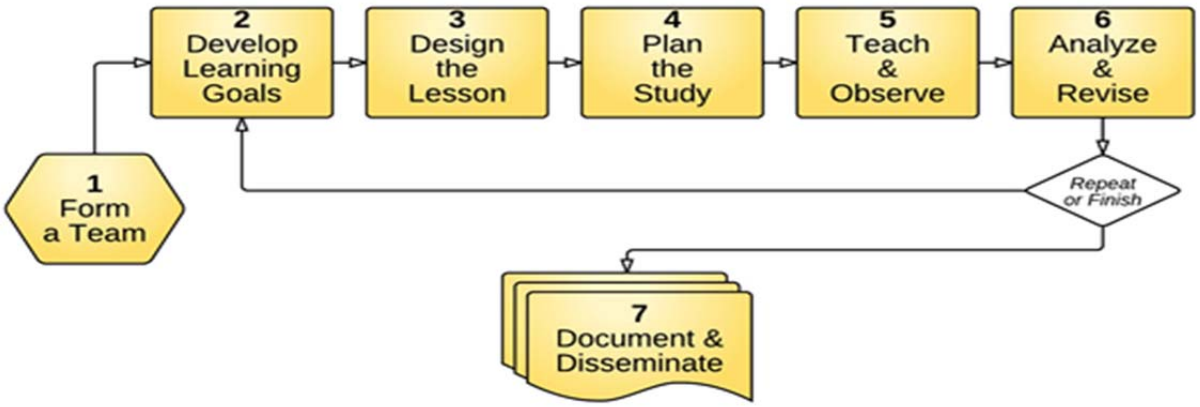
Reflections on Practice



Lesson Plan: Solving Simultaneous Equations
Class: Second Year
Lesson Date: 27-11-2014
Location: Loreto College, Cavan
Teacher: Áine Shannon
Developed by: Ger Sharpe, Pádraic Kavanagh, Dymphna Graham, Natalie Noone, Áine Shannon

Title of the Lesson:

An Introduction into solving linear simultaneous equations algebraically.

Brief Description of the Lesson:

To help students realize the need for solving linear simultaneous equations algebraically.

Aims of the Lesson:

I'd like my students to appreciate that mathematics can be used to solve real world problems.
I'd like my students to appreciate that mathematics can be used to communicate thinking effectively.
I'd like my students to appreciate that algebra is a tool for making sense of certain situations.
I'd like to foster my students to become independent learners.
I'd like to emphasise to students that a problem can have several equally valid solutions.
I'd like my students to connect and review the concepts that we have studied already.

For students to understand the limitations of solving simultaneous equations graphically or by extending the table. To recognise that solving simultaneous equations graphically is at best an estimation. To understand that it is another representations for equality and this being the most accurate.

Learning Outcomes:

To consolidate their understanding of the concept of equality.
Solve first degree equations in one or two variables, with coefficients in elements of $\mathbb{Z}$ and solutions also elements of $\mathbb{Z}$.

Background and Rationale:

In a typical lesson on solving simultaneous equations, the lesson often starts with the following: Today we are going to find the intersection of two lines; this is called solving simultaneous equations. When we solve two simultaneous equations we get the point of intersection of two lines. Using this approach what is most likely is that students learn the algorithm of solving simultaneous equations in other words the "how" without understanding the "what" and the "why". Moreover students do not get to investigate the motivation of why we need to be able to solve simultaneous equations.

As students discover that there are limitations regarding the accuracy of an answer from a graph or a table, their ability to think algebraically will be fostered. Therefore, in planning this lesson, the following perspective was taken: the lesson would develop students' abilities not only to solve simultaneous equations but to appreciate why and placed in a problem solving context develop their ability to make discoveries on their own.

Research:

Handbooks developed by the Project Maths Development Team and Sample Examination Papers.

About the Unit and the Lesson

Work done in previous lesson will be similar to the question for today's class below but one where the point of intersection can easily be read from graph. The previous lesson will also have focused on all parts of the question printed below but problems should arise when students don't get the same answer to both parts in (v) as it can't read it accurately from graph.
Students will leave class that day with the following question to think about: what does the point of intersection mean?

[The question is attached at the back.]

