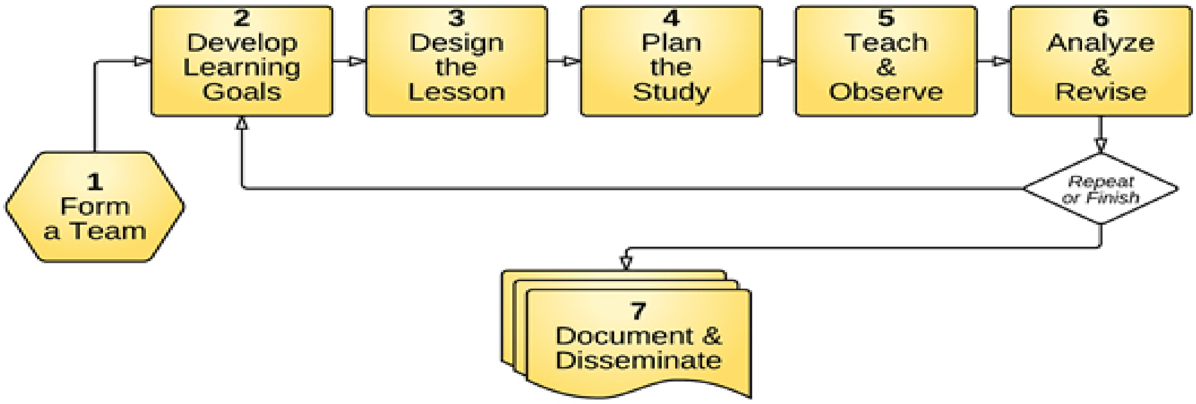
Reflections on Practice



Project Maths
Tionscadal Mata
Development Team

Lesson Plan for First Year: Equivalent fractions

Lesson plan developed by: Eamonn Conroy (Coláiste Einde, Galway)
Lorraine Corcoran (Scoil Mhuire, Strokestown)
Joanna Pres-Jennings (Coláiste Iognáid, Galway)

Title of the Lesson: **Fraction equivalence**

1. **Brief description of the lesson:** Using visual models (such as the fraction wall, sliced rectangles and fraction strips) to help the students gain a better understanding of equivalent fractions and proportional reasoning.

2. **Aims of the Lesson:**

Short-term goal We wish for the students:

- to gain a robust understanding of equivalent fractions,
- to develop meaning for fractions using concrete models.

Long-term goal We wish for the students:

- to understand rational numbers so that they are able to apply the relevant concepts in algebra,
- to develop strategies for dealing with rational numbers that would lead to a deeper understanding of operations associated with algebraic fractions,
- to develop capacity for logical explanation, justification and communication.

3. **Learning Outcomes:** At the end of the lesson students should be able to:

- develop, understand and model the algorithm for finding equivalent fractions by multiplying numerator and denominator by the same number
- reverse this process when reducing a fraction to its simplest form by dividing the numerator and denominator by the same number
- discuss and justify their findings.

4. **Background and Rationale**

According to the (Junior Certificate Mathematics Syllabus, 2015, p.22) students need to learn the following.

Strand/Description of topic	Learning Outcomes Students will be able to
Strand 3 : 3.1 Number systems Students learn strategies for computation that can be applied to any number; implicit in such computational methods are generalisations about numerical relationships with the operations being used. **Students articulate the generalisation that underlies their strategy**, firstly in vernacular language and then in symbolic language. Problems set in context, using diagrams to solve the problems so they can appreciate how the mathematical concepts are related to real life. Algorithms used to solve problems involving fractional amounts.	– generalise and articulate observations of arithmetic operations – **investigate models to help think about the operations of addition, subtraction, multiplication and division of rational numbers** – **consolidate the idea that equality is a relationship in which two mathematical expressions have the same value** – calculate percentages – **use the equivalence of fractions**, decimals and percentages **to compare proportions** – consolidate their understanding of the relationship between ratio and proportion – check a result by working the problem backwards

The major goal of this lesson is to act at an early stage on students' learning of fractions in view of potential future misconceptions related to operations on algebraic fractions. We have chosen this topic because we have come across many examples of mishandling algebraic fractions by Leaving Certificate students (including higher level classes). We identified the following issues:

a) Students lack strong mental representations of equivalence of fractions.

b) Students struggle in algebra from an incomplete understanding of fraction concepts.

$$\frac{x+3}{y+3}=\frac{x}{y}$$ (common misconception)

In the planning of this lesson we take into account that our first year students have already been taught fractions as the first year Teacher Handbook (Project Maths Development Team, 2012) suggests to teach this topic early in the first year programme (Sets and number systems). We are therefore going to use this lesson plan as a means of reinforcing the concepts that students met in the first semester. Our lesson plan is supplemented with Equivalent Fractions Puzzles to be used if the lesson goes very smoothly and students finish their work early.

