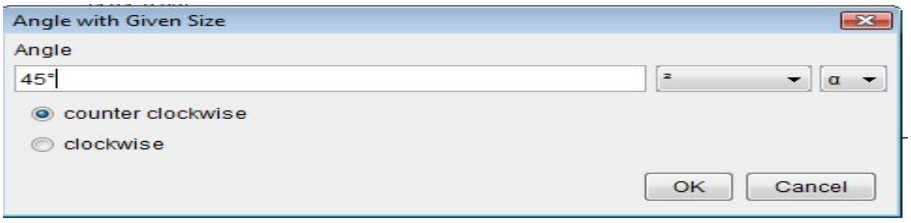
1. Create the point A.

2. Select the Segment with Given Length from Point tool  . Click on A. In the new dialogue box that appears type in the value 5 and click OK.



3. If the labels have not appeared on the points A and B, click on the points, right click and select Show Label.

4. Select the Angle with Given Size tool  . Click on the point A followed by the point B. In the new dialogue box that appears, replace the 45^0 with 40^0 (do not remove the ^{0} or use the ^{0} from the first drop-down menu of the dialogue box) and check that clockwise is selected. Click OK. The new point that appears should be called A'.

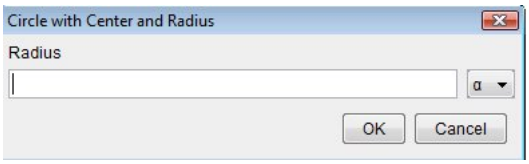


5. Select the Circle with Centre and Radius tool  . Click on the point B and in the new dialogue box insert 4 for the radius. Click OK.

6. Select the Ray through Two Points tool  and click on B followed by A'.
7. Find the point of intersection C of the ray and the circle.

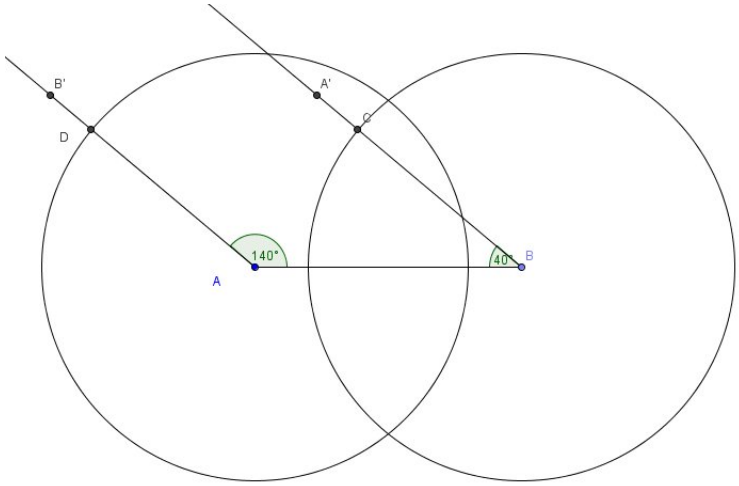
8. Select the Angle with Given Size tool  . Click on the point B followed by the point A. In the new dialogue box that appears, replace the 45^0 with 140^0 and check that counter clockwise is selected. Click OK. The new point that appears should be called B'.

9. Select the Circle with Centre and Radius tool  , click the point A and in the new dialogue box insert 4 for the radius. Click OK.



10. Select the Ray through Two Points tool  and click on A followed by B'.

Find the point of intersection C of the ray and the circle formed in No 9 and No 10.



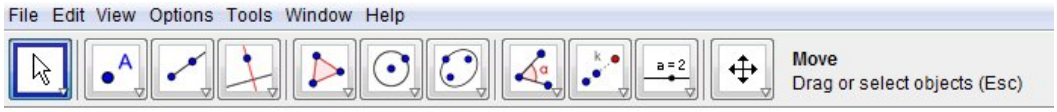
11. Create the line segment DC.

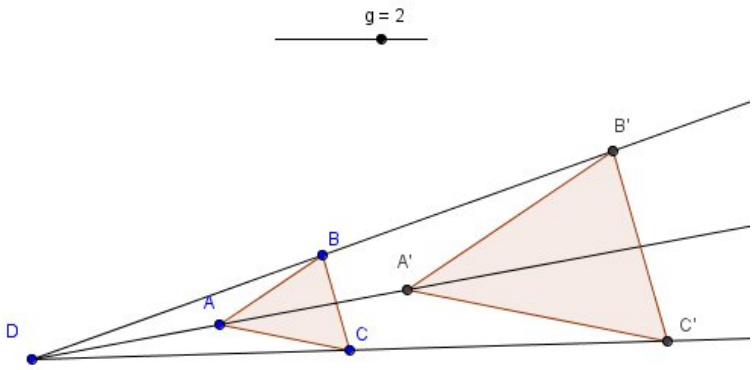
12. Hide unnecessary objects (the two circles and the remaining lines outside the parallelogram from points C and D). (To hide an object, right click on it and click on Show Object.) Draw line segments as required to give the parallelogram ABCD.