Flow of the Unit:

Handbooks would be useful here

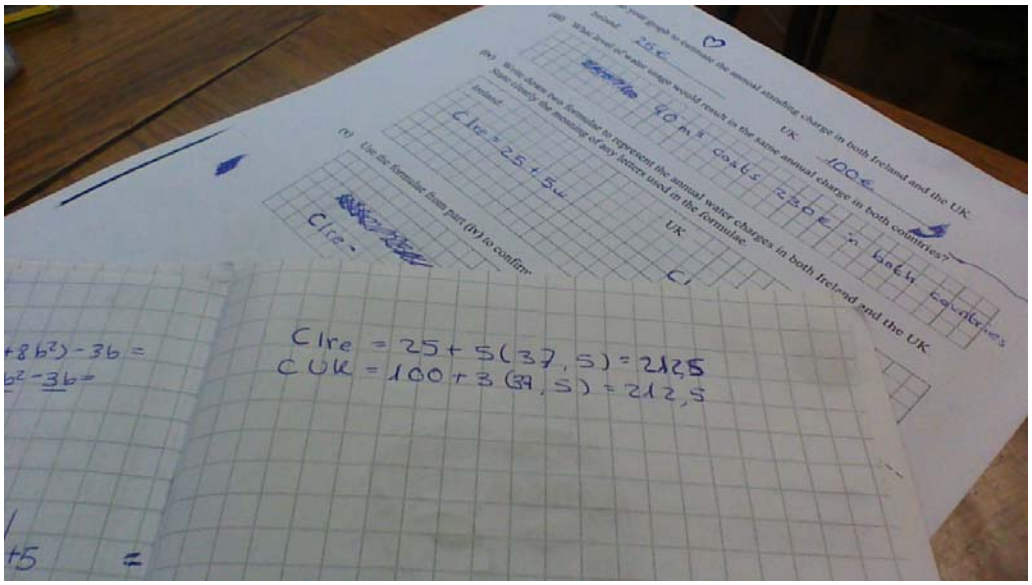
Lesson	Section 4: Algebra	# of lesson periods
1	Revision and extension of algebraic expressions and simple linear equations from first year	4 classes
2	Linear equations in one and two variables and linear inequalities in one variable	8 classes #3 research lesson
3	Adding algebraic fractions	2 classes
4	Algebraic factors	8 classes
5	Solve quadratic equations	5 classes

Flow of the Lesson:

Teaching Activity	Points of Consideration
1. Introduction Review problem from previous lesson	Give students a few minutes to review problem from previous lesson
2. Posing the Task Opening question: What do we know about the point of intersection of the graph? What information does the point of intersection tell us?	Put graph on board.
3. Anticipated Student Responses Possible responses: **R1**: They are equal at the point of intersection. **R2:** It tells us where the two lines cross **R3:** It tells us where the two lines meet	
Talk about the point of intersection relating it back to the graph. ***When is it equal?*** **R1:** When the lines cross ***What is equal?*** **R1:** the x-value is the same as the y-value	To clarify this show on the graph that the same x-value can correspond to different y-values, when the same x-value gives the same y-value and the case we want when the graphs meet the value of x and the corresponding value for y. This can be helped by drawing various vertical lines before and after the POI to indicate that the two lines give different y-values except at the point of intersection.

