ICT & MATHS

Module 2

GeoGebra
for Co-ordinate Geometry

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GeoGebra for Co-ordinate Geometry

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Please note

- Screenshots used in this manual may appear different from those on computer screens used by participants; variations in versions of the software and differing operating systems may be in use.
- The World Wide Web is constantly evolving and content and URLs (Universal Resource Locators - website addresses) change over time. It is possible that the content located at some of the URLs listed throughout this manual may change over time.
- Screenshots and software titles used throughout the manual are from a PC using Windows Vista©.
- Participants using other operating systems may encounter some differences in screen presentation and layout.

Throughout this module reference may be made to software titles and suppliers of Internet services. These references are made purely to illustrate or expound course content. Any such reference does not imply any endorsement by the NCTE of a product or company. The reader should be aware that typically there are many products and companies providing similar services in areas related to ICT. Participants should be as informed as possible before making decisions on purchases of ICT products or services.

Duration

2.5 hours

Objectives

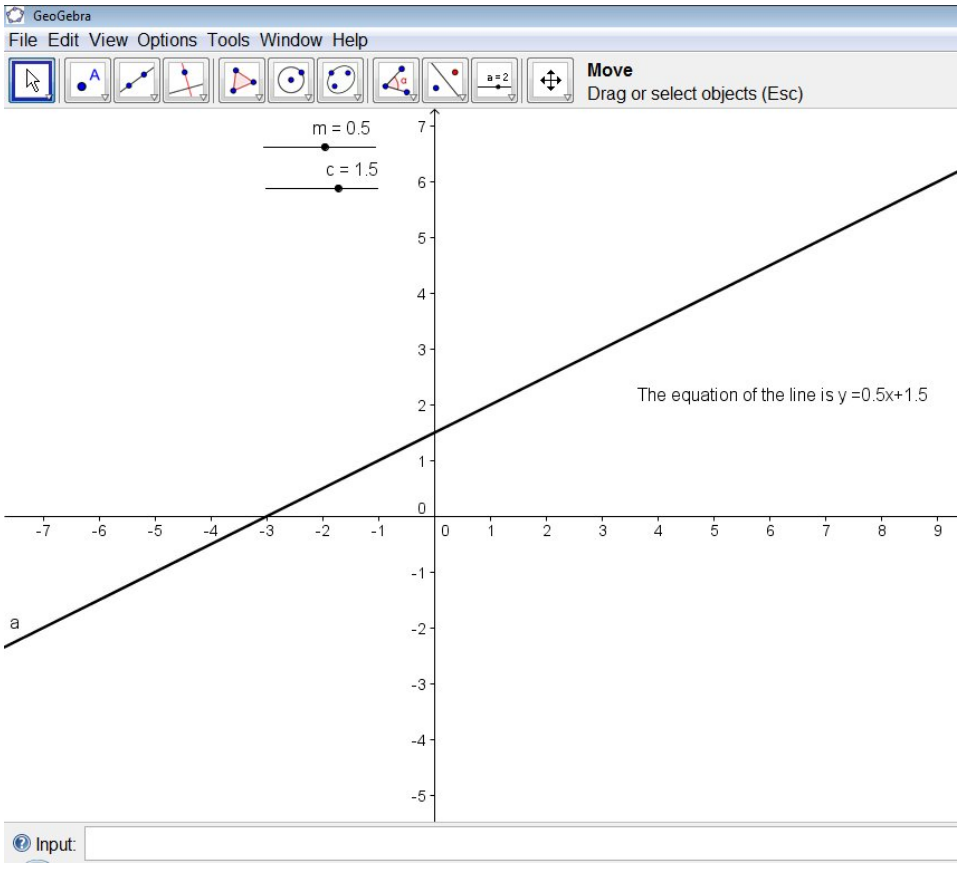
This module aims to enable the participant to:

This module aims to enable the participant to:

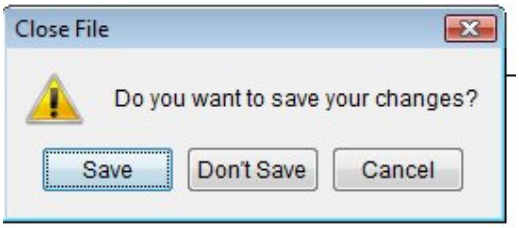
- plot points and draw lines
- examine the properties of points and lines
- investigate lines of the form $y = mx + c$ using sliders
- compare the properties of lines of the form $ax + by + c = 0$ using sliders
- examine their properties of objects by finding their images under a translation
- demonstrate that a circle is a set of points which are equidistant from a given point
- investigate circles of the form $x^2 + y^2 = r^2$ using a slider
- plot circles and intersecting lines
- investigate circles of the form $x^2 + y^2 = r^2$ using a slider
- investigate circles of the form $x^2 + y^2 + 2gx + 2fy + c = 0$ using sliders
- explore geometrical properties of circles.

GeoGebra

Co-ordinate Geometry of the Line



Opening a new file using File and New: Click on File and select New. One is then asked if they want to save the changes made in the current file.

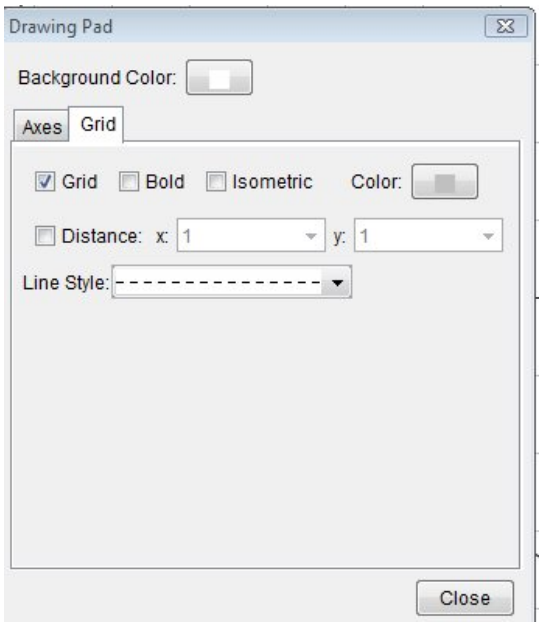


If Don't Save is clicked here all the changes since the last save are lost and the new GeoGebra file replaces the one you had been using.

Opening a new file using File and New Window: Click on File and select New Window. In this case the original file remains open and a new GeoGebra file opens.

To add a Grid to the Drawing Pad

Go to View and click Grid. The grid is useful when using GeoGebra for co-ordinate Geometry. To change the colour of the grid, go to Options and choose Drawing Pad. On the new dialogue box that appears, open the Grid tab. Change the Background Color, Grid Color, Line Style, etc.



