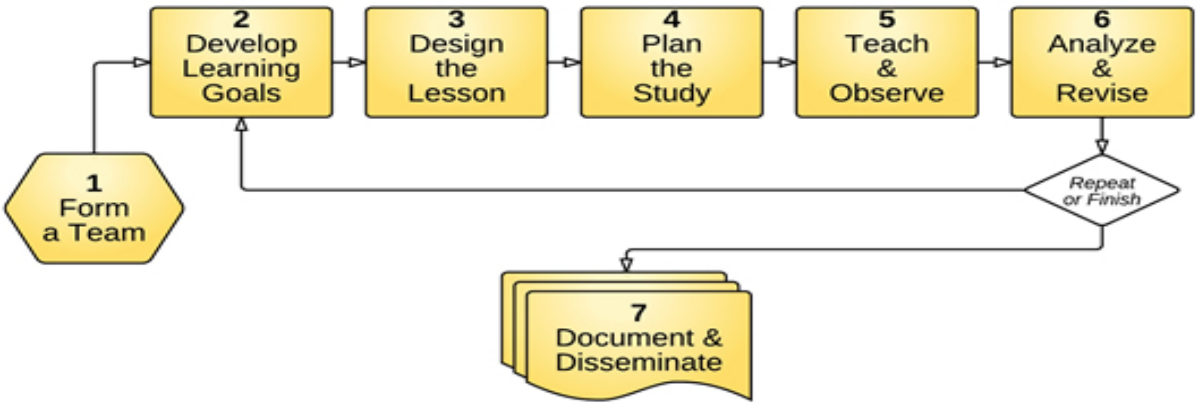
Reflections on Practice

The Difference of Two Squares



Project Maths Tionscadal Mata
Development Team

Lesson Plan for Second Year Maths: The Difference of Two Squares

For the lesson on *2-03-2015*
At *Laurel Hill Secondary School, Limerick, 2nd Year Maths* class
Teacher: *Noreen McKeogh*
Lesson plan developed by: *Noreen McKeogh, Maria Nolan, Olive Dillon*

1. **Title of the Lesson:** *Teaching the Difference of Two Squares for understanding*

2. **Brief description of the lesson:** *To help students reason towards and discover the difference of two squares formula using a hands-on geometric approach.*

3. **Aims of the Lesson:**

   I'd like my students to appreciate that mathematics can be used to communicate thinking effectively.
   I'd like my students to appreciate that algebra is a tool for making sense of certain situations.
   I'd like to foster my students to become independent learners
   I'd like to emphasise to students that a problem can have several equally valid representations.
   I'd like my students to connect and review the concepts that we have studied already

4. **Learning Outcomes**:

   Students to understand how the difference of two squares looks in a visual sense

   Students to understand patterns more deeply through the use of hands on materials

   Students to see Algebra as a tool to generalise from patterns

5. **Background and Rationale**

   The difference of two squares, a key algebraic skill, is usually taught in a procedural manner in that students are typically given the formula and practice applying it over and over without any real appreciation for what they are doing. For this reason, teachers find that they have to reteach this concept to students a few times before they "get it". This lesson attempts, through a geometrical and patterns based approach, to give students a deeper conceptual understanding from the outset. Hopefully this will lead to winning back some vital time as the teacher won't have to repeat the teaching as often.

6. **Research**

   Handbooks developed by the Project Maths Development Team and Sample Examination Papers.

7. **About the Unit and the Lesson**

This material is usually covered with students when they learn to factorise algebraic expressions in Second Year. This lesson is the introductory one on the difference of two squares.

8. Flow of the Unit:

(Project Maths Handbook)

Lesson	Section 4: Algebra	# of lesson periods
1	Revision and extension of algebraic expressions and simple linear equations from first year	4 classes
2	Linear equations in one and two variables and linear inequalities in one variable	8 classes
3	Adding algebraic fractions	2 classes
4	Algebraic factors	8 classes ***(Including Research Lesson)***
5	Solve quadratic equations	5 classes