5. Research

The learning outcomes for this lesson are aligned with those of the Junior Certificate Mathematics Syllabus as highlighted in Section 5 above. In the planning of the lesson Teaching & Learning Plan: Equivalent Fractions (Project Maths Development Team, 2014) has been adapted. It places the emphasis on building integrated mathematical understanding and capacity for logical explanation, justification and communication. It revisits the topic of equivalent fractions that students have been studying in fifth and sixth classes of primary school (Government of Ireland, 1999).

The target game at the beginning of the lesson is designed to engage students' interest. The idea comes from (Rooke, 2014). It has an active element and it stimulates thinking and discussion around ordering and equivalence of fractions.

Visuals such as the fraction wall, sliced rectangles and fraction strips are used in the lesson to enhance students' understanding of the concept of equivalent fractions. The Open University unit *Using visualisation in maths teaching* focuses upon how visualisation can be used to improve learning: *"If you really want to grasp a concept or idea, struggling to visualise it is worthwhile. There are many aids to visualisation. Diagrams or symbols on paper often help, or physical apparatus. Trying to 'say what you see' (or cannot see!) can be helpful, too. Visualisation and articulation go hand in hand."* (Open University, 2012)

A popular book *How to solve it* (Pólya, 1945) states: *"One of the most important tasks of the teacher is to help his students. […] The teacher should help, but not too much and not too little, so that the student shall have a reasonable share of the work"*. This lesson uses student centred investigative approach to teaching and intends to promote peer learning. Students are asked to discuss ideas in pairs, lead group investigation that addresses common misconceptions related to generating equivalent fractions and explain their findings to the rest of the class.

Mathematics can be viewed as a language in itself with its own vocabulary and grammar. Some everyday words can cause confusion for students, for example *cancel.* This lesson encourages the use of the more accurate word *simplify* when talking about writing fractions in equivalent but lower terms.

6. About the Unit and the Lesson

The lesson is designed in line with the objectives of (Junior Certificate Mathematics Syllabus, 2015, page 6) to develop learners' conceptual understanding, procedural fluency and adaptive reasoning seen as capacity for logical thought, reflection, explanation, justification and communication. The learning outcomes for the lesson correspond to those highlighted in Section 5 above (Junior Certificate Mathematics Syllabus, 2015, page 22).

Learning outcome of the lesson	Lesson design
• develop, understand and model the algorithm for finding equivalent fractions by multiplying numerator and denominator by the same number • reverse this process when writing a fraction in its simplest form by dividing the numerator and denominator by the same number	Group work: students investigate if the operation of multiplication, division, addition, subtraction applied to both numerator and denominator of a given fraction changes the value of the fraction. Each student generates a different fraction equal to 2/3. Students' answers are written on the board. Students realise that infinitely many fractions equivalent to 2/3 can be produced by multiplying numerator and denominator by the same number. In reverse, students are asked how one can make a simpler fraction equivalent to one of the fractions listed on the board. Student Activity sheet: students move between the concrete and symbolic representations of equivalent fractions.
• discuss and justify findings	Who was the best at the game? Students discuss in pairs and explain their reasoning. Group investigation addressing common misconceptions in generating equivalent fractions. One person from each group gives an account of the group's findings.

7. Flow of the Unit:

Lesson	Number system Q (11 class periods)	# of lesson periods
1	Fraction diagnostic test	1
2	Partitioning and ordering	1
3	Equivalent fractions	2
4	Addition and subtraction of fractions	2
5	Division and multiplication of fractions	2
6	Decimals	1
7	Percentages	2

8. Flow of the Lesson (detailed Lesson Plan attached to this document)

Teaching Activity	Points of Consideration
1. Introduction A target game for 3 students: tossing paper balls into a classroom paper bin from a distance. Student 1 gets 3 attempts, Student 2 gets 9 attempts, Student 3 gets 12 attempts. **(5 minutes)** Who was the best at this game? Students discuss in pairs and explain their reasoning. **(5 minutes)**	• Can students reason correctly on ordering the fractions? • Remind students to use 0, 1/2, 1 as benchmarks.
2. Posing the Task and Discussing What can we do to a fraction so that it looks different but has the same value? Investigate if the value of the fraction 4/12 changes after you: Group 1 add the same number to the numerator and denominator; Group 2 subtract the same number from the numerator and denominator; Group 3 multiply the numerator and denominator by the same number; Group 4 divide the numerator and denominator by the same number. (**5 minutes**)	• Ask students to look at the fraction wall if they are having difficulty. • Suggest to use a calculator as a checking tool. • Students who have good fraction sense should be able to explain it to the other persons in a group.
3. Anticipated Student Responses Example of possible explanation given by Group 1: We added 1 above and below. $\frac{4+1}{12+1}=\frac{5}{13}$.	• Ask representatives of four groups to report on their investigation results.

