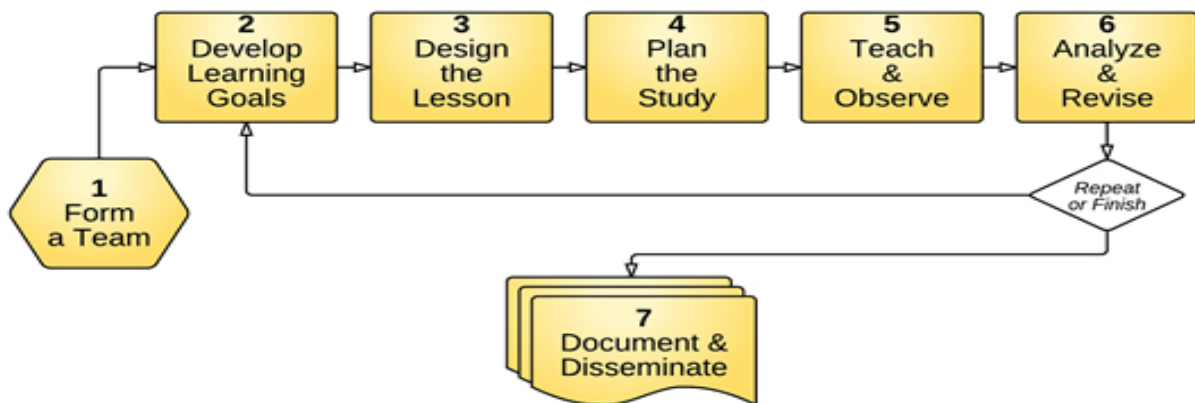


Reflections on Practice



Lesson Plan for First Year: Division of fractions

For the lesson on 4th February 2015
At St Joseph's College, (The Bish), Galway: PJ Folan's 1st Year Class
Teacher: PJ Folan
Lesson plan developed by: [PJ Folan, Stefanie Carr, Derek Murray]

1. **Title of the Lesson:** Visualising and Making Sense of the "Invert and Multiply" Algorithm for Fraction Division: *A Junior Cycle Approach*

2. **Brief description of the lesson:** To get students to visually represent the operation of division of fractions thereby giving them a better understanding of why the "Invert and Multiply" algorithm works.

3. **Aims of the Lesson:**

a. *Long-range/thematic goals:*

I'd like my students to recognise that the division of fractions is an extension of the division of whole numbers.

I'd like to encourage my students to become independent learners.

I'd like my students to become more creative when devising methods to solve problems.

I'd like to emphasise to students that a problem can have several equally valid solutions.

I'd like my students to experience meaningful mathematics, i.e. that they see a need for what they are studying.

I'd like to build my students' enthusiasm for the subject by engaging them with stimulating activities.

I'd like my students to connect and review the concepts that we have studied already.

b. *Short-term goals:-*

Students should understand and demonstrate division of fractions using a variety of methods.

4. **Learning Outcomes:**

As a result of studying this topic students will be able to:

- Explain and justify methods of division to each other.
- Divide a whole number by a whole number.
- Represent division of fractions using diagrams.
- Divide a whole number by a fraction.
- Divide a proper fraction by a proper fraction.
- Divide an improper fraction by an improper fraction.

5. **Background and Rationale**

(a) Students need to be able to "investigate models to help think about the operations of addition, subtraction, multiplication and division of rational numbers (Strand 3, Number Systems, 3.1 Junior Certificate Syllabus).

(b) Difficulties encountered by students:- pupils are too attached to the rote learning approach of division of fractions without having any real understanding of the reasons why. Students also have difficulties representing fractions using diagrams.

(c) To develop a relational understanding of division of fractions. This is fundamental to all aspects of the syllabus.