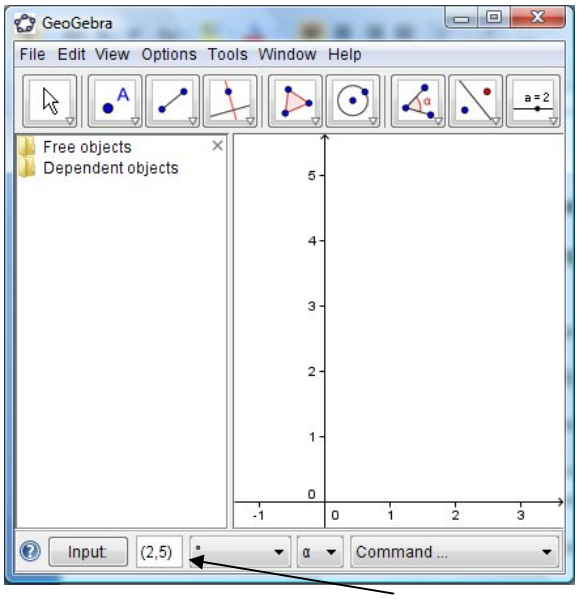
To ensure all points appear on the Grid, go to Options, Point Capturing and On (Grid).

Note: for clarity the screen shots in this module have been taken without the grid showing.

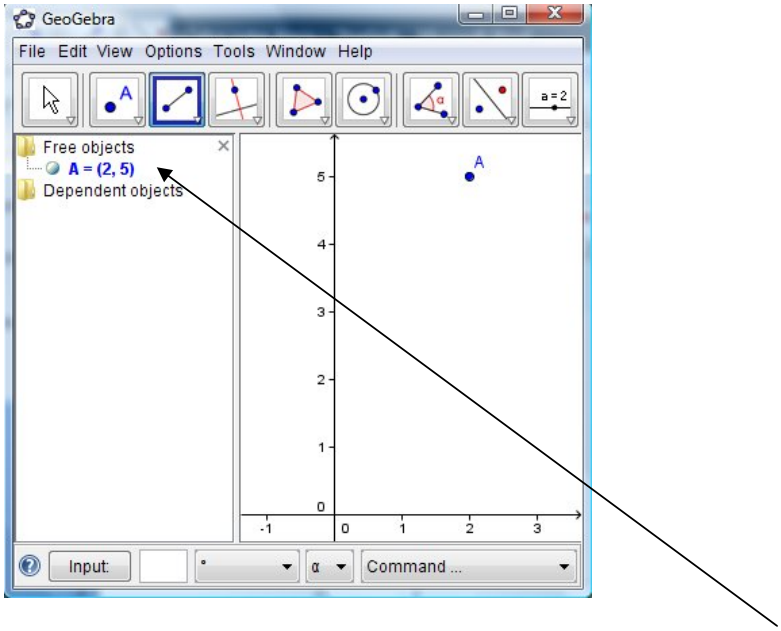
Note: in co-ordinate Geometry to get the accurate location of points when points are not set to appear on the grid, it is important that the decimal points are set to at least 2 decimal places. Go to Options, Rounding and pick the number of Decimal places required.

Investigating Co-ordinate Geometry of the Line

Plotting points with fixed coordinates, reading coordinates, drawing line segments and finding their lengths and midpoints.



Enter the point (2, 5), on the Input line, and press Enter.

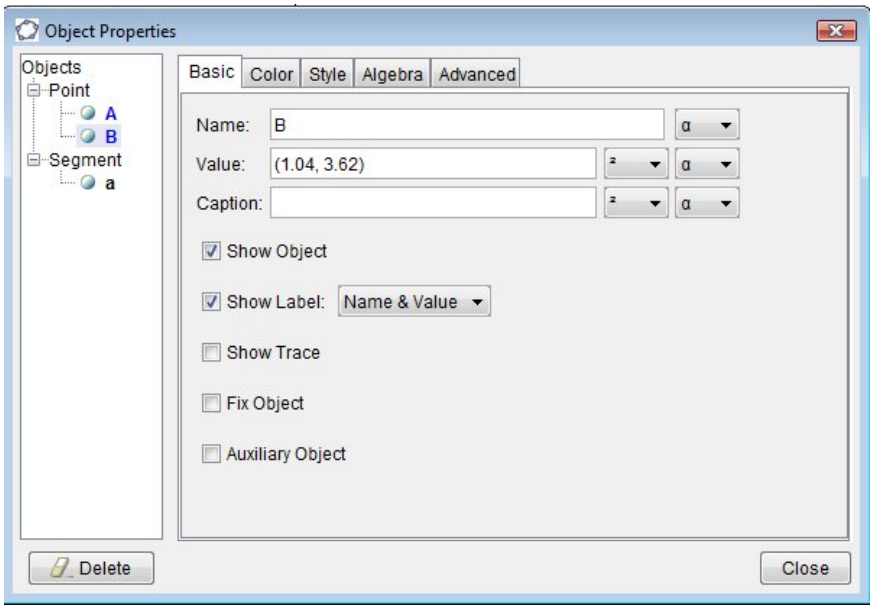


Note that the coordinates of points are displayed in the Algebra View once Enter is pressed.

In the Algebra View, objects are divided into two categories: Free Objects and Dependent Objects. If you create a new object without using any existing objects, it is classified as a Free Object and objects created using existing objects are classified as Dependent Objects.

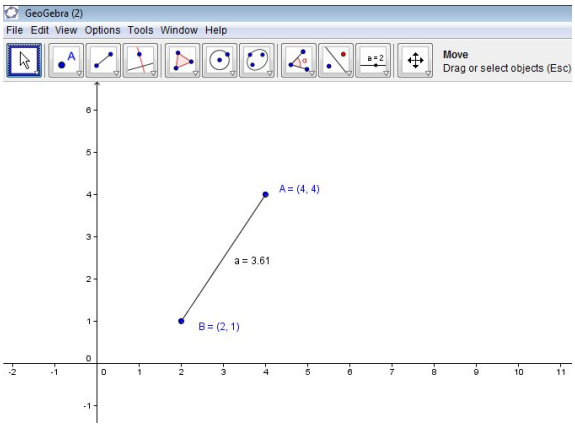
Plot the point (-1, 2), displaying its coordinates on the Drawing Pad.

To display the coordinates on the Drawing Pad: Right click on the point and select Object Properties. In the new dialogue box that appears, with the Basic tab open in the drop down menu beside Show Label, choose Name & Value.



Select the Segment between Two Points tool and then select points A and B to plot segment [AB].

The length of line segment [AB] i.e. |AB|, is displayed in the Algebra View as a=



(To display the length of a line segment, right click on it, choose Object Properties and with the Basic tab open, and the box Show Label ticked, choose Name and Value from the drop down arrow beside it.)

Select the Midpoint or Centre tool . Then select the line segment [AB] or points A and B to get the midpoint, which is plotted and the coordinates are displayed in the Algebra View.

Click on File, Save as and choose the location you wish to save your GeoGebra file to. Enter a filename that describes the file content and click on Save.

To start a new file, click on File, New.

Using the New Point tool to plot points with variable coordinates

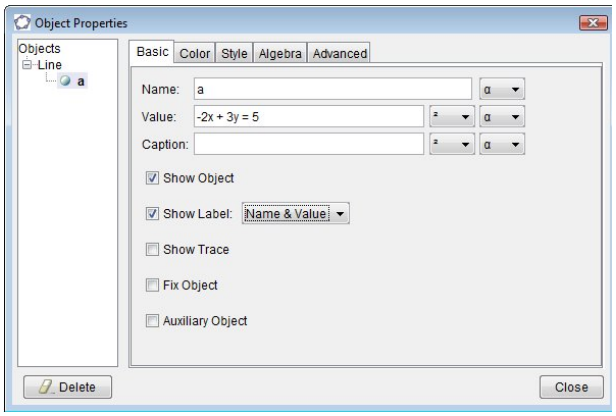
With the New Point tool selected and clicking anywhere on the drawing pad, points may be plotted whose coordinates are displayed automatically in the Algebra View and which may be displayed on the Drawing pad as above.