9. Flow of the Lesson

Teaching Activity	Points of Consideration
1. Opening **Teacher distributes the 4 by 4 square grid to each student in the class.**	*Students are seated in groups of 5 but expected to work in pairs or threes.* *They also need a pair of scissors each.*
2 Prior Knowledge ***Opening Questions:*** **What shape is this?** **Why is it a square?** **What is the area?** **How do you know that is the area?** **16 is an example of a "square number". Can you give me some other examples of square numbers?**	*Teacher takes individual feedback quickly to assess prior knowledge* *Any student who gives "4 by 4" as an answer is asked for an efficient way to write this (4^2)* *Teacher should list the first 8/9 square numbers on the board.*
3. Student Activity 1 (SA1) **Ask each student to cut one square from the bottom right hand corner of the grid.** ***Teacher Questions:*** **What is the area of the square you have cut**	*Teacher needs to include "square units" in any discussion of area* *Students are asked to justify any answers they give.*

out? **What is the area of the remaining shape?** **How did you get this answer?** **Can you write "16 minus 1" using square numbers?** **Teacher challenges the group with this question:** **Can anybody confirm it's 15 by cutting what you have once and making another shape that we know how to get the area of?**	*Anticipated Student Responses* *"16 subtract 1 gives 15"* *"Just count them"* *It is really important that $4^2 - 1^2$ comes up in this discussion.* *Teacher circulates checking students' attempts.* *Students can show correct rearrangement to a rectangle to the rest of the class using a visualizer.* *It is important here that students realise that the difference of two squares can also be written as the product of sides of a rectangle.*
4. Student Activity 2 (SA2) **Teacher distributes the 10 by 10 square grid to each student.** **Teacher assigns different side lengths to each group to cut from this square grid.** **The results are collated in the table attached.**	*Teacher puts a table on the board like this:* 1. *Area of Large Grid (10^2)* 2. *Area Cut Out* 3. *Area of remaining rectangle (L X W)*
Teacher Questions **Can anyone see a relationship between the sides of the two squares and the dimensions of the rectangle formed?** **Can we describe the relationship that we found in words?**	*Teacher may need to guide students to see that the sides of the rectangle are formed by the product of the sum of the side lengths by the difference of the side lengths of the squares* *Students need to use the phrases "difference" and "square numbers" in their answers*
Possible Extension Questions: **How could we calculate the value of 57^2-55^2 ?** **What about 129^2-127^2?**	*Teacher needs to circulate and check how students are coping with this question.* *Students may need guidance with the second question.* *Students need to be given time to think about this question.*

10. Board Plan

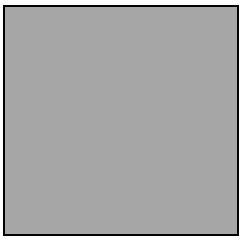
All Word handouts were projected on to an ordinary whiteboard and the teacher wrote over or discussed the key points of each one.