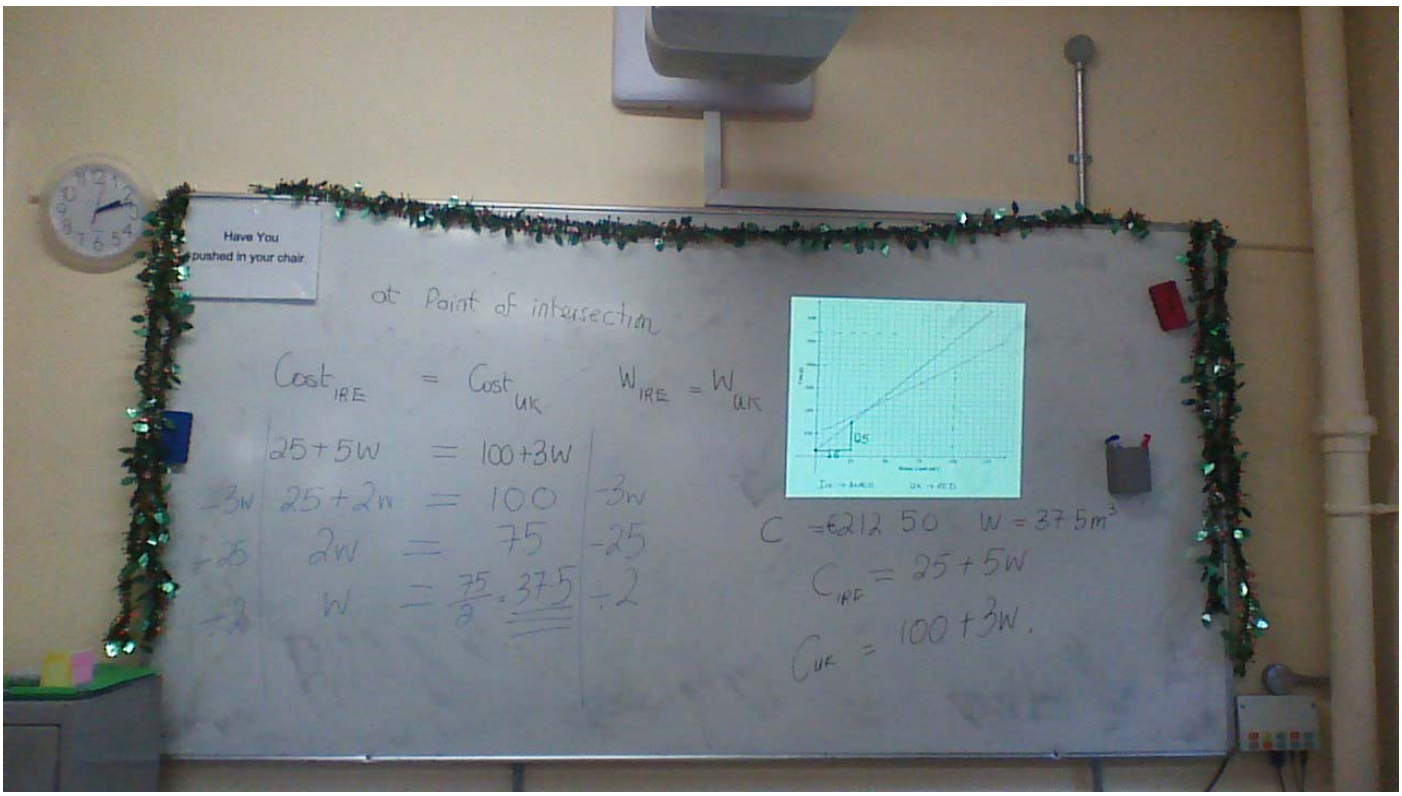
Posing the Task Ask can we write what is equal at the point of intersection **Anticipated Student Responses** Students will be able to give the formulas found in the previous lesson **R1:** $y = 5x + 25, y = 3x + 100$ **R2:** $Cost_I = 5(water\ used)_I + 25$ $Cost_U = 3(water\ used)_U + 100$ **R3:** $Cost_I = Cost_U$ $(water\ used)_I = (water\ used)_U$ ***Can we write this information a different way without words?***	Write on board beside graph the student responses. Students will be able to give the formulas found in the previous lesson Write these on board **R1:** y=5x+25, y=3x+100 **R2:** $Cost_I = 5(water\ used)_I + 25$ $Cost_U = 3(water\ used)_U + 100$ **R3:** $Cost_I = Cost_U$ $(water\ used)_I = (water\ used)_U$
Does it matter which we use? **R1:** No (ask student to explain why it does not matter) **R2:** Yes (ask student to explain why it does matter) ***Which looks like the easier to use?*** **R1:** Cost as the formulas are set up that way $Cost_I = Cost_U$	During the class discussion help students recognise and arrive at an understanding of equality
If we use $Cost_I = Cost_U$ how can we use this information to find the point of intersection?	Students are given time to solve this equation If help is required revisit the idea of equality Ask a student to explain their work or come to board and explain it?
Anticipated Student Responses Students solve the equation and use the answer to found to find the cost Students can't solve it and will ask for help Students solve equation but don't know what to do with the information to fine the cost	Listen to students. Help as requires
4. Comparing and Discussing Ask students to compare the answers they found to their answers found in previous lesson when reading from graph. ***Why do we need to do what we have just done?*** **R1:** It is more accurate. ***Why can't we read the answer from the graph?*** **R1:** Reading from graph is only an estimate.	Listen to feedback Write answers on board.
5. Summing up ***Why do we need to do what we have just done?*** **R1:** It is more accurate. **Why can't we read the answer from the graph?** **R1:** Reading from graph is only an estimate.	

Explain to students that what they have discovered is how to solve simultaneous equations. Ask them to write into their copybooks what they think this means. Compare and discuss what they have written. Distribute non-context simultaneous equations and ask students to find the point of intersection. (see below) Ask them to verify their solutions in both equations. **More Difficult Questions to Think About for Next Lesson:** ***What do we do when the coefficients are not the same?*** Give student an example to work on for the next day. Listen to students. Help as required Encourage to students to explain the work to others if required.	