Then we used a calculator/looked at the fraction wall and found out that 5/13 is not the same as 4/12. (**5 minutes**)	
4. Summing up the investigation What can we do to a fraction to make it look different but keep the same value? What are we not allowed to do? In how many ways can I write the fraction 2/3? Can I have one for everyone in class? What do we call all these fractions that look different but have the same value? How can we produce "more complicated looking" equivalent fractions? How can we simplify a "more complicated looking" fraction? Is there a way we could use equivalent fractions to compare the results of the game that we had at the beginning of the lesson? **(5 minutes)** Working in pairs complete the Student Activity sheet. **(10 minutes)**	• Students should realise that there are infinitely many fractions equivalent to any one fraction (link to an infinite set). • Avoid the word *cancel* as students often use it erroneously. • Can students move between the concrete and symbolic representation? • Students who know the rule for forming equivalent fractions may use the rule first and then decide how to partition the picture. The exercise should give meaning to the rule for them.

9. Evaluation

Observational strategy: using notes to lesson plan (Appendix 1: Student Observation Record).

Evidence collected: student work (activity sheets, photographs of student work).

10. Board Plan

Game results (*for example*): Student 1: 1/3 Student 2: 2/9 Student 3: 5/12 Investigation results: Group 1: Group 2: Group 3: Group 4:	Fraction wall on the screen. Student Activity sheet displayed on the screen in case common problems need to be addressed.

11. Post-lesson reflection

Our overall reflection is that the lesson worked. Quite a lot of pre planning was necessary to make it successful.

Learning outcomes
According to the lesson plan the learning outcomes are:

a) Students develop, understand and model the algorithm for finding equivalent fractions by multiplying the numerator and denominator by the same number.
b) Students reverse this process when simplifying a fraction by dividing the numerator and denominator by the same number.
c) Students discuss and justify their findings.

One of our classes was fortunate to have an undergraduate to observe the learning. This person was familiar to the students and this meant that the students were quite comfortable in opening up and discussing various aspects of the lesson. Giving the weaker student the opportunity to be the spokesperson worked really well. Taking the more capable students out and allowing them to work with those who had difficulties was successful. It was noticed in one of our schools that the students liked board writing and that those who are better at maths tended not to doubt themselves up at the board.

The students were able to apply their knowledge to other problems by the end of the lesson. In one of our classes they were able to tell that the concept of equivalent fractions could be applied to a sporting context: - if a footballer scored 7 out of 9 penalties, was that better than a footballer who scored 5 out of 6 penalties? The students were able to answer which was best by comparing the fractions $\frac{14}{18}$ and $\frac{15}{18}$.
The majority of students were able to complete the Student Activity sheet. It was felt, however, that the students benefited from being guided through the questions.

Overall, the lesson seems to have achieved its learning outcomes.

Learning tasks
The lesson incorporated the active, investigative and visual types of tasks. The game at the beginning of the lesson worked really well and contributed to students seeing mathematics as sensible and useful (fostering productive disposition). We felt that it was an interesting task that stimulated good discussion. It helped to determine if students can reason correctly on ordering the fractions and linked with forming equivalent fractions in the summary section of the lesson. Having the fraction wall reflected up on the board was beneficial. Students also had a copy of it on their desks.

<u>Student difficulty</u>: In one of our classes the groups investigating the effect of adding/subtracting the same number to the numerator and the denominator did not seem to understand the task very well as if they did not know why they were doing it being used to using multiplication and division for forming equivalent fractions.

Although it was felt that the students needed to be guided through the visual Question 1 in Student Activity, the task might be useful when a rectangular model is used later for operations on fractions.

Instrumental and conceptual learning
Procedural and conceptual learning did take place during the lesson.

<u>Student misconception</u>: Our discussion revealed that one group were ordering the fractions by only looking at the denominators – “The bigger the denominator, the smaller the fraction”- without considering the numerators. It was noted in another school that the ordering of the game results was explained by the students in a visual way (fraction wall) and one student came up with the idea of bringing all game results to twelfths straight away.

The students were able to make equivalent fractions by the end of the lesson.