Experiment with using the New Point tool to plot points, find midpoints and draw line segments connecting the points.

Drag the points and note changes in midpoint and length of line segments.

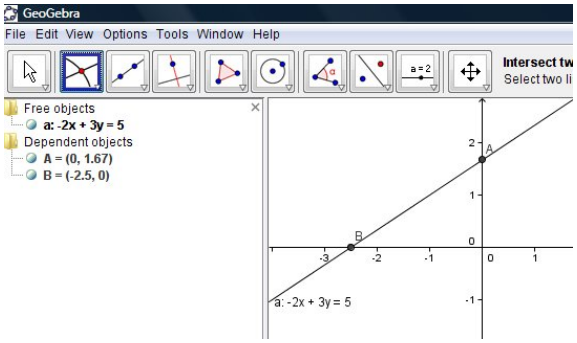
Plotting a line, finding its points of intersection with the axes, finding its slope using the slope tool, then the angle made by the line with the positive x-axis, and the slope of the line from the tan of this angle.

1. Click on File, New.
2. In the Input line type in 2x-3y+5=0. In the Algebra View it will be written as -2x+3y=5.
3. Right click on the line and select Object Properties, and with the Basic tab open choose Name & Value in the drop down menu beside Show Label, to have the equation appear on the line.



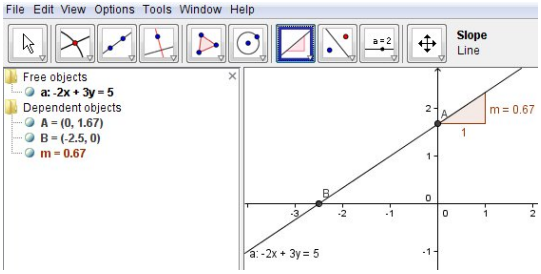
To find the points of intersection with the axes

1. Select the Intersect Two Objects tool , the line a and the *y axis*. The point of intersection A is marked on the line and its coordinates are displayed in the Algebra View.



2. Repeat this for the point of intersection with the *x axis*.

To find the slope of a line (Method 1)

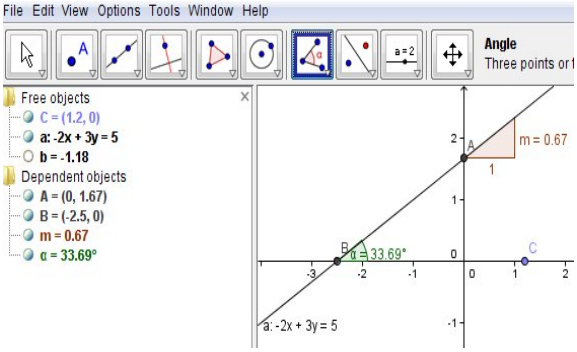


1. Select the Slope tool, as shown, and the line, to find its slope.
2. The value of the slope is displayed in the Algebra View.

To find the slope of the line (Method 2)

1. Select the New Point tool and construct a point C to the right of B.

2. Select the Angle tool and the points A, B , C to construct <ABC. Ensure it is not the reflex angle which is given. (To show an angle as a non reflex angle, right click on the angle, go



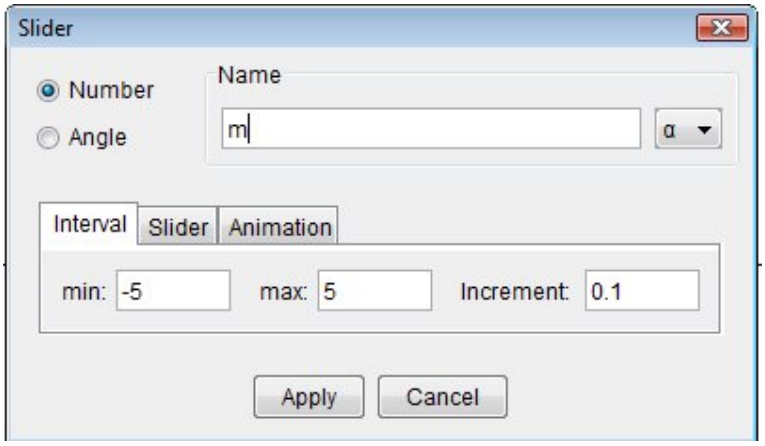
to Object Properties and with the Basic tab open unclick the Allow reflex angle button and click Close.)
(Right click the angle, choose Object Properties *and with the* Decoration tab open select the decoration. This allows you to put an arrow on the angle giving direction of rotation.) Find tan(a) using your calculator and note that it is equal to the slope of the line a.

Using sliders to see the effect of varying *m* and *c* in the equation of the line *y=mx+c.*

Go to file and select New.

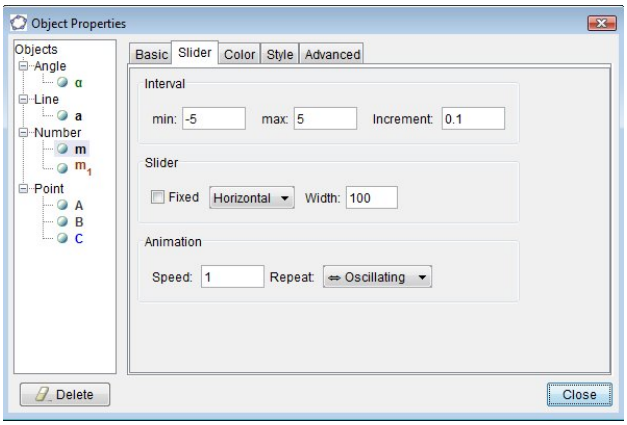
(i) Creating a slider for *m*

1. Go to the Slider tool and click on the screen where you want your slider located.
2. A new dialogue box appears. Choose number rather than angle. Give it a name m. Choose the min, max values and the increment you want it to go up in. Click Apply.



3. Right click on slider m.

4. Select Object Properties and the Slider tab to vary max and min values, the increment, and the width of the line representing the slider.



5. Selecting the Color tab allows you to vary the colour of the slider.

6. The Style tab allows you to vary the thickness of the slider.

7. Select Close when finished.

(ii) Creating a slider for c

1. Create a slider c as above.

2. Change colour and thickness of slider c as above.

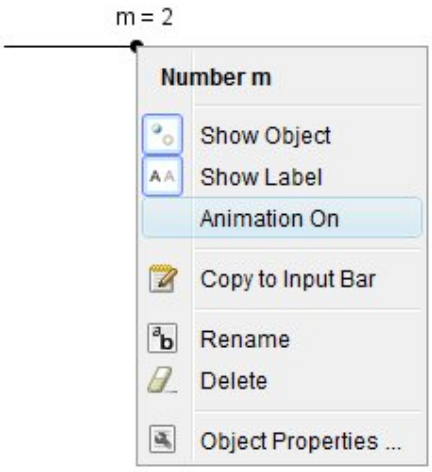
3. Type in *y=m x+c* into the Input line (space between *m* and *x or m*x for m multiplied by x*).