13. Click on File, Save as and choose the location you wish to save your GeoGebra file to. Enter a filename that describes the file content and click on Save.

14. To start a new file, click on File, New.

To demonstrate the enlargement of a triangle by the ray method, where the scale factor k is greater than 0

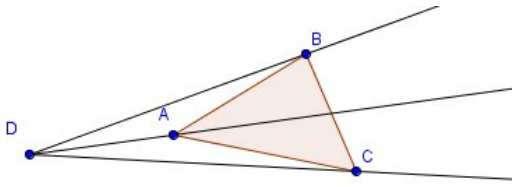




1. Create the polygon ABC in the shape of a small triangle.

2. Create the point D, to the left of A.

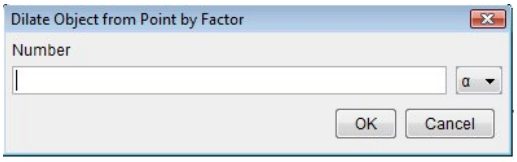
3. Draw rays through DA, DB and DC.



4. Create a slider g with min value 0, max 5 and increment 0.1 and click on Apply.

5. Set the slider to 2.

6. Select the Dilate Object from Point by Factor tool  . Click on the polygon that forms the triangle. Click on point D and in the new dialogue box that appears type in the name of the slider. Click OK.



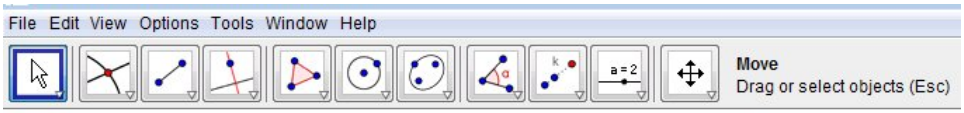
7. Move the original polygon to enable the enlargement to be demonstrated clearly.

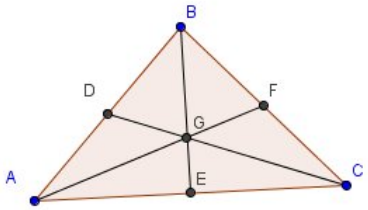
8. Click on File, Save as and choose the location you wish to save your GeoGebra file to. Enter a filename that describes the file content and click on Save.

9. To start a new file, click on File, New.

Constructing Medians and constructing the Centroid of a triangle

(A median is a line segment connecting any vertex of a triangle to the midpoint of the opposite side)





1. Click on File and select New Window.

2. Draw a triangle using the Polygon tool  as above, and using the Midpoint or Centre tool  and Segment between Two Points tool  , construct the medians of each side of the triangle.

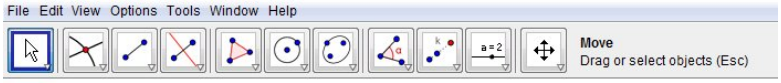
3. Construct the intersection of the medians by selecting the Intersect Two Objects tool  .

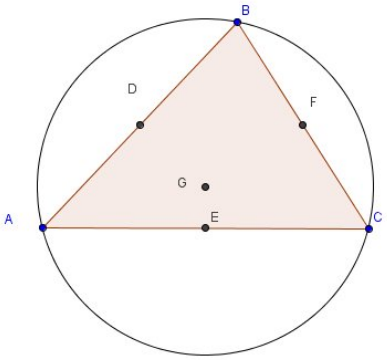
4. Drag any of the vertices of the triangle and note that the 3 medians remain concurrent, at the CENTROID.

5. Click on File, Save as and choose the location you wish to save your GeoGebra file to. Enter a filename that describes the file content and click on Save.