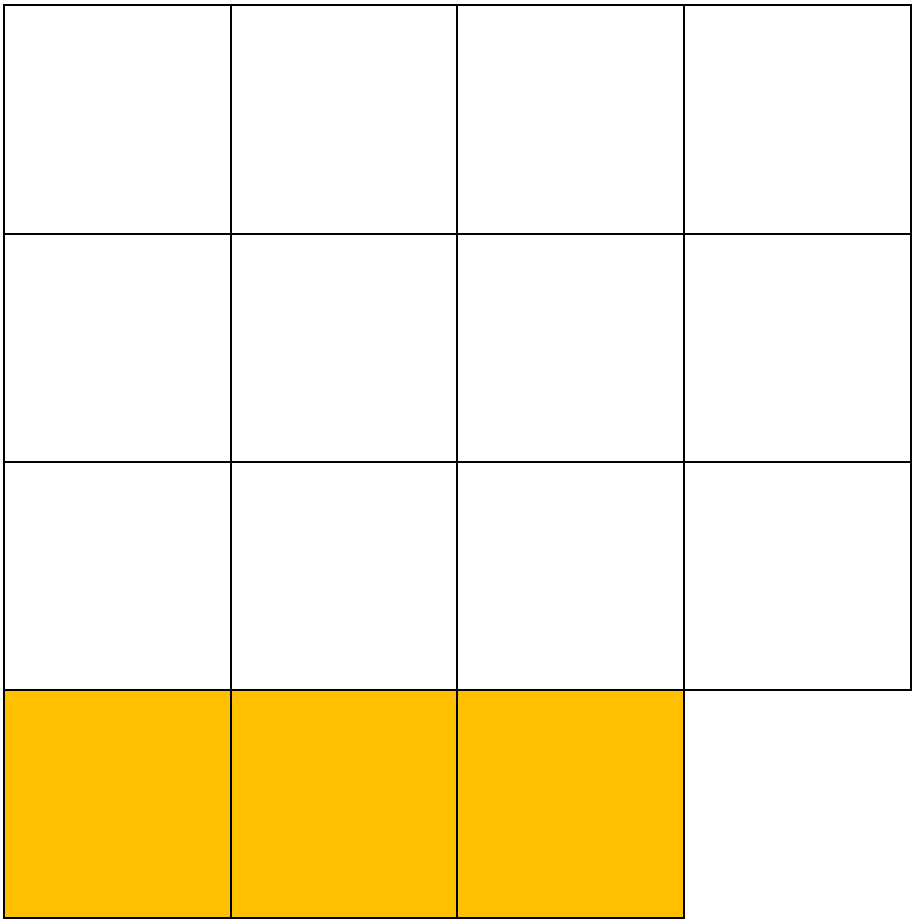
11. Post-lesson reflection

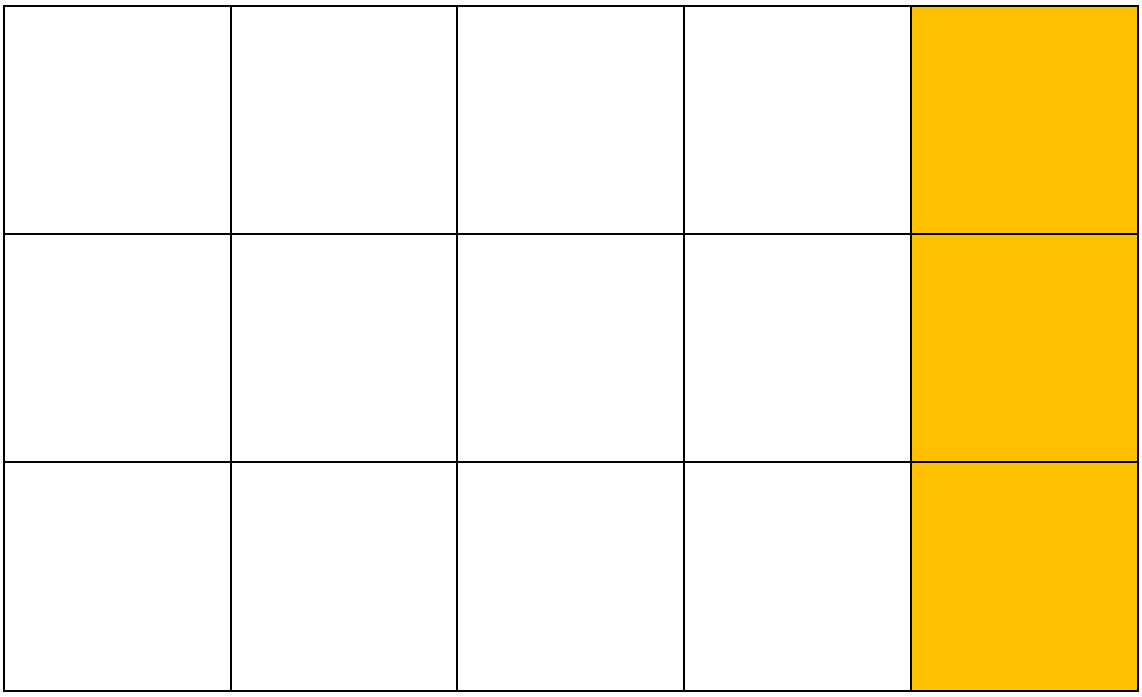
- *Students seemed to enjoy the hands-on approach more than just blindly applying a formula*
- *The planning before-hands enabled us to get quite a lot covered during the class and it seemed to flow very smoothly*
- *We weren't sure before the class if we would get to the general case but the planning probably enabled this to happen.*
- *Student identified the pattern quickly and they were able to apply it to problems straight away.*
- *There was some confusion regarding the remaining shape.*
- *Some students over complicated the task for themselves – the students with $10^2 – 6^2$ tried to make an 8x8 (a square number) out of the remaining shape.*
- *The lesson re-emphasised the fact that x squared represents a square of side x, this is another area in algebra that causes a lot of problems.*
- *Language can be an issue! It took a bit of questioning to get students to use the word "difference"*
- *The activity with the 100 square needs to be explained well to the class but is a powerful way to get to the general formula through a patterns approach.*
- *Before the lesson, students could work individually on an assessment task designed to reveal their current understanding and difficulties.*
- *Get students to clean up after first activity*
- *If you wished to shorten the class, demonstrate the first activity and get students to complete second activity (10 x 10) only*
- *The condensed version of the flow of the lesson was useful to have in the class itself*
- *We feel that this lesson would be perfect for a subject inspection visit!*
- *A physical model for the general formula $(x+y)(x-y)=x^2-y^2$ could also be used at the end of the lesson*