4. Vary the value of *m* by selecting the Move tool , and dragging the slider m. Observe the effect.

5. Investigate the effects of varying c by dragging c along the slider.

6. What do you conclude about m and c?

7. To animate an object created with the use of a slider, for example, m or c in this case. Right click on that particular slider and select Animation On.

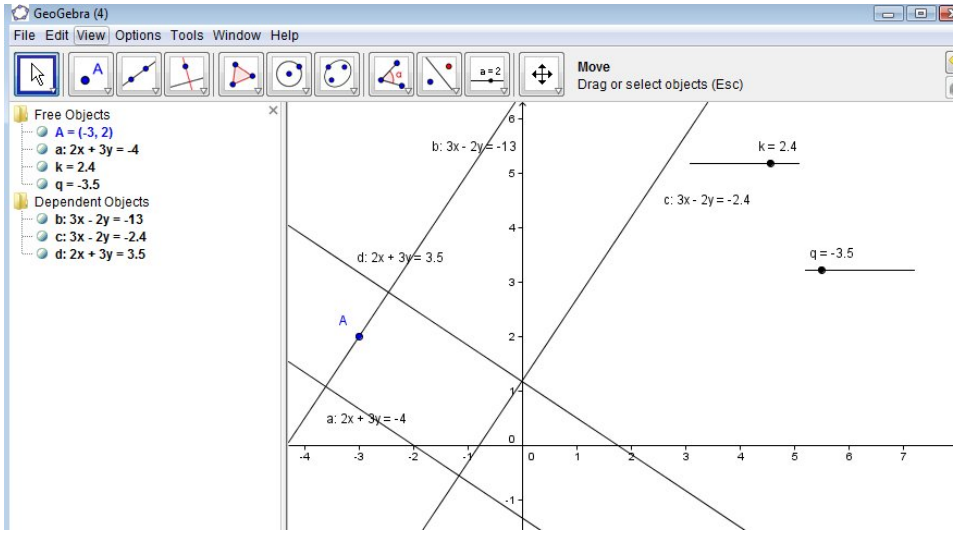


8. To turn the animation off, right click on the slider and deselect Animation On.

9. Click on File, Save as and choose the location you wish to save your GeoGebra file to. Enter a filename that describes the file content and click on Save.

10. To start a new file, click on File, New.

Investigate the relationship between lines of the form *ax+by+c=0* and *bx-ay+d=0*



1. Plot the point (-3, 2)

2. Plot the line 2x+3y+4=0. Right click, select Object Properties, with the Basic tab open, in the drop down menu beside Show Label select Name & Value.

3. With the Perpendicular Line tool selected, plot the line containing the point (-3, 2), which is perpendicular to 2x+3y+4=0.

4. Display its equation also on the Drawing pad as above. Its equation is 3x-2y =-13

5. Select the Slider tool and click on the Drawing pad.

6. Set up a slider called k to investigate lines with equation 3x-2y +k = 0.

7. Type in 3x-2y+k =0 into the Input window.

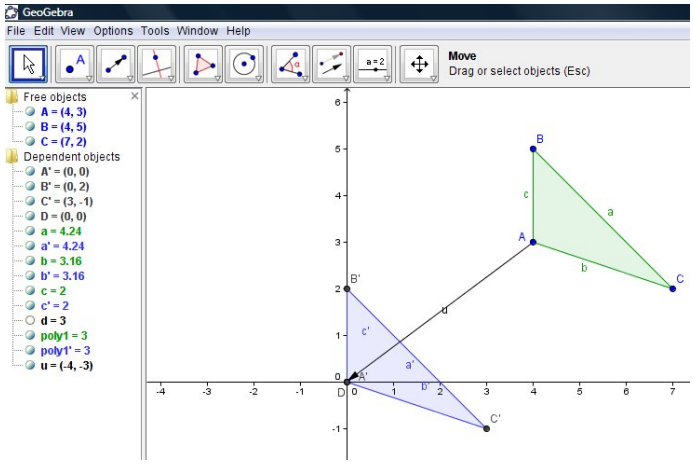
8. Drag the slider to see the relationship between 2x+3y+4=0 and 3x-2y+k=0

9. Now set up a slider q and investigate lines of the form 2x+3y +q =0.

10. Click on File, Save as and choose the location you wish to save your GeoGebra file to. Enter a filename that describes the file content and click on Save.

11. To start a new file, click on File, New.

Drawing triangles, finding area, finding area of the image of a triangle under a translation



Triangles may be plotted using the New point tool or by typing in specific coordinates in the Input line and joining the points with line segments or else by using the Polygon tool.

1. Use the Polygon tool to make triangle ABC (not necessary to construct 3 points first).

2. The area is given in the Algebra View as "poly1 = " or alternatively you may click on Command on the Input line, select the word Area which will be followed by closed square brackets i.e. Area[ ].

3. Inside the brackets type A,B,C and the area is printed in the Algebra View as d= .

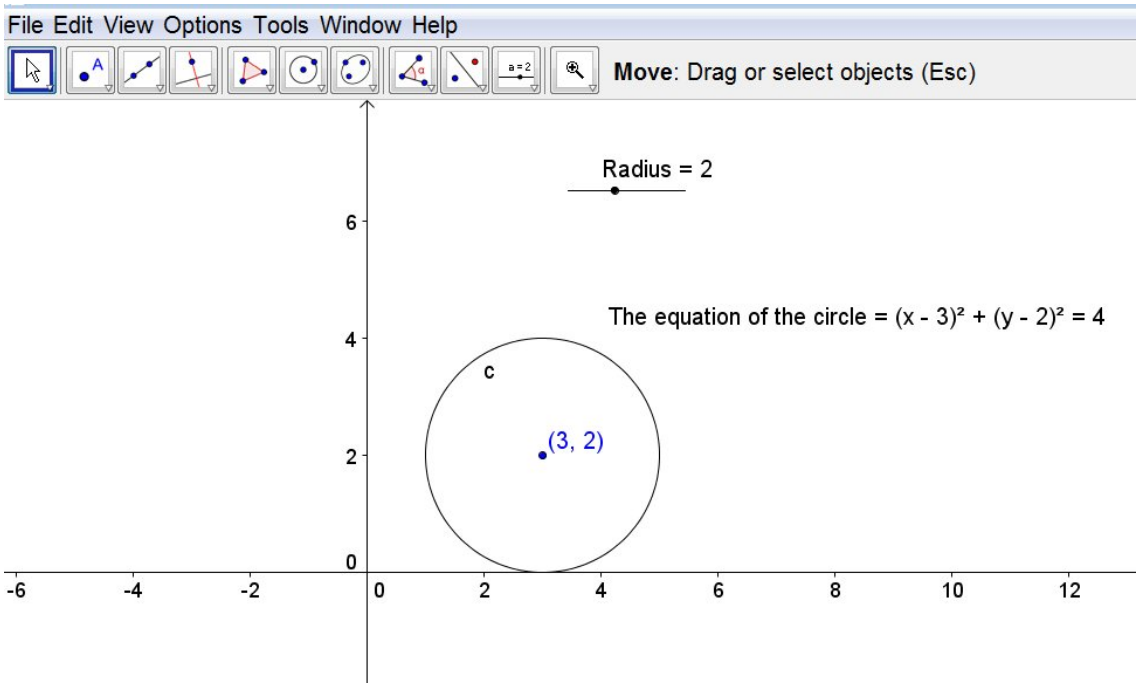
4. Define a vector between two points first using the Vector between Two Points tool selecting the initial point and final point for the translation.