Evaluation

Board Plan



Post–Lesson Reflection

The timing for the lesson was great and furthermore the teacher never directed the student to hurry up. The lesson was well prepared and built on a question asked at the end of the previous lesson.

The solutions were not limited and varied for different students. The students definitely recognised the limitations to the graph and the table for finding a point of intersection when it falls outside the set of natural numbers (which tend to have obvious POI from the table and the graph.) There was a desire for the students to learn what caused this discrepancy in their answers; this was obvious from the buzz around the room.

The students did grasp the concepts of equality of both variables but they did struggle with the resulting linear equation. This could be easily remedied by a short recap on linear equations before the lesson.

A big part of the lesson was for students to get more from simultaneous equations rather than just applying the algorithm, after realising the potential of simultaneous equations pupils will then perfect the algorithm before returning to more questions in context.

Positives of RoP

It was great to learn from other practising teachers, and see the different student potentials and their thinking processes.

Insights:

- Used rise over run instead of slope formulae
- Relied on where the graph cut the y-axis for c as opposed to putting $x = 0$ into equation of line
- Different colours for different lines helped
- All automatically put up stabilisers for solving linear equation: not a panacea

Surprising Ideas:

- some knew that reading from the graph would at best give an estimate

- mathematical language better than expected, could articulate what the point of intersection meant in this context
- Surprised at how many did understand equality,
- how talkative students are and can stay on task

Misconceptions:

- variables on both sides of the equals sign caused problems,
- When they did solve linear equation, ($x = 37.5$), they felt that it was adequate to read the y value from the graph

Difficulties:

- Could identify the correct formulae but didn't make the connection of equality of $y =$ to form a linear equation

Confusion:

- Could not solve the linear equation

Student Observation Record.

BEGINNING OF LESSON:

Observe level of difficulty with homework/previous class. If no difficulty tick the box for each student. If student has difficulty please identify issues.

	Student 1	Student 2	Student 3	Student 4	Student 5	Student 6
(i) Graphing						
(ii) Standing charge						
(iii) Point of Intersection						
(iv) Formulating formulae						
(v) Subbing POI into formulae						
Questions asked by students:						

DURING LESSON:

Observe student interaction. If no difficulty tick the box for each student. If student has difficulty please identify issues.

	Student 1	Student 2	Student 3	Student 4	Student 5	Student 6
(i) Questions asked to teacher						
(ii) Questions asked to other group members						
(iii) Identify when						

	student understood ***reading from graph is not most accurate method***					
(iv)	Identify when student understood ***Simultaneous equations***					
Other observations						

LESSON CONCLUSION:
Observe student interaction. Check student level of confidence with simultaneous equations question.

	Student 1	**Student 2**	**Student 3**	**Student 4**	**Student 5**	**Student 6**
Rate student understanding of POI/Reading from graph, Scale 1-3 where : 1= poor 2 = some understanding 3 = competent						
Rate student understanding of Simultaneous Equations, Scale 1-3 where : 1= poor 2 = some understanding 3 = competent						
Rate student understanding of non-context simultaneous equations, Scale 1-3 where : 1= poor 2 = some understanding 3 = competent						

Other observations						
LESSON CONCLUSION: **Observe student interaction.**						
	Student 1	**Student 2**	**Student 3**	**Student 4**	**Student 5**	**Student 6**
Other observations						
Issues that need to be addressed in the next class						
Recommended changes to lesson plan						