6. To start a new file, click on File, New.

Constructing Mediators and constructing the circumcentre and circumcircle of a triangle





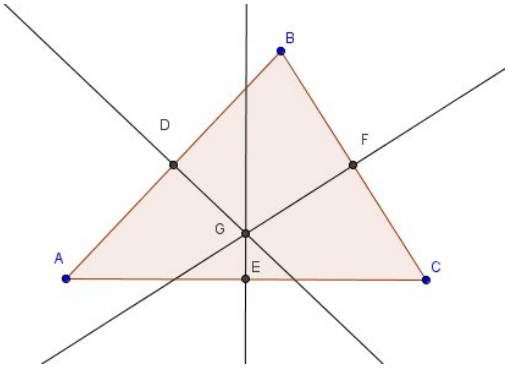
(A mediator is a perpendicular bisector of a line segment)

1. Click on File, New Window, and draw a triangle using the Polygon tool  .

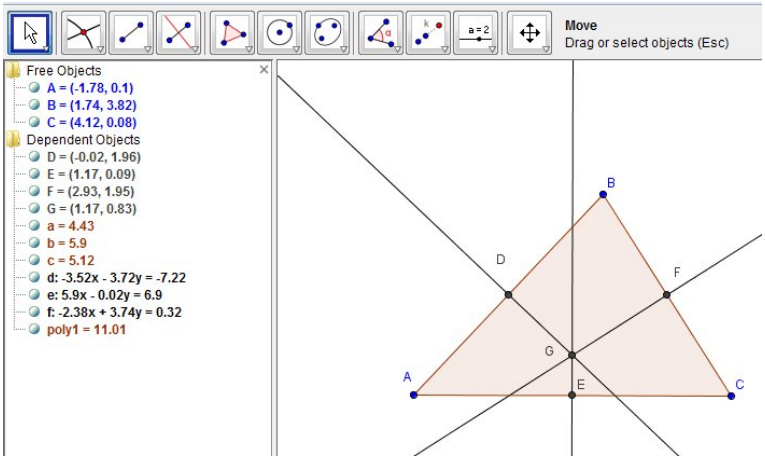
2. Select the Midpoint or Centre tool  and selecting each side of the triangle in turn, construct the midpoints of each side.

3. Using the Perpendicular Bisector tool  , select each side to construct perpendicular bisectors (mediators) of each side.

4. Select the Intersect Two Objects tool  , and then 2 of the mediators to construct the circumcentre.



5. The equations of the 3 mediators are shown in the Algebra View.



6. Hide the mediators by right clicking on each one and clicking on Show object. Drag the vertices to see the circumcentre change position.

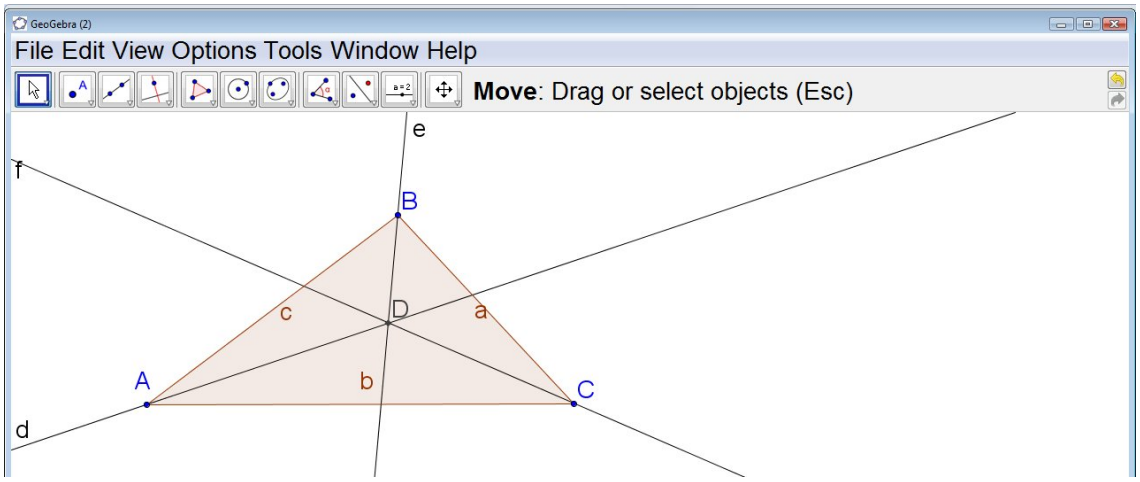
7. Click on the Circle with Centre through Point tool  , then the circumcentre (point of intersection of the mediators) and one of the vertices of the triangle and construct the circumcircle, which passes through the 3 vertices.

8. Drag the vertices of the triangle to confirm the construction.

9. Click on File, Save as and choose the location you wish to save your GeoGebra file to. Enter a filename that describes the file content and click on Save.