5. Next use the Translate Objects by Vector tool to translate the triangle under the defined translation.

6. Drag the points of the original triangle and note the area of the original triangle and its image. The area of both triangles will be displayed in the Algebra View as "Poly 1 = " and "Poly 1' = " .

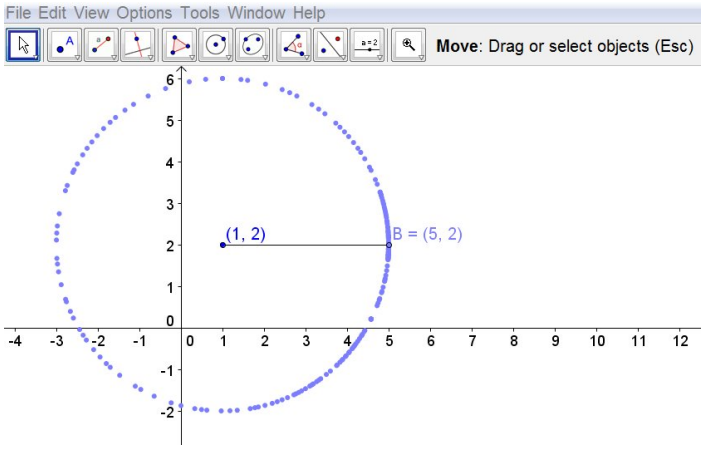
7. Click on File, Save as and choose the location you wish to save your GeoGebra file to. Enter a filename that describes the file content and click on Save.

Co-ordinate Geometry of the Circle



Coordinate geometry of the Circle

To demonstrate a circle as a set of points equidistant from a given point.

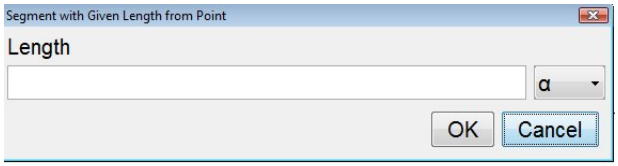


1. Plot the point (1, 2) by typing (1, 2) in the Input line and pressing Enter.

2. Display its coordinates on the Drawing Pad – right click, Object Properties, Name and Value.

3. Select Segment with Given Length from Point tool .

4. Click on the point A (1,2) and type 4 in for length in the dialogue box which appears and click OK.



5. Point B is drawn 4 units away from A.

6. Right click on point B and select Object Properties, Show Trace.

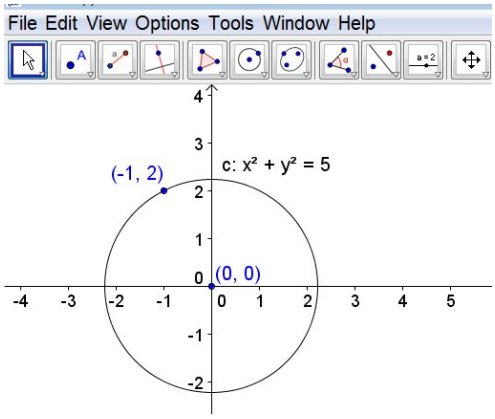
7. Drag B and observe.

8. Select View on the Menu bar, then Refresh Views to get rid of the traces.

9. Click on File, Save as and choose the location you wish to save your GeoGebra file to. Enter a filename that describes the file content and click on Save.

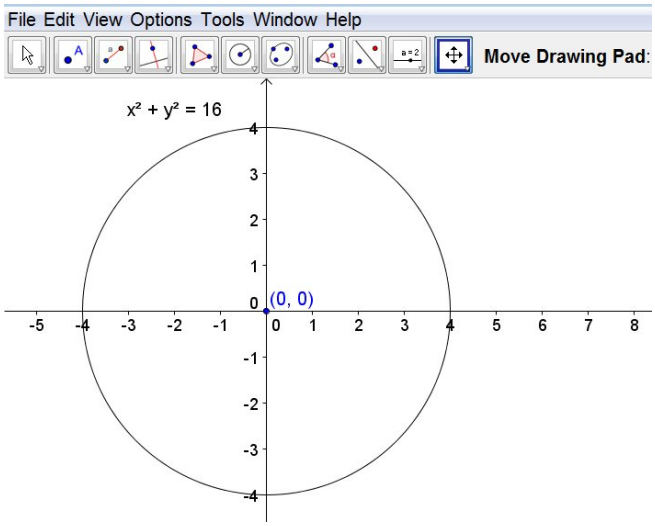
10. To start a new file, click on File, New.

Plotting a circle given 2 points



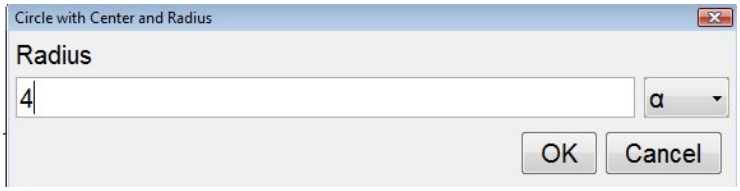
1. Plot points A(0,0) and B(-1,2)
2. Select the Circle with Centre through Point tool and the 2 points A and B and draw a circle.
3. The equation of the circle is displayed in the Algebra View.
4. Display the equation on the Drawing Pad.
5. Click on File, Save as and choose the location you wish to save your GeoGebra file to. Enter a filename that describes the file content and click on Save.
6. To start a new file, click on File, New.

Plotting a circle given a centre (0,0) and radius 4



1. Select the Circle with Centre and Radius tool .

2. Select centre (0,0) and type in 4 into the dialog box which appears and click on OK.



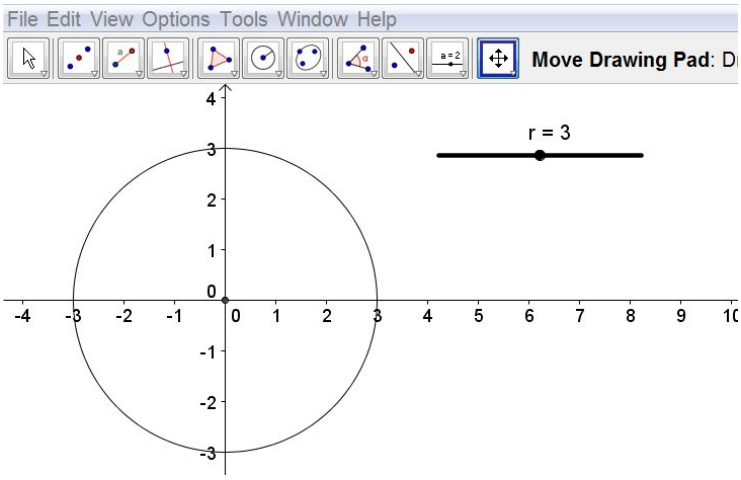
3. Display the equation on the Drawing Pad.

Exercise : Draw more circles with centre at (0,0) and with different radii and observe their equations.