$\frac{2}{3} = \frac{4}{6} = \frac{6}{9} = \frac{8}{12} = \frac{10}{15}$

Student interpretation: Some students could see the pattern of equivalence but explained it as adding 2 to the numerator and adding 3 to the denominator. In the end they did use the word multiply.

The visual nature of the activity worksheet contributed to their conceptual learning of this topic.
Possible student difficulty: The students benefited from being guided through these questions, otherwise time may have been an issue. It was found that the weaker students struggled with the third problem where they were provided with the opportunity to create their own equivalent fractions.

In a summary of the investigation students found it difficult to comprehend that what they were working on was equivalence. They used the word “equal” as opposed to the word “equivalent”.
It was also found that the more capable students used the correct terms of numerator and denominator as opposed to the “top” and “bottom”. In one of our classes students did not see “simplify a fraction” as meaning to form an equivalent fraction.

As the lesson progressed it was brilliant to observe that the students did not use the word “cancel” at all during the lesson.

Revising the lesson: Possible extensions
To further consolidate the lesson on equivalent fractions, a set of playing cards was made with a fraction on each card. These fractions formed pairs of equivalent fractions, eg. $\frac{1}{3}$ and $\frac{3}{9}$. A game was played based on the card game “Old Maid”. One card was taken out of the pack and the rest of the cards were dealt out to the students. Each student looked at their cards, paired up any equivalent fractions and put these pairs down on the desk. One at a time a student would pick a card from their neighbour on their left and if it paired up with anything in their hand this pair would be put down on the desk. The object of the game is to get rid of all your cards by pairing them up. The winner is the person who has no cards left and the loser is the person left with one card at the end.

In one school a group showed great competency at this game having experienced equivalent fractions in the lesson. Another teacher’s class played this game prior to the lesson on equivalent fractions. Only the really good students were able for it. After teaching the lesson and going back to the game again the students did improve.

As another assessment task one could use a fraction domino activity in the following lesson (included in Appendix 2: Lesson Plan for First Year: Equivalent Fractions). Instruction: cut along the heavy lines giving pairs of fractions. Ask students what strategies they were using to match up the pairs.

Alternatively, if a teacher cuts each fraction out singly, leaving out the START and FINISH cards from the dominoes, they will have 22 cards. With a group of 3, one student can be the dealer and deal out 7 cards to each person in the group including themselves. This leaves one card which they can leave face down. The person left with the card matching this one at the end of the game is the loser.
Rules:

The dealer deals out all the cards to the players leaving one card face down in the centre of the table. The players all look at their cards and discard any pairs they have (a pair is two cards which are equivalent fractions).

The dealer begins. At your turn you must offer your cards spread face down to the player to your left. That player selects a card from your hand without seeing it, and adds it to her hand. If it makes a pair in her hand she discards the pair. The player who just took a card then offers her hand to the next player to her left, and so on.

If you get rid of all your cards you are safe and you take no further part. The turn passes to the next player to your left, who spreads his or her cards for the following player to draw one. Eventually all the cards will have been discarded except one and the holder of this card loses. This card should match the card in the centre of the table.

If at the end the odd card does not match the centre card, then an error has been made and the students have to collectively check each pair formed and decide if they have been correctly matched. They should be left down tidily in pairs as the game progresses to ensure that they are easy to check and also to allow the teacher to do random checks as the game is progressing.

Students could be exposed to the highest common factor and the lowest common multiple in this lesson. When simplifying fractions students divide the numerator and denominator by a common value (common divisor) which is a common factor of both. When deciding who won the game, students would see that when all the fractions were written with denominator 36 (the LCM) they could be compared.

Depending on the pace of the lesson/capability of the students time might be an issue. Replacing the group investigation with a quicker pair work and class discussion might be worth considering. Also an alternative worksheet could be used (included in Appendix 2: Lesson Plan for First Year: Equivalent Fractions) where problems 1 and 3 could be done in class and problems 2 and 4 left as homework.

$\frac{10}{12}$	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{20}{25}$
$\frac{2}{3}$	$\frac{56}{80}$	$\frac{1}{2}$	$\frac{1}{3}$
$\frac{3}{9}$	Finish	$\frac{2}{5}$	$\frac{20}{30}$
$\frac{1}{6}$	$\frac{6}{8}$	$\frac{7}{10}$	$\frac{2}{12}$
$\frac{4}{16}$	$\frac{8}{20}$	$\frac{3}{10}$	$\frac{10}{20}$
$\frac{4}{5}$	$\frac{9}{30}$	Start	$\frac{5}{6}$

12. Bibliography

[1] Government of Ireland, 1999. *Mathematics Teacher Guidelines.* Dublin: Stationary Office.