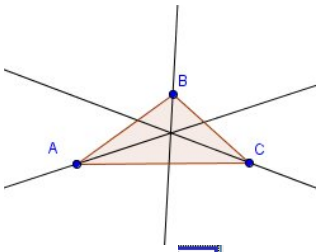
10. To start a new file, click on File, New.

Constructing the bisectors of the angles and constructing the incentre and incircle of a triangle.



As in the last 2 examples, construct a triangle ABC in a new window.

1. Select the Angle Bisector tool  . Select the points B, A and C, in that order, to construct the angular bisector of <BAC. Repeat for the other two angles in the triangle.



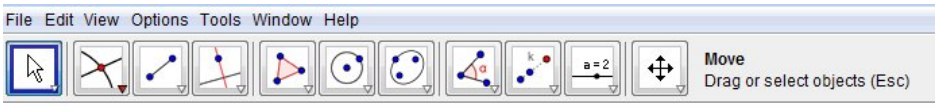
2. Select the Intersect Two Objects tool  and 2 of the angle bisectors to construct the incentre.

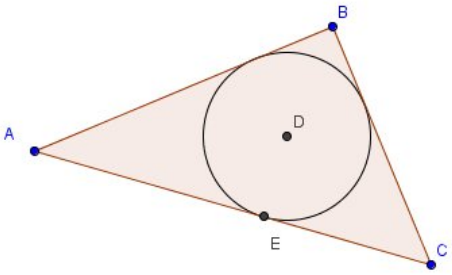
3. Hide the angle bisector lines as in the previous example on the circumcircle.

4. Each side of the triangle will be a tangent to the incircle and should remain as such if we were to drag the vertices. It is important therefore not to just use the Circle with Centre through Point tool  , selecting the incentre and moving outwards until we touch the circle to draw the incircle.

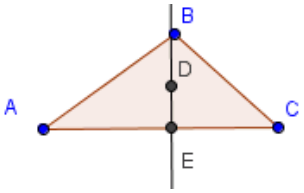
5. The circle constructed in this way will not remain an incircle as we drag the vertices. Try it to see.

To construct the incircle



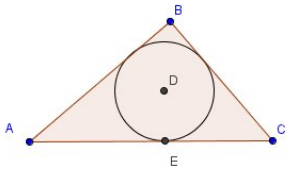


1. Selecting the Perpendicular Line tool  , draw a perpendicular line from the incentre D, to line AB or any of the 3 sides of the triangle.



2. With the Intersect Two Objects tool  selected construct the intersection E of side AB and this perpendicular line.

3. Hide the perpendicular line. Select the Circle with Centre through Point  , and with D as centre and E as the point on the circle, construct the incircle.



4. Drag the vertices to confirm the construction.

5. Click on File, Save as and choose the location you wish to save your GeoGebra file to. Enter a filename that describes the file content and click on Save.

6. To start a new file, click on File, New.

Exercise: Construct the altitudes (perpendiculars from the vertices to the opposite sides) of a triangle and show that they are concurrent and construct the orthocentre.

Using the same triangle for all constructions, construct the centroid, circumcircle, incentre and orthocentre. Label each one. Drag the vertices and look at the way these points move.

Which ones move farthest away? Are the four points always collinear? Sometimes? When? When do all 4 of them coincide?
You may have noticed that 3 of them are always collinear. Which ones?
Answer: The circumcentre (and orthocentre) are inside the triangle if all the angles are acute. If one angle becomes greater than 90 degrees, the circumcentre moves out of the triangle by crossing over the hypotenuse at its midpoint. (The orthocentre crosses over the right angle on its way out. The more obtuse the angle the further away they go.) Note: The incentre and the centroid are always inside the triangle and the four points (circumcentre, orthocentre, centroid, and incentre) coincide if and only if the triangle is equilateral, and the four are collinear if and only if the triangle is isosceles. Apart from the incentre, the other 3 points are always collinear, and this line is called the Euler line of the triangle.

Appendix A: How to Download GeoGebra.

The GeoGebra software package is available free at http://www.geogebra.org/cms/. It is recommended that the Download GeoGebra version be used, rather than the GeoGebra Webstart version.