6. Research

The following materials were used to research this area:-

- Junior Certificate Mathematics Syllabus
- Project Maths Teaching and Learning Plans specific to fractions
- Maths Trailblazers, Grade 5 (Dubuque: Kendall/Hunt, 1997) pp. 290, 291
- Exploring Project Maths- Book 1 – Seamus McCabe
- The following websites:
<http://fawnnguyen.com/fraction-division-via-rectangles/>
<http://www.mathsisfun.com>
<http://www.nrichmaths.co.uk>

7. About the Unit and the Lesson

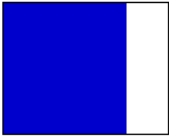
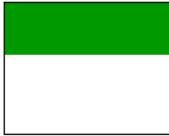
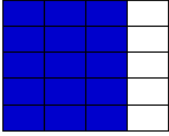
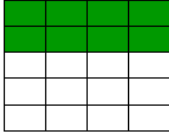
This lesson is part of a unit on fractions. Prior to this unit, pupils would have covered addition, subtraction, multiplication and division of both natural numbers and integers (Strand 3: Number 3.1, Number Systems).

The goal is to help students visually represent division of fractions by fractions. This will lead them to a deeper understanding and appreciation of the standard algorithm. By drawing diagrams students will quickly realise how best to represent fractions. They will learn that rectangles are better than pie-chart diagrams. They will see the need for a common denominator for operations on fractions and the importance of equivalent fractions in getting a common denominator.

8. Flow of the Unit:

Sequence	Title: Fractions	
1	<ul style="list-style-type: none">• Overview of Fractions	
2	<ul style="list-style-type: none">• Partitioning the Whole / Unit into Equal Parts	
3	<ul style="list-style-type: none">• Ordering Fractions	
4	<ul style="list-style-type: none">• Fraction Equivalence	
5	<ul style="list-style-type: none">• Adding and Subtracting Fractions	
6	<ul style="list-style-type: none">• Multiplying and Dividing Fractions	#1 = research lesson
7	<ul style="list-style-type: none">• Decimals	
8	<ul style="list-style-type: none">• Percentages	

9. Flow of the Lesson

Teaching Activity	Points of Consideration
<p>1. Introduction Begin the lesson with a discussion about what division means.</p> <p>Pose the question: “What does $6 \div 2$ mean?” asking for an answer in words.</p> <p>Pose a second question: “Does division always result in a smaller number?”</p> <p>Put students into groups of 3 or 4, with an A2 sheet, markers and a mini-whiteboard.</p>	<p>Expected responses: “$\frac{1}{2}$ of 6”, “How many 2’s in 6?”</p> <p>Expected responses: “Yes”, “No, division by 0”, “No division by a fraction.”</p>
<p>2. Activities Pictorially represent $1 \div \frac{1}{3}$.</p> <p>Display some student work that best represents the concepts. This will encourage participation and foster confidence among others.</p> <p><i>Proceed with tasks in the order given:</i></p> <p>Pictorially represent $2 \div \frac{1}{5}$. <i>Multiple units to ensure that students represent each unit as the same size.</i></p> <p>Pictorially represent $3 \div \frac{3}{4}$. <i>Numerator greater than 1.</i></p> <p>Pictorially represent $2 \div \frac{2}{5}$. <i>Repetition.</i></p> <p>Pictorially represent $\frac{1}{2} \div \frac{1}{3}$. <i>Fraction divided by a fraction. Key Task</i></p> <p>Pictorially represent $\frac{2}{3} \div \frac{1}{4}$. <i>Fraction divided by a fraction. Key Task</i></p> <p>Pictorially represent $\frac{3}{4} \div \frac{2}{5}$. <i>Fraction divided by a fraction. Key Task</i> E.g.</p> <div style="display: flex; justify-content: space-around; align-items: flex-start;"> <div style="text-align: center;">  </div> <div style="text-align: center;">  </div> </div> <div style="display: flex; justify-content: space-around; align-items: flex-start; margin-top: 10px;"> <div style="text-align: center;">  </div> <div style="text-align: center;">  </div> </div>	<p>Expected responses: Students simply write “$1 \div \frac{1}{3}$”.</p> <p>Students use rectangles or pie charts to model units and fractions.</p> <p>During the process, hopefully students will discover that using rectangles lends itself to an easier and better representation of the problem.</p> <p>It will also lead to discovering that the rectangles need to be divided into a number of parts corresponding to the common denominator.</p> <p>When the denominator becomes sufficiently large, students should realise that using “bar division” only of rectangles is less efficient than rows and columns.</p> <p>Examples of students work during the tasks should be displayed on the board and students should be asked to evaluate the usefulness of the representation shown.</p> <p>These class discussions and tasks should help students realise that they accomplish division of fractions by re-writing the fractions with a common denominator and then dividing the numerators.</p> <p>Students who are finished early are asked to pick their own examples and represent them pictorially first and then in numbers.</p> <p>Some students may even discover the common algorithm of “invert and multiply”.</p>