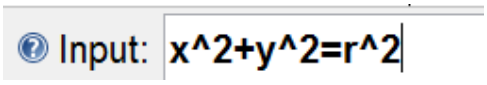
4. Click on File, Save as and choose the location you wish to save your GeoGebra file to. Enter a filename that describes the file content and click on Save.

5. To start a new file, click on File, New.

Use a slider to vary r and to investigate equations of the type $x^2 + y^2 = r^2$



1. Click on File, New Windo*w.*
2. Select the Slider tool and click on the Drawing pad.
3. Type in r for the slider name, 0 to 6 for the limits and 1 for the increment.
4. Change the colour and thickness of the slider – right click – Object Properties etc. Type in x^2+y^2=r^2 into the Input line and press Enter.

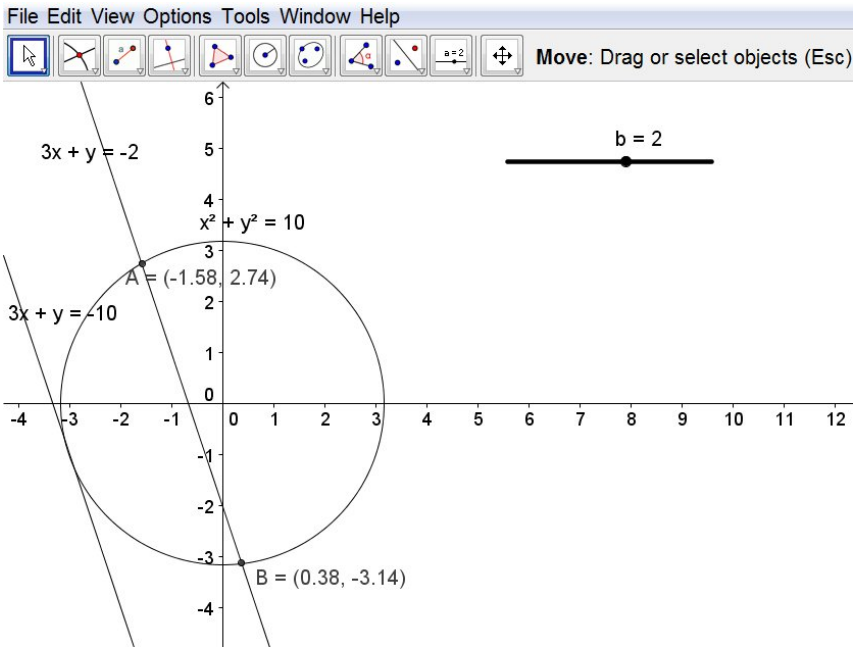


5. Drag the slider to vary the value of r. What is the centre of all of these circles?
6. Select the Midpoint or Centre tool and the circle to mark the centre.
7. Open Microsoft Word and type the equations of circles with radii from 1 to 6.

Exercise: Given the circle $x^2 + y^2 = 36$, *and using* and not the equation of the circle, plot a circle with one quarter of this area.

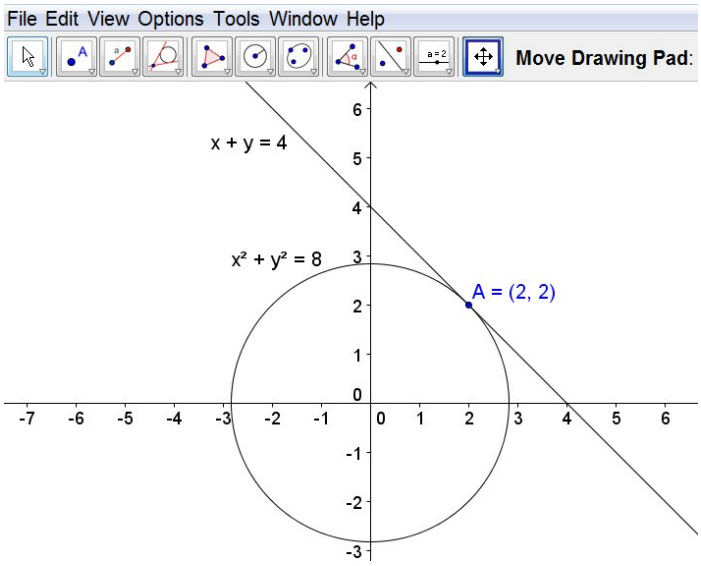
Click on File, Save as and choose the location you wish to save your GeoGebra file to. Enter a filename that describes the file content and click on Save. To start a new file, click on File, New.

Intersection of lines and circles



1. Plot the circle x^2+y^2=10 and the line 3x+y+10 = 0.
2. Select the Intersect Two Objects tool and make a conclusion about the relationship between the line and the circle.
3. Create a slider for the constant b in the equation 3x+y+b = 0. Set the values of b from -12 to +12.
4. Plot the line 3x+y+b =0 and vary the value of b by dragging the slider.
5. Select the Intersect Two Objects tool and note the points of intersection of the line and the circle - given as A and B in the Algebra View.
6. How many possible points of intersection are there?
7. Click on File, Save as and choose the location you wish to save your GeoGebra file to. Enter a filename that describes the file content and click on Save.
8. To start a new file, click on File, New.

To find the equation of the tangent to circle $x^2 + y^2 = 8$ at the point (2,2).

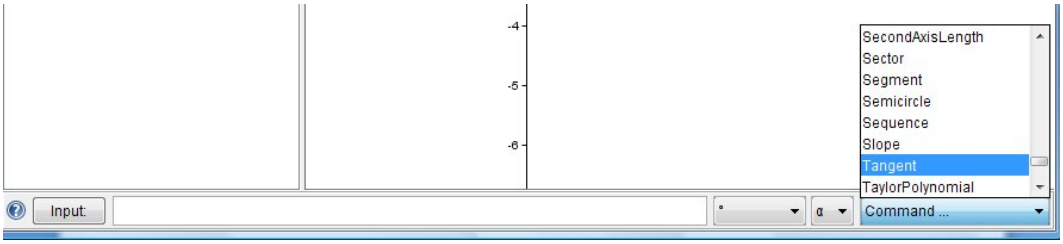


1. Click on File, New Window.

2. Type x^2 + y^2 = 8 into the Input line and press Enter to plot this circle, called c.

3. Plot the point (2,2) , called A

4. Choose "Tangent" from the list of Commands.

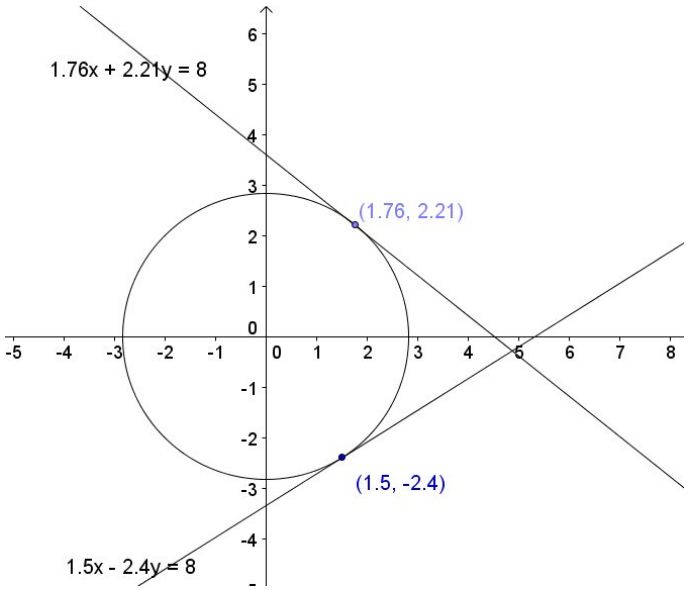


5. The word Tangent, followed by closed square brackets is then displayed in the Input line. Type in A,c in between the square brackets.