[2] Junior Certificate Mathematics Syllabus, 2015. *Junior Certificate Mathematics Syllabus (For examination from 2016).*

[3] Open University, 2012. *Using visualisation in maths teaching.* [Online]
Available at: http://www.open.edu/openlearn/education/using-visualisation-maths-teaching/content-section-0
[Accessed 15 January 2015].

[4] Pólya, G., 1945. *How to solve it.* s.l.:Princeton University Press.

[5] Project Maths Development Team, 2012. *First Year Teacher Handbook.* [Online]
Available at: www.projectmaths.ie
[Accessed 15 January 2015].

[6] Project Maths Development Team, 2014. *Teaching and Learning Plan: Fractions Equivalence.* [Online]
Available at: www.projectmaths.ie
[Accessed 15 January 2015].

[7] Rooke, R., 2014. *Teaching Ideas.* [Online]
Available at: www.teachingideas.co.uk
[Accessed 15 January 2015].

Appendix 1: Student Observation Record

<table>
<tr><td colspan="3">LESSON ON EQUIVALENT FRACTIONS</td></tr>
<tr><td></td><td></td><td>Observations</td></tr>
<tr><td rowspan="2">1. Prior knowledge</td><td><u>Target game</u>
• Are students interested in what is going on?
• Are they willing to get involved in the game?
• Are they discussing the problem of ordering of with each other?</td><td rowspan="2"></td></tr>
<tr><td><u>Ordering fractions</u>
• Are students having difficulty with ordering fractions?
• Can students reason correctly on ordering the fractions?
• Do they need more practice with “ordering” type questions?</td></tr>
</table>

Questions asked to teacher

Questions asked to other classmates

		Group 1 (+)	Group 2 (-)	Group 3 (x)	Group 4 (÷)
2. Investigation (group work)	<u>Identifying misconceptions</u> • Do students understand the task? • Are they cooperating? • If students have difficulty please identify issues				
	Students' explanations re equivalence • Are students surprised by other groups' results? • Are the misconceptions clarified? • Are students using the fraction wall or a calculator? • Please check students' confidence and clarity when reporting to the class. • What questions do students ask to other groups/teacher?				

		Observations
3. Summary of investigation	• Are students able to answer teacher's questions? • Are they able to summarise findings at conclusion of investigation? • Do students see the point of other groups work? • Do students realise that there are an infinite number of equivalent fractions for any one fraction? • Can every student produce a fraction equivalent to 2/3? • Can students explain how to simplify a fraction? • Do they use the word 'cancel' or 'simplify'? • Are they able to compare the results of the target game using equivalent fractions?	

Appendix 1: Student Observation Record

	Rate student understanding of Student Activity sheet	Student 1	Student 2	Student 3	Student 4	Student 5
4. Work on student activity sheet	**Problem 1** Scale 1-3 where : 1= poor 2 = some understanding 3 = competent					
	Problem 2 Scale 1-3 where : 1= poor 2 = some understanding 3 = competent • **Can students move between the concrete and symbolic representation?** • **Do students use the rule first and then decide how to partition the picture? Or the other way around?**					
	Problem 3 Scale 1-3 where : 1= poor 2 = some understanding 3 = competent					

OTHER OBSERVATIONS	
Issues that need to be addressed in the next class	
Recommended changes to lesson plan	

Students' Learning Tasks: Teacher Input	Student Activities: Possible Responses	Teacher's Support and Actions	Checking Understanding
Introduction			
Set up a target game for 3 pupils. This could be tossing a paper ball into a classroom waste paper bin from a set distance. Give each pupil in the group a try at the target game BUT restrict how many attempts they can have e.g. Student 1 - 3 attempts Student 2 - 9 attempts Student 3 - 12 attempts. Record the successes on the board e.g. Student 1 – 1/3 (1 on target, out of three attempts) Student 2 – 2/9 (2 on target, out of 9 attempts) Student 3 – 5/12 (5 on target, out of 12 attempts). T: Who was the best at this game? Discuss in pairs and explain your reasoning. Use the fraction wall/number line to help visualise the answers. I want one person from each pair to give an explanation for the ordering they have used. Does everyone agree with the final positions and reasoning?	5/12 is larger than 1/3 for example, because it is closer to 1/2	Give each of the three students a bag with 3, 9 and 12 paper balls, respectively. Give each student a sheet with a **fraction wall**. Move around the room and check students' work. Where there are difficulties ask: • Which is the smallest? • Which is bigger 2/9 or 5/12? Remind students to use 0, ½ and 1 as benchmarks. Teacher writes the fractions in order as given by the students.	If students can reason correctly on ordering the fractions then they can move on to equivalent fractions and the operations of addition and subtraction. If they are having difficulty they must use drawings of fraction strips/ the number line and get more practice with these "ordering" types of questions. Students who have good fraction sense should be able to mentally visualise fractions in relation to 0, ½ and 1. They should be able to explain it to the other person in a pair.
Name a fraction which is closer to 1 than 6/7? Why is 6/8 not bigger than 6/7?	7/8, 8/9, 9/10 Same number of parts but each one is smaller in the case of 6/8	Ask students to look at the fraction wall if they are having difficulty. (Fraction wall on the screen also.)	Can students reason that it should have smaller parts and more of them?