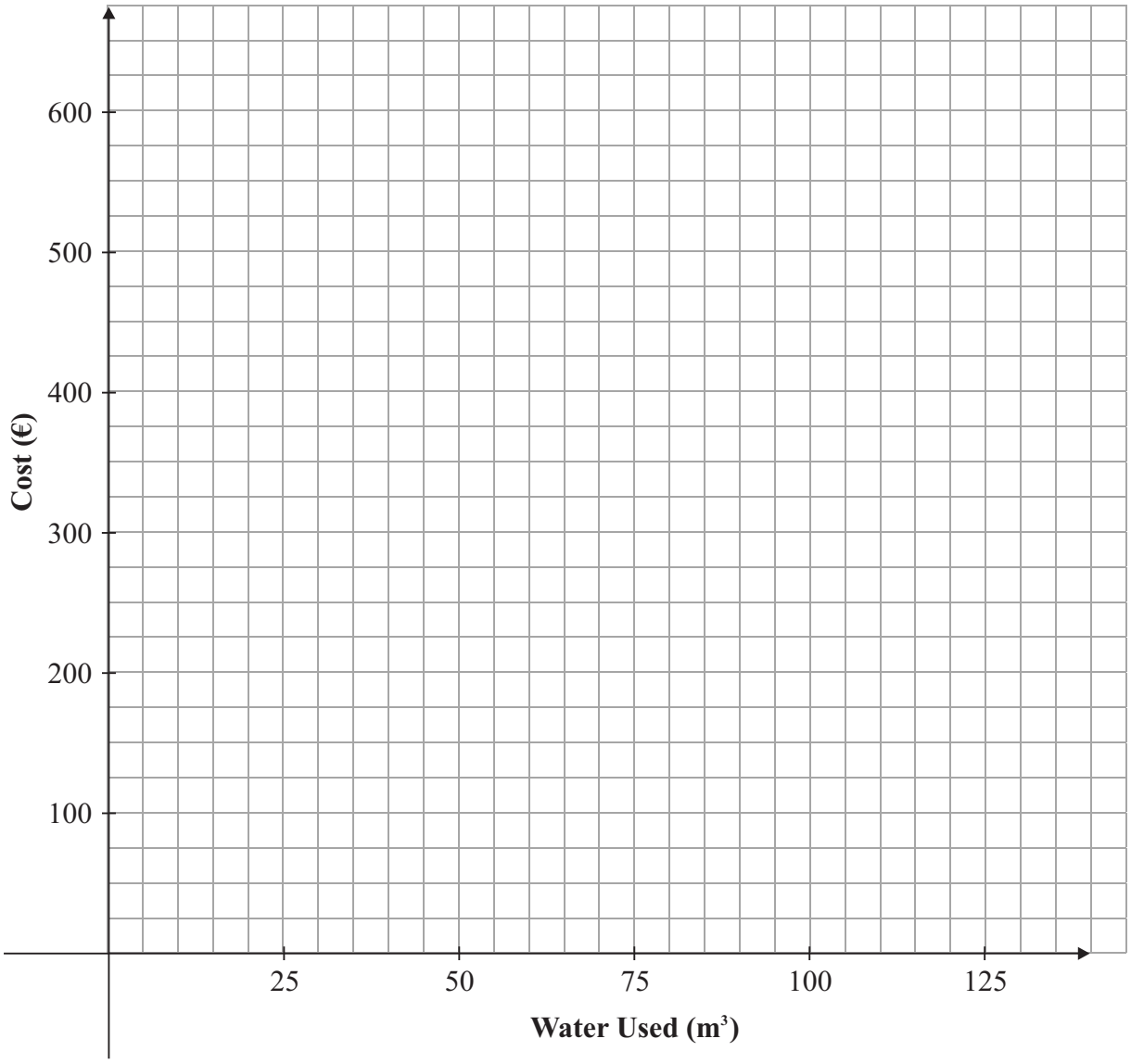
Question 6 **(suggested maximum time: 20 minutes)**

Peter and his family plan to move home to Ireland from the UK next year. He is concerned about the cost of water charges being introduced in Ireland.

The table below shows the proposed pricing structure for water in Ireland and the pricing structure of Peter's current water provider in the UK. Both pricing structures include an annual standing charge, even if no water is used, and the cost of different amounts of water used (in m^3).

Water Used (m^3)	IRL Cost (€)	UK Cost (€)
25	150	175
50	275	250
75	400	325
100	525	400
125	650	475

(i) Draw a graph to show the relationships between the amount of water used and the cost in both Ireland and the UK.



(ii) Use your graph to estimate the annual standing charge in both Ireland and the UK.

Ireland: _____ UK: _____

(iii) What level of water usage would result in the same annual charge in both countries?

(iv) Write down two formulae to represent the annual water charges in both Ireland and the UK. State clearly the meaning of any letters used in the formulae.

Ireland: UK:

(v) Use the formulae from part **(iv)** to confirm that your answer in part **(iii)** is correct.

(vi) Peter decides that he will install a rainwater harvester in his new home in Ireland. Based on his UK usage, he estimates that his household water usage will be 45 m^3 per quarter in Ireland. The rainwater harvester company claims that Peter will save one-third of his current water usage. The cost of installing the rainwater harvester is €1500.

How long will it take before Peter benefits financially from installing the rainwater harvester, based on current market rates?

(vii) Under what circumstances would Peter benefit earlier?

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