6. The tangent to the circle is then drawn and its equation is displayed in the Algebra View.

7. Plot any point on the circle and plot the tangent at that point. What do you notice about the form of the equation of the tangent? (Look at the coordinates of the point of tangency.)

8. Open Microsoft Word and type the points and the equations of the tangents to see the pattern.

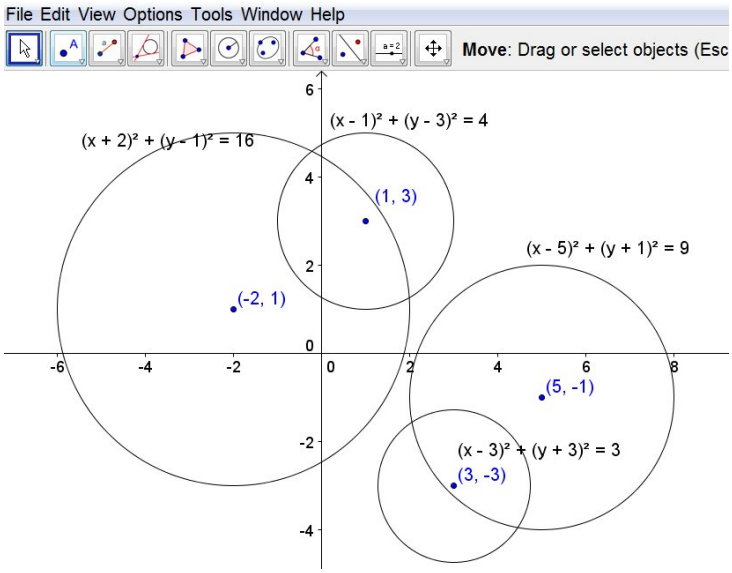


8. Click on File, Save as and choose the location you wish to save your GeoGebra file to. Enter a filename that describes the file content and click on Save.

9. To start a new file, click on File, New.

Exercise: Given circle C_1: $x^2+y^2=18$ and the point (3,-3), write down the equation of the tangent to C_1 at the point (3,-3). Check out the answer by plotting the tangent, using the equation and the circle, selecting the Intersect Two Objects tool and verifying the equation by getting (3, -3) as the point of intersection.

Circles whose centres are not at (0,0)



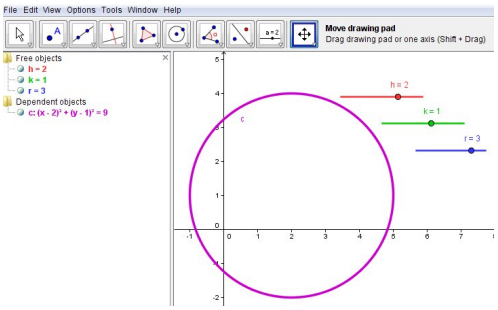
1. Click on File, New Window.

2. Select the Circle with Centre and Radius tool and plot the circle with centre (-2,1) and radius 4. Show its equation in the Drawing Pad. (Right click, Object Properties, Name and Value)

3. Plot the following circles and note their equations:
   (i) Centre (1,3) Radius 2
   (ii) Centre (5,-1) Radius 3
   (iii) Centre (3,-3) Radius $\sqrt{3}$ (typed in as sqrt(3))

4. What would you expect to be the equation of a circle with centre (h,k) and radius r?

5. Set up sliders for h, k, and r and investigate the equations of circles with different values of h,k and r, typing in (x-h)^2+(y-k)^2 =r^2 in the input window.



6. What is the effect of varying h?

7. What is the effect of varying k?

8. To display the circle equation in the form $x^2 + y^2 + 2gx + 2fy = -c$, right click on the circle equation in the Algebra View and click on the option $ax^2 + bxy + cy^2 + dx + ey = f$

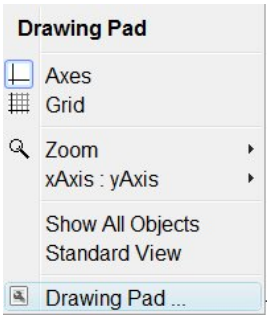
Transformations of the circle

1. Click on File, New Window.

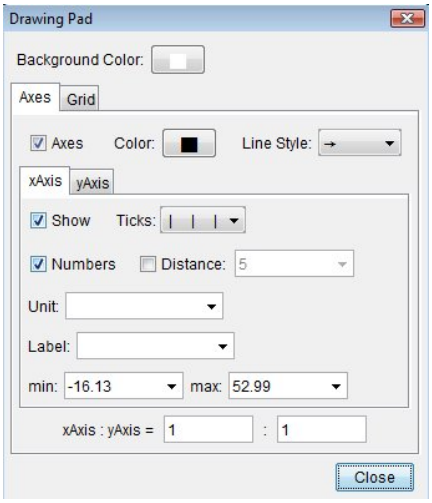
2. Plot the circle $x^2 + y^2 + 4x - 6y + 4 = 0$

3. The equation will automatically be displayed in the Algebra View in the form $(x-h)^2 + (y-k)^2 = r^2$.
   Note: Geogebra uses $(x-m)^2 + (y-n)^2 = r^2$. By right clicking on the equation it is possible to toggle back and forth between the two forms of the equation.

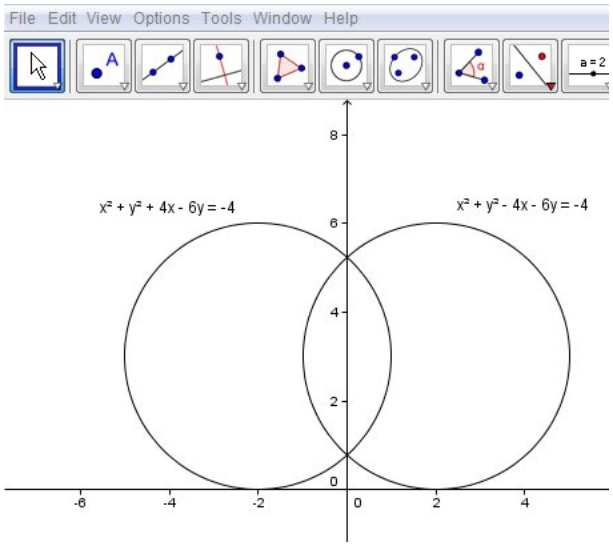
4. The scale on the axes may need to be changed so that the circle and its image can be clearly seen. This is achieved by right clicking on the Drawing Pad, selecting Drawing Pad, and then selecting the xAxis tab or the yAxis tab.



The max and min values can then be set for each axis. Be sure that the ratio of the *x* axis to *y* axis is 1:1 to maintain the circular shape (equal aspect ratio).