Equivalent fractions – investigation			
T: What can I do to a fraction so that it looks different but has the same value? I want you to investigate if the value of the fraction 4/12 changes after you: Group 1 add the same number to the numerator and denominator; Group 2 subtract the same number from the numerator and denominator; Group 3 multiply the numerator and denominator by the same number; Group 4 divide the numerator and denominator by the same number. I want one person from each group to give an account of your investigation.	Group 1: We added 1 above and below. $\frac{4+1}{12+1} = \frac{5}{13}$. Then we used a calculator and found out that 5/13 is not the same as 4/12. Group 2: We subtracted 3 above and below. $\frac{4-3}{12-3} = \frac{1}{9}$. Then we looked at a fraction wall and found out that 1/9 is not the same as 4/12. Group 3: We multiplied the numerator and denominator by 2. $\frac{4\times 2}{12\times 2} = \frac{8}{24}$ and we verified using a calculator that 8/24=4/12. Group 4: We divided the numerator and denominator by 2. $\frac{4\div 2}{12\div 2} = \frac{2}{6}$. Then we looked at a fraction wall and found out that 2/6 is the same as 4/12.	Group work: Students investigate if the operation of multiplication, division, addition, subtraction applied to both numerator and denominator of a given fraction changes the value of the fraction. Move around the room and check students' work. Suggest to use a calculator as a checking tool.	Are the misconceptions concerning creating equivalent fractions clarified? Can students clearly explain their findings?

Summary of the investigation			
We will **summarise** our investigation now: what can we do to a fraction to make it look different but keep the same value? And what can we not do? In how many ways can I write the fraction 2/3? Can I have one for everyone in class? What do we call all these fractions that look different but have the same value? How can we produce equivalent fractions using bigger numbers? How can we create a simpler equivalent fraction for any given fraction? Explain how you do it. Is there a way we could use equivalent fractions to compare the results of the game that we had at the beginning of the lesson?	We can multiply or divide the numerator and denominator by the same number. But when we add or subtract the same number above and below we get a fraction that is not equal to the original fraction. Each student generates a different fraction by multiplying the numerator and the denominator of 2/3 by the same number. Students realise that there are infinitely many ways of writing one fraction. Equivalent fractions. We multiply the numerator and denominator by the same number. We divide the numerator and denominator by the same number. *Example:* 1/3 is equivalent to 3/9 and 3/9 is bigger than 2/9, so 1/3 is better than 2/9. On the other hand, 1/3 is equivalent to 4/12 which is smaller than 5/12, so 5/12 is the best result.	Teacher writes down all the fractions on the board/flipchart. Students may say equal and not equivalent which is the opportunity to point out that equivalent means equal in value but not necessarily identical. Best to avoid the word “cancelling” as students often use it erroneously.	Can students see the point of other groups considerations? Do students realise that there are an infinite number of equivalent fractions for any one fraction? (Link to an infinite set.) Do students realise that they can only divide the numerator and denominator by a factor of both in order to generate a simpler equivalent fraction? (Link to the highest common factor.) Link to the lowest common multiple when comparing the game results.
Student Activity			
Working in pairs complete the **Student Activity** sheet. **Problem 1** What fraction is shaded in each rectangle? Possible questions for students having trouble with the task: Look at this rectangle divided into fourths by vertical lines. What fraction is shaded? (3/4)		Teacher displays the **Student Activity** sheet on the screen in case common problems need to be addressed. If there are pairs of students who finish the activity sheet early the teacher may give them the **Equivalent Fraction**	