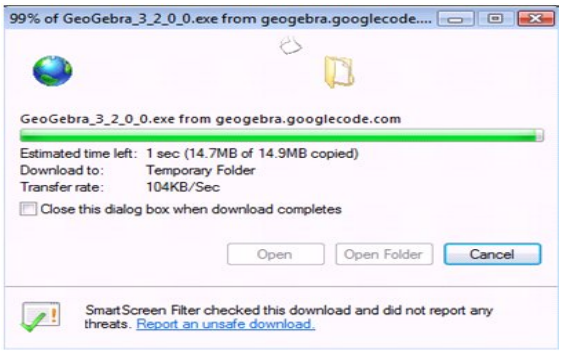
Note: in order for GeoGebra to work on a computer, one also needs one of the latest versions of a program called Java; this is available free at http://www.java.com/en/ . The latest current version of Java is version 6. See instructions on how to download Java in Appendix B.

Instructions for downloading GeoGebra: (Note these may vary if the layout of the website http://www.geogebra.org/cms/ changes.) The current version available on this webpage is 3.2.38.0.

1. Go to http://www.geogebra.org/cms/.

2. Click the Download button.

3. Click the version suitable for your computer for example GeoGebra 3.2 for Windows (Server 2).



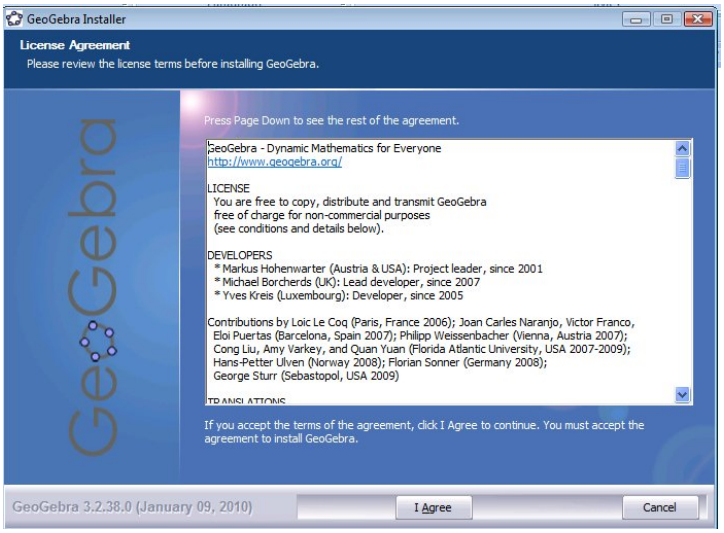
4. A new window appears. Click Run. The installation process will start and this will continue for a few minutes depending on download speed.

5. When the following window appears click Run. This process will again take a few minutes depending on download speed.

6. Follow any instructions that appear.

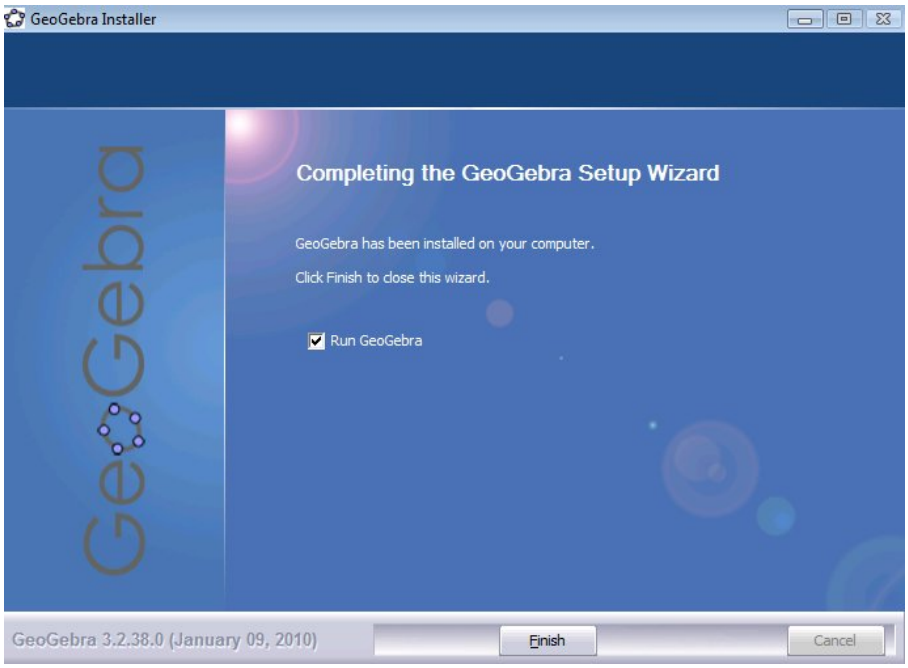
7. A new dialogue box appears. Choose “English” and click Next.