12. Handouts

Student Activity 1







Student Activity 2

Table for Collecting Data

Area of Large Square Grid (10^2)	Area of Cut Out Square	Area of remaining rectangle (l x w)
10^2	1^2	
10^2	2^2	
10^2	3^2	
10^2	4^2	
10^2	5^2	
10^2	6^2	
10^2	7^2	
10^2	8^2	
10^2	9^2	

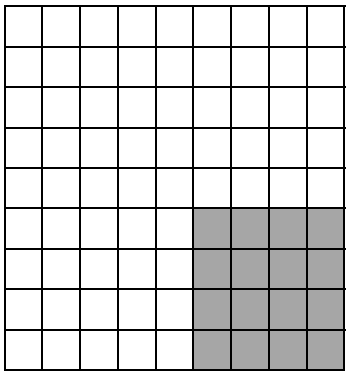
Post Lesson Worksheet

What have we discovered? Explain in your own words_____

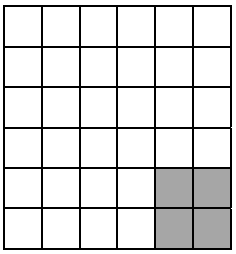
_____

Use what we have discovered to find the area of the remaining shape in the following questions (show all your work);

(i) $9^2 - 4^2$



(ii) $6^2 - 2^2$



(iii) $96^2 - 94^2$

(iv) $25^2 - 20^2$

(v) $23^2 - 17^2$

(vi) $59^2 - 57^2$

Can you write an expression for the remaining area in the following questions?

(i) $x^2 - y^2$

(ii) $9x^2 - 4y^2$

13. Condensed Flow for Reference

Distribute the 4 by 4 square grid & scissors to each student

Prior Knowledge: (Put 4x4 up on board) *Opening Questions:* What shape is this? Why is it a square? What is the area? How do you know that is the area? 16 is an example of a "square number". Can you give me some other examples of square numbers?	"4 by 4" as an answer, asked for an efficient way to write this (4^2) List the first 8/9 square numbers on the board. Discuss "square units" in area
SA1: (Put 4x4 up on board) Ask students to cut one square from the bottom right hand corner of the grid. *Teacher Questions:* What is the area of the square you have cut out? What is the area of the remaining shape? How did you get this answer? Can you write "16 minus 1" using square numbers? Challenge: Can anybody confirm it's 15 by cutting what you have once and making another shape that we know how to get the area of?	Ask students to justify any answers they give "16 subtract 1 gives 15" "Just count them" It is really important that $4^2 - 1^2$ comes up in this discussion It is important here that students realise that the difference of two squares can also be written as the product of sides of a rectangle.
SA2: (Put 10x10 up on board) Distribute the 10 by 10 square grid to each student. Assigns different side lengths to each group to cut from this square grid. The results are collated in the table on the board and get students to fill in their own table. Teacher Questions: Can anyone see a relationship between the sides of the two squares and the dimensions of the rectangle formed? Can we describe the relationship that we found in words? On the worksheet students write answers in their own words	Teacher puts a table on the board like this: 1. Area of Large Square Grid (10^2) 2. Area of Cut Out Square 3. Area of remaining rectangle (LxW) Teacher may need to guide students to see that the sides of the rectangle are formed by the product of (the sum of the side lengths) by (the difference of the side lengths) of the squares Students need to use the phrases **"difference"** and **"square numbers"** in their answers
Possible Extension Questions: *Worksheet*	Teacher needs to circulate and check how students are coping with these questions. Students may need guidance Students need to be given time to think about this question.