<p>3. Summing up</p> <p>Put up posters in the classroom of student work on the A2 sheets of the pictorial representation of division of fractions.</p> <p>Provide a homework activity sheet to solidify the main ideas.</p>	<p>Get students to give a re-cap of the main ideas visited in the class using the posters.</p> <p>The posters can serve as a reminder of the process and concepts.</p>
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10. Evaluation Sample Questions

This section includes questions that the planning team hopes to explore through this lesson and the post-lesson discussion. Examples

- What is your plan for observing students? *PJ is to teach the class. The class is to be divided into groups of 3 or 4. Liaise with management to organise cover for observing teachers.*
- Discuss logistical issues such as who will observe, what will be observed, how to record data. *Stefanie will take one half of the class and Derek the other for observation. Student A2 sheets are to be collected and a photographic record will be made of the students' work on white boards.*
- What observational strategies will you use? *Notes will be made about the students' engagement and their depth of understanding of the tasks set.*
- What types of student thinking and behaviour will observers focus on to measure student interest and active student learning? *Record the interaction and communication between students.*
- What additional kinds of evidence will be collected? *The homework worksheet in the next class*

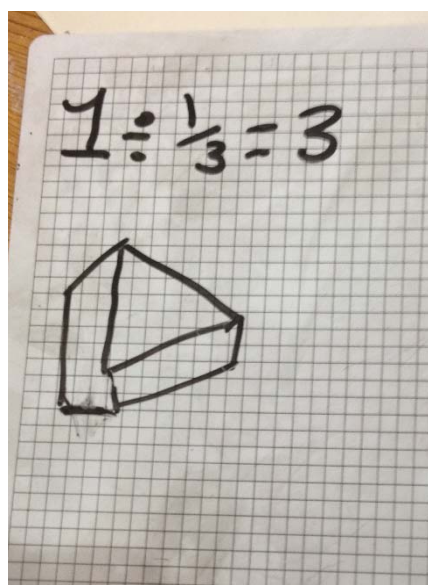
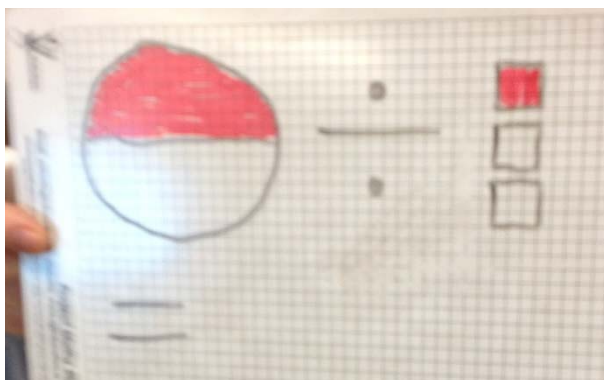
11. Suggested Board Use

In separate sections of the board, include:

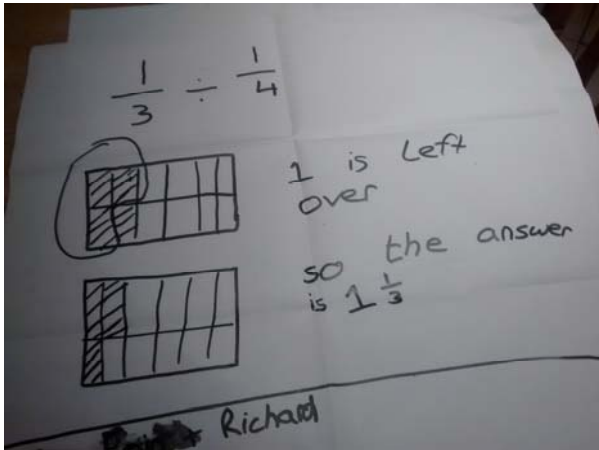
- *Key Words:*
 - Equivalent Fractions, Common Denominator
- *Key Tasks:*
 - Examples from students work of the key tasks to be put on the board.

12. Post-Lesson Reflection

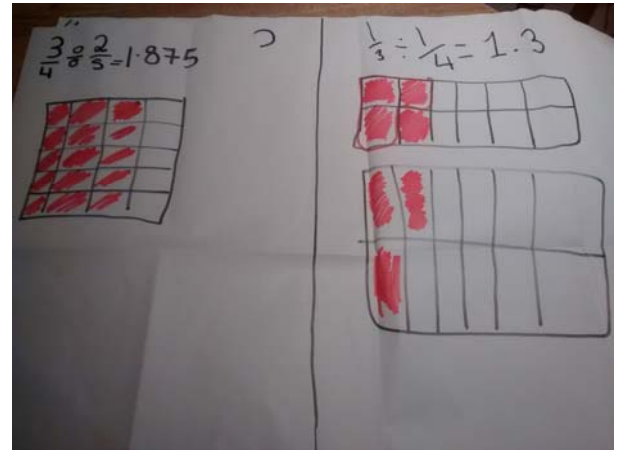
- What are the major patterns and tendencies in the evidence? Discuss. *Starting with what exactly was meant by whole number division allowed students to make sense of fraction division. However **as shown below** the visualisation of fractions was not initially well developed.*
- E.g. At the beginning of the lesson:



- What are the key observations or representative examples of student learning and thinking?
As shown in the examples below there was a progression during the class to using rectangles to represent fractions. If the denominators of the fractions were m and n , the students realised that the dimensions of the rectangles representing each fraction should be $m \times n$.



This student is using prior knowledge of the common denominator of 12. He did not draw a 3 by 4 array but a 6 by 2 array which still gave a common denominator of 12. The student realised that the referent unit was $1/3$ and correctly realised that he had 1 and $1/3$ lots of $1/3$ in $1/4$.



When showing $3/4$ divided by $2/5$ the student used just one diagram.

- In what ways did students achieve or not achieve the learning goals?
Students were able to make sense of fraction division but we did not get time to make the explicit link to the invert and multiply algorithm. They were able to find the quotient of two fractions from first principles.

The problem given here was $3/4 \div 2/5$ → This student clearly shows 1 and $7/8$ lots of $2/5$ in $3/4$.



- Based on your analysis, how would you change or revise the lesson?

The students should meet this way of modelling equivalent fractions earlier (i.e. the pictorial representation) and use it not only for making sense of equivalent fractions but also for addition and subtraction of fractions. For multiplication of fractions only one rectangle is needed.

Some other observations

- The most important part was sitting down and deciding the problems and their sequencing that we were going use for the class.
- One 40 minute lesson is inadequate, however as already stated this could be remedied if the students were used to representing fractions pictorially
- It was one of the first occasions in which members of the Maths Department in our school have sat down to plan a lesson together.
- The whole exercise was non-threatening because of the collaboration and it would be much easier to do it again.
- The most difficult part was writing up the template, but this would become less onerous with experience.