8. A new dialogue box appears. Click, “I Agree”.

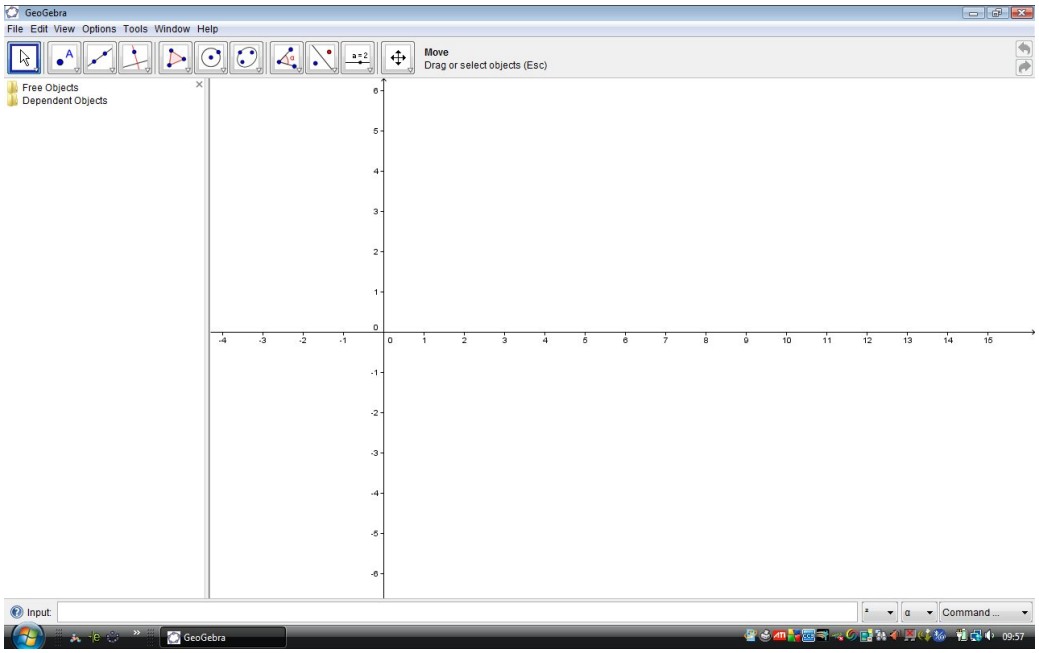


9. Follow the wizard by clicking Next until Install appears and click Install.

10. When this window appears click Finish.



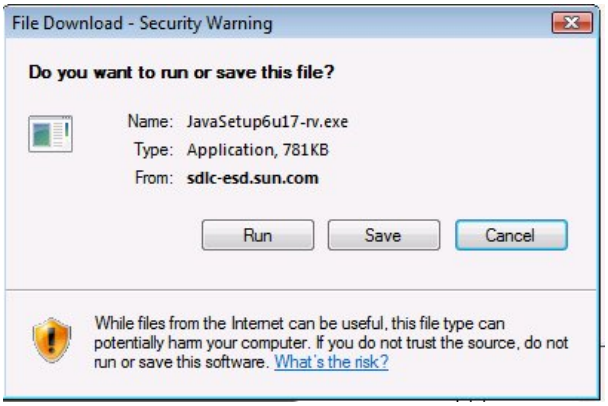
11. The GeoGebra window should now open and GeoGebra is installed on your computer. GeoGebra can now be opened by the usual method you open programs on your computer.



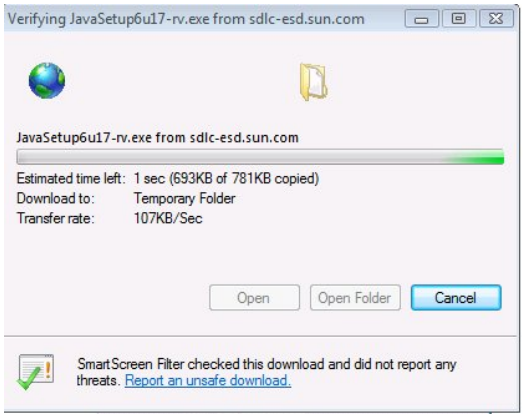
Appendix B: How to download Java:

1. Go to http://www.java.com/en/ and click “Free Java Download”.

2. The following window appears. Click Run.



3. The following window will appear.

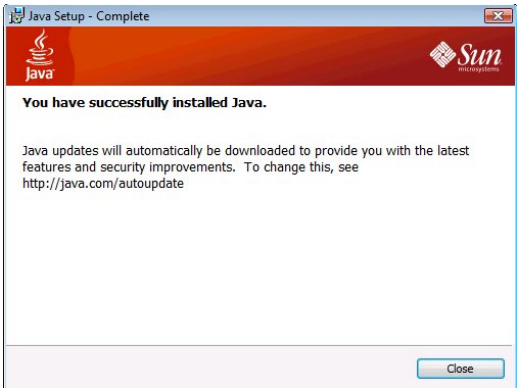


4. Eventually another this window will appear.

5. Click Run.

6. When the following window appears click Install and follow the wizard.

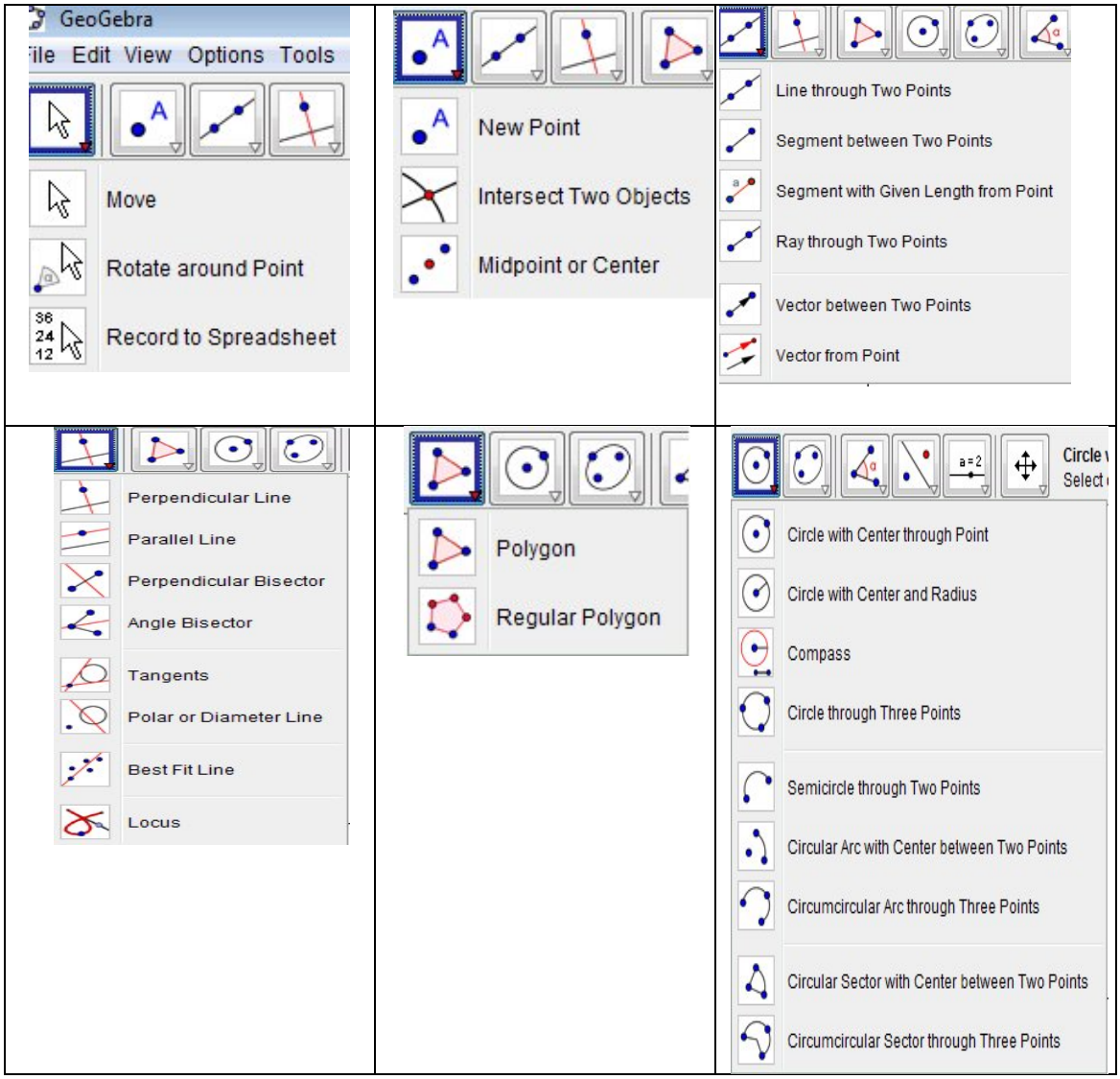
7. When the "You have successfully installed Java"window below appears click Close.