Now you sliced this rectangle with 1 horizontal line. Into how many pieces is the original rectangle divided? (8) How many eighths equal 3/4? You divided the second rectangle using 2 horizontal lines. How many equal pieces do we have now in the rectangle? (12) You now have 3 times as many parts in the whole rectangle and in the shaded part. How many pieces will make 3/4? (9) What happens to the numerator and the denominator of 3/4 to make 9/12? Can you explain in the same way why 3/4=24/32? **Problem 2** (can be left as homework) **Problem 3** Generate equivalent fractions. Fill in the missing numbers in the boxes below and justify your reasoning. **Problem 4** (can be left as homework)	Both numerator and denominator are multiplied by 3. There are 8 times as many parts in the whole rectangle after dividing it into 8 equal horizontal parts compared to the original rectangle with four parts. To choose the same shaded amount we need 8 times the number of equal parts. Both numerator and denominator are multiplied by 8. Students either use the rule or partition first to find the missing number.	**Puzzles.** Students who know the rule for forming equivalent fractions will use the rule first and then decide how to partition the picture. The exercise will give meaning to the rule for them.	Do students see the pattern (fixed ratio as opposed to fixed difference) between the numerator and denominator with equivalent fractions? Can students move between the concrete and symbolic representations of equivalent fractions to increase their understanding?
Closing			
Summarise what you mean by equivalent fractions.	Fractions representing the same amount even if the partitions (divisions of the whole into equal parts) are different. To generate them you either multiply or divide the numerator and denominator by the same number.	Teacher asks a few students to state in their own words what equivalent fractions mean.	

Equivalent Fraction Puzzles

1. I am equivalent to ½ .
The sum of my numerator and denominator is 15.
What fraction am I?

2. I am equivalent to $\frac{2}{5}$.
The product of my numerator and denominator is 40.
What fraction am I?

3. I am equivalent to $\frac{2}{3}$.
My denominator is 10 more than my numerator.
What fraction am I?

4. I am equivalent to $\frac{80}{100}$.
My denominator is a prime number.
What fraction am I?

1 unit or 1 “whole”														
$\frac{1}{2}$	$\frac{1}{2}$													
$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$												
$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$											
$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$										
$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$									
$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$								
$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$							
$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$						
$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$					
$\frac{1}{11}$	$\frac{1}{11}$	$\frac{1}{11}$	$\frac{1}{11}$	$\frac{1}{11}$	$\frac{1}{11}$	$\frac{1}{11}$	$\frac{1}{11}$	$\frac{1}{11}$	$\frac{1}{11}$	$\frac{1}{11}$				
$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$			
$\frac{1}{13}$	$\frac{1}{13}$	$\frac{1}{13}$	$\frac{1}{13}$	$\frac{1}{13}$	$\frac{1}{13}$	$\frac{1}{13}$	$\frac{1}{13}$	$\frac{1}{13}$	$\frac{1}{13}$	$\frac{1}{13}$	$\frac{1}{13}$	$\frac{1}{13}$		
$\frac{1}{14}$	$\frac{1}{14}$	$\frac{1}{14}$	$\frac{1}{14}$	$\frac{1}{14}$	$\frac{1}{14}$	$\frac{1}{14}$	$\frac{1}{14}$	$\frac{1}{14}$	$\frac{1}{14}$	$\frac{1}{14}$	$\frac{1}{14}$	$\frac{1}{14}$	$\frac{1}{14}$	
$\frac{1}{15}$	$\frac{1}{15}$	$\frac{1}{15}$	$\frac{1}{15}$	$\frac{1}{15}$	$\frac{1}{15}$	$\frac{1}{15}$	$\frac{1}{15}$	$\frac{1}{15}$	$\frac{1}{15}$	$\frac{1}{15}$	$\frac{1}{15}$	$\frac{1}{15}$	$\frac{1}{15}$	$\frac{1}{15}$

1 unit or 1 “whole”

$\frac{1}{2}$ | $\frac{1}{2}$

$\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$

$\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$

$\frac{1}{5}$ | $\frac{1}{5}$ | $\frac{1}{5}$ | $\frac{1}{5}$ | $\frac{1}{5}$

$\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$

$\frac{1}{7}$ | $\frac{1}{7}$ | $\frac{1}{7}$ | $\frac{1}{7}$ | $\frac{1}{7}$ | $\frac{1}{7}$ | $\frac{1}{7}$

$\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{8}$

$\frac{1}{9}$ | $\frac{1}{9}$ | $\frac{1}{9}$ | $\frac{1}{9}$ | $\frac{1}{9}$ | $\frac{1}{9}$ | $\frac{1}{9}$ | $\frac{1}{9}$ | $\frac{1}{9}$

$\frac{1}{10}$ | $\frac{1}{10}$ | $\frac{1}{10}$ | $\frac{1}{10}$ | $\frac{1}{10}$ | $\frac{1}{10}$ | $\frac{1}{10}$ | $\frac{1}{10}$ | $\frac{1}{10}$ | $\frac{1}{10}$