Transformations of the circle using Axial Symmetry



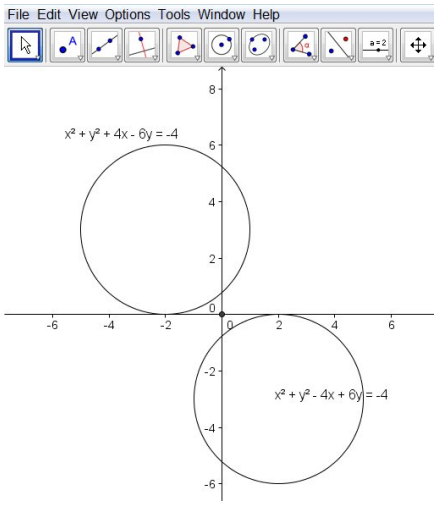
1. Select Reflect Object abot Line tool and find the image of the circle under axial symmetry in the *x* axis.

2. Compare the equations of the circle and its image.

3. Click on File, Save as and choose the location you wish to save your GeoGebra file to. Enter a filename that describes the file content and click on Save.

4. To start a new file, click on File, New.

Transformations of the circle using Central symmetry



1. Plot the point (0,0).

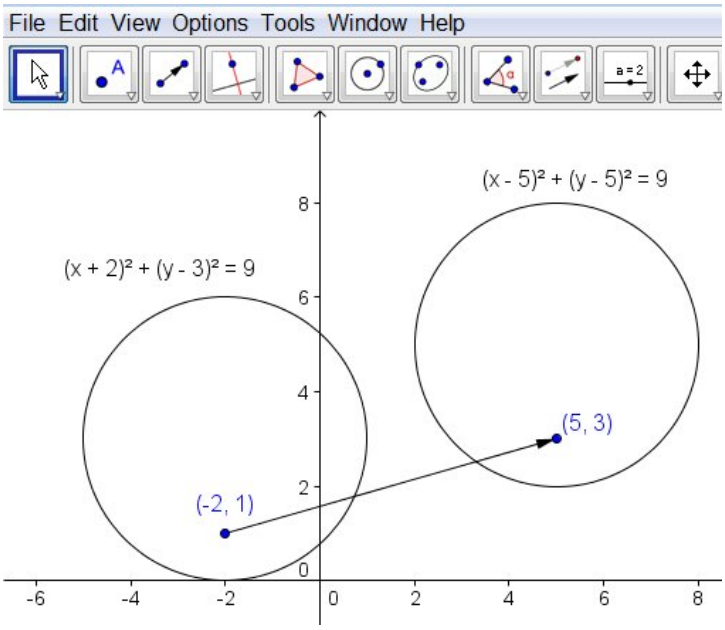
2. Select the Reflect Object about Point tool and find the image of the circle by central symmetry in the origin.

3. Compare the equations of the circle and its image.

4. Click on File, Save as and choose the location you wish to save your GeoGebra file to. Enter a filename that describes the file content and click on Save.

5. To start a new file, click on File, New.

Transformations of the circle using a Translation



1. Plot the points (-2,1) and (5,3)

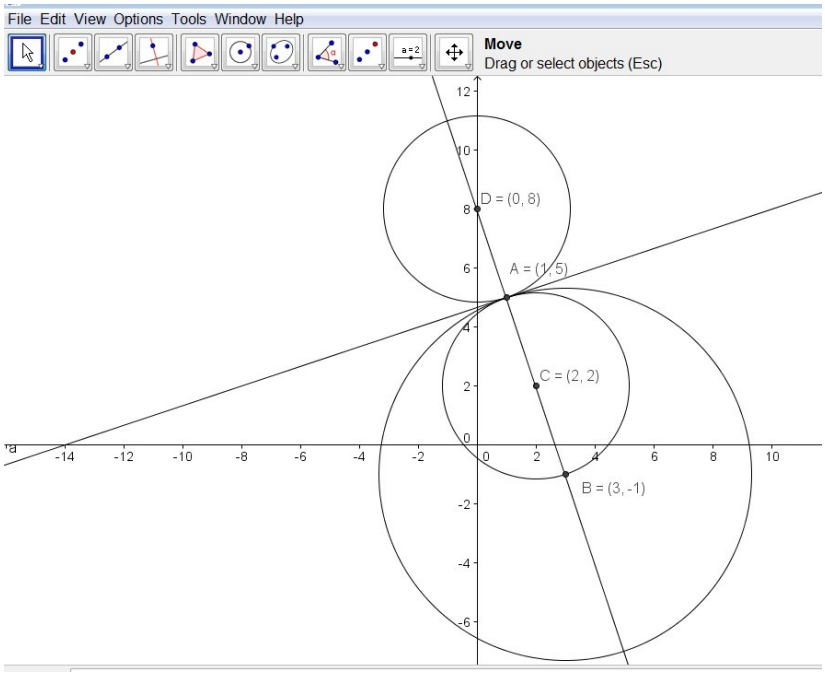
2. Select the Vector between Two Points tool and select each of the points (-2,1) and (5,3) to define the translation (-2,1) ⟶ (5,3).

3. Select the Translate Object by Vector tool , followed by the circle and the defined vector to translate the circle. Compare the equations of the circle and its image.

4. Click on File, Save as and choose the location you wish to save your GeoGebra file to. Enter a filename that describes the file content and click on Save.

5. To start a new file, click on File, New.

A case of Theorem 20 ,Corollary 6 – If two circles intersect at one point only, then the 2 centres and the point of contact are collinear.



1. Click on File, New Window.

2. By typing their equations into the Input line, plot the two circles
   c: x^2+y^2-6x+2y-30=0 and d: x^2+y^2 -4x-4y-2=0

3. Set the min and max values of the x and y axes to -20 to +20 respectively, keeping the 1:1 ratio.

4. Selecting the Intersect Two Objects tool , find the point of intersection and show it on the Drawing Pad. (Right click, Object Properties, Name and Value).

5. Plot the common tangent of the two circles (Tangent{A,c] or Tangent[A,d] in the Input line).

6. Selecting the Reflect Object about Point tool find the image of circle d in the common point of intersection A.

7. Selecting the Midpoint or Center tool and each circle c,d and d' in turn, plot the centre of each circle and show the coordinates on the Drawing pad.

8. Selecting the Line through Two Points tool and selecting any of the centres of the 3 circles (all of whch intersect in one point) and the point of contact and draw a line – the other 2 centres also lie on this line. Check by choosing another combination of 2 out of the 4 points and drawing a line through them and checking that the equation of the line is the same.

Exercises:

1. The line 4x+y-6 = 0 is a tangent to a circle c with centre (3,1). Plot circle c and display its equation on the Drawing pad. Use Theorem 20.
2. Verify, by drawing and by measuring the lengths of their respective radii and the distances between their centres that the circles $x^2 + y^2 - 6x - 4y + 11 = 0$ and $x^2 + y^2 + 4x + 6y - 19 = 0$ touch externally.
3. Verify, by drawing and by measuring the lengths of their respective radii and the distances between their centres that the circles $x^2 + y^2 - 6x + 2y - 30 = 0$ and $x^2 + y^2 - 4x - 4y - 2 = 0$ touch internally.
4. Given 3 points (2,1), (0,2), (4,1.5)which lie on the circumference of a circle find the centre of the circle using Theorem 21. Verify the answer by selecting the Circle through Three Points tool .