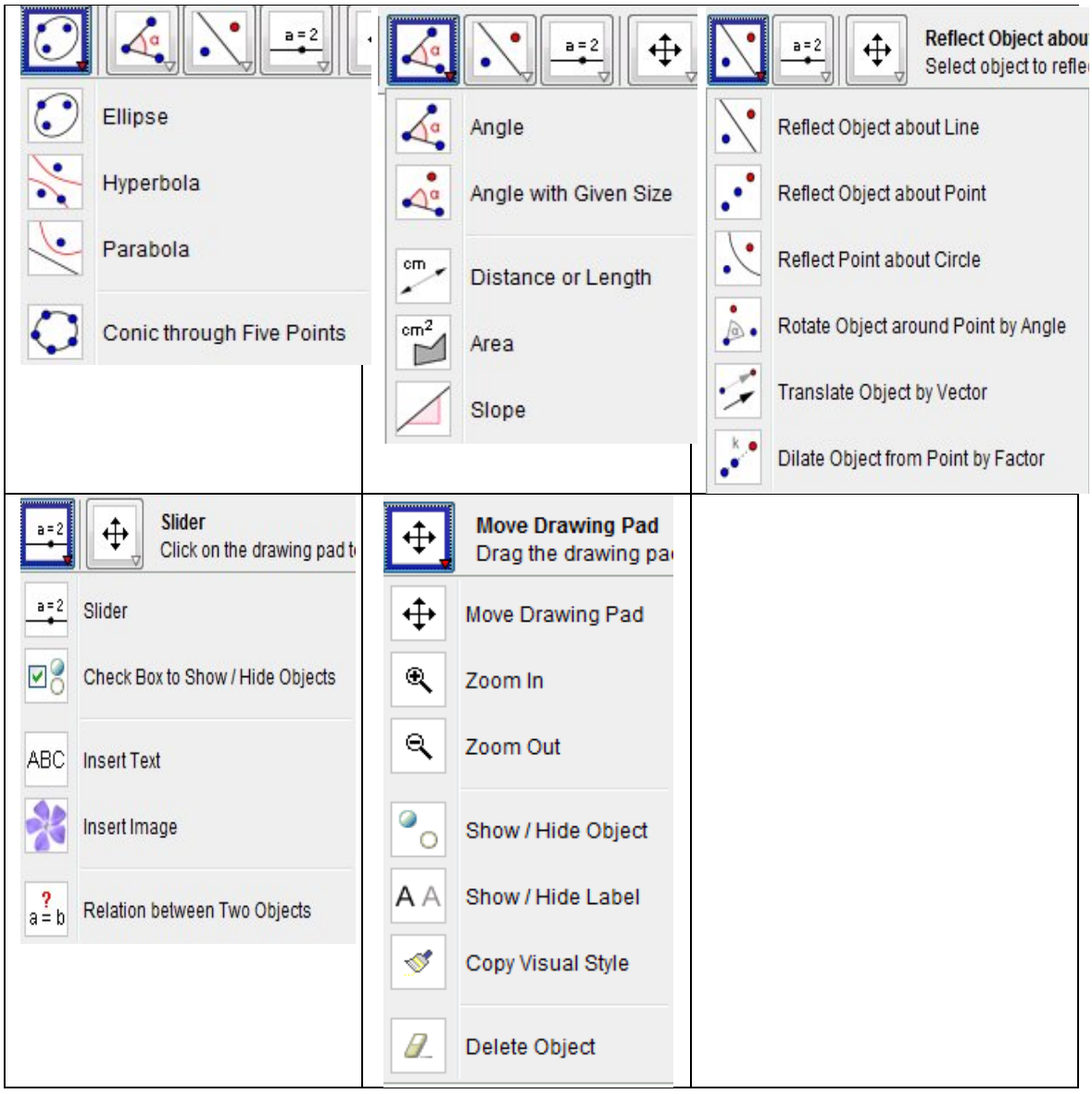


8. The latest version of Java should now be installed on your computer.

Appendix C

Underneath are the various Tools available in GeoGebra by clicking the appropriate arrow on the right hand side.




Ellipse
Hyperbola
Parabola
Conic through Five Points
Angle
Angle with Given Size
Distance or Length
Area
Slope
Reflect Object about Line
Reflect Object about Point
Reflect Point about Circle
Rotate Object around Point by Angle
Translate Object by Vector
Dilate Object from Point by Factor
Slider
Check Box to Show / Hide Objects
Insert Text
Insert Image
Relation between Two Objects
Move Drawing Pad
Zoom In
Zoom Out
Show / Hide Object
Show / Hide Label
Copy Visual Style
Delete Object