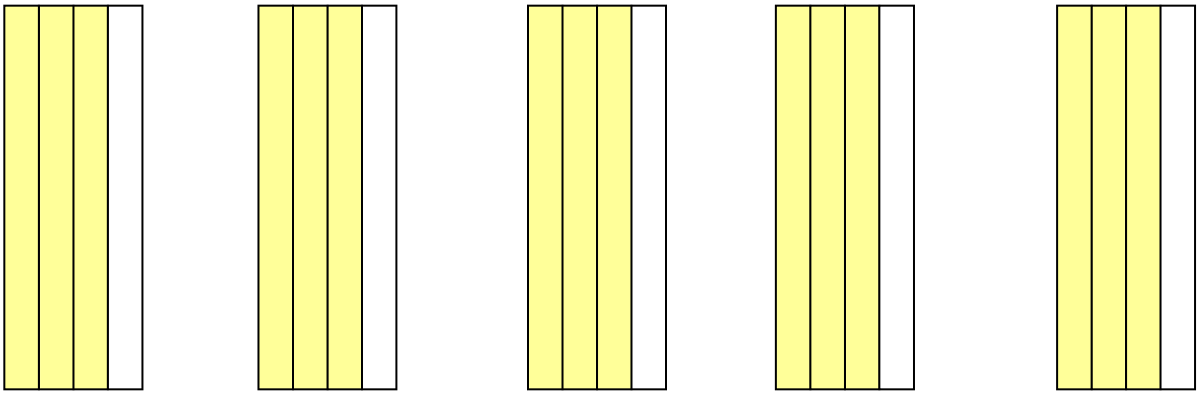
$\frac{1}{11}$ | $\frac{1}{11}$ | $\frac{1}{11}$ | $\frac{1}{11}$ | $\frac{1}{11}$ | $\frac{1}{11}$ | $\frac{1}{11}$ | $\frac{1}{11}$ | $\frac{1}{11}$ | $\frac{1}{11}$ | $\frac{1}{11}$

$\frac{1}{12}$ | $\frac{1}{12}$ | $\frac{1}{12}$ | $\frac{1}{12}$ | $\frac{1}{12}$ | $\frac{1}{12}$ | $\frac{1}{12}$ | $\frac{1}{12}$ | $\frac{1}{12}$ | $\frac{1}{12}$ | $\frac{1}{12}$ | $\frac{1}{12}$

$\frac{1}{15}$ | $\frac{1}{15}$ | $\frac{1}{15}$ | $\frac{1}{15}$ | $\frac{1}{15}$ | $\frac{1}{15}$ | $\frac{1}{15}$ | $\frac{1}{15}$ | $\frac{1}{15}$ | $\frac{1}{15}$ | $\frac{1}{15}$ | $\frac{1}{15}$ | $\frac{1}{15}$ | $\frac{1}{15}$ | $\frac{1}{15}$

Student Activity (Answer Q1, Q3 in class and Q2, Q4 at home)

1. **What fraction is shaded in each rectangle below?**



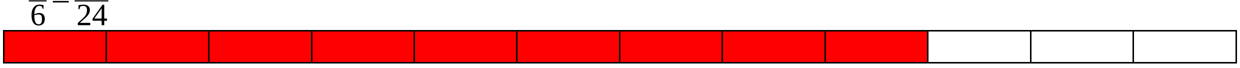
Now, use horizontal lines to divide each rectangle into a **different** number of equal parts.
Under each sliced rectangle write an equation showing how many smaller pieces will make 3/4.

2. **Find the equivalent fraction in each case. Show on the fraction strip why your answer makes sense.**

$\frac{3}{5} = \frac{\square}{15}$

$\frac{2}{3} = \frac{\square}{12}$

$\frac{5}{6} = \frac{\square}{24}$



$\frac{9}{12} = \frac{\square}{4}$

3. **Fill in the missing numbers in the equivalent fractions below.**

$\frac{3}{\square} = \frac{1}{12}$	$\frac{3}{\square} = \frac{12}{16}$
$\frac{5}{\square} = \frac{\square}{\square}$	$\frac{4}{\square} = \frac{\square}{\square}$
$\frac{1}{\square} = \frac{\square}{12}$	$\frac{5}{\square} = \frac{\square}{4}$

4. Jane got 10 out of 15 for her test and Mark got 15 out of 20 in his test. Anita said that they both did equally well because they both got 5 wrong. Is Anita correct? Explain your answer.