Reflections on Practice

Project Maths Tionscadal Mata
Development Team

Lesson Plan for [First year, introduction to algebraw**]**

For the lesson on [18/03/2015]
At [Colaiste Bride, Enniscorthy, Co. Wexford], [Ms. McGuinness's] class
Teacher: [Olive McGuinness]
Lesson plan developed by: [Aidan Roche, Olive McGuinness, Julie Duggan]

1. **Title of the Lesson:** My first algebra class

2. **Brief description of the lesson:** Students will be introduced to algebra and the concept of a variable through some simple visual patterns.

3. **Aims of the Lesson:**
   - I'd like my students to appreciate that maths can be used to solve real life problems.
   - I'd like to emphasise that there are several correct ways to solve that same problem.
   - I'd like my students experience meaningful mathematics
   - I'd like my students to see algebra as a tool that can be used to make sense of concrete problems.
   - I'd like to give my students opportunities to discuss mathematical ideas in small groups and in front of the class.
   - That students understand that a pattern exists if you can predict the next stage.
   - That students understand that patterns can be approached numerically or by considering changing shapes/pictures.
   - That students could predict the shape/value of any stage in a given pattern.

4. **Learning Outcomes:**

As a result of actively participating in this lesson my students should be able to:
   - Convert pattern images into numbers and visa-versa.
   - Recognise that there is a pattern.
   - Recognise what changes and what stays the same in a pattern.
   - Understand that as stages got bigger it was easier to use a formula rather than the picture to predict the value/number of tiles in a stage.

5. **Background and Rationale**

This lesson was designed to:
   - Meet syllabus needs as students and teachers engage with new maths syllabus in a more hands on, interactive, real life approach.
   - Maths is seen as something that can be applied to understand the world around us.
   - Approach algebra in a new way that has been suggested might increase students understanding, by considering changing stages in a pattern.
   - Approach algebra in an active, practical and visual way to engage all learners in a mixed ability class.

6. **Research**

This relates to Section 4.1 and 4.2 of the Common Introductory Course for 1st Year maths students.

Strand /Topic Title	Learning outcomes Students should be able to
Strand 4: 4.1Generating arithmetic expressions from repeating patterns Students examine patterns and the rules that govern them and so construct an understanding of a relationship as that which involves a set of inputs, a set of outputs and a correspondence from each input to each output.	– use tables and diagrams to represent a repeating-pattern situation – generalise and explain patterns and relationships in words and numbers – write arithmetic expressions for particular terms in a sequence
Strand 4: 4.2 Representing situations with tables diagrams and graphs Students examine relations derived from some kind of context – familiar, everyday situations, imaginary contexts or arrangements of tiles or blocks.They look at various patterns and make predictions about what comes next.	– use tables, diagrams and graphs as a tool for analysing relations – develop and use their own mathematical strategies and ideas and consider those of others – present and interpret solutions, explaining and justifying methods, inferences and reasoning

7. About the Unit and the Lesson

The lesson is designed to

- Help students realise that you can create numerical patterns using tiles and vice versa.
- Students will look at various patterns and make predictions about what comes next.
- Help students understand that in patterns somethings remain constant while other things change.

8. Flow of the Unit:

Prior to this lesson the students will have studied Strand 3.1 Number Systems.
This is an introductory lesson to algebra. The next lesson will look at further patterns and introduce the concept of the nth term.

Lesson		# of lesson periods
	Patterns and algebra	
1	These lessons will involve the students in investigating and understanding: • The use of tables, graphs, diagrams and manipulatives to represent and analyse patterns (e.g. using unifix cubes) and introduce concepts of variables and constants • A relationship as that which involves a set of inputs, a set of outputs and a correspondence from each input to each output (e.g. “money box” problems as in the teaching and learning plan), identifying variables and constants • Relations derived from some kind of context – familiar, everyday situations, imaginary contexts or arrangements of tiles or blocks. • The use of tables, diagrams and graphs as tools for analysing	6 x 40min. of which the research lesson is the first lesson.

	relations • How to use patterns to make predictions about what comes next • How to develop and use their own generalising strategies and ideas and consider those of others • How to present and interpret solutions, explaining and justifying methods, inferences and reasoning • How to generalise and explain patterns and relationships in words and numbers • How to write arithmetic expressions for particular terms in a sequence, linear only • Change and rate of change linked to slope	

9. Flow of the Lesson

Teaching Activity	Points of Consideration
1. Introduction Students are presented with 3 stages of a geometrical pattern and are asked what they see. The teacher asks students to explain what they mean by pattern?	Do students recognize the presence of a pattern? Can students explain what a pattern is? Do students understand that a pattern is predictable?
2. Posing the Task The teacher asks the students to use their tiles to construct the missing stages of the pattern. The teacher asks students to explain how they reasoned to construct their patterns.	Can students complete the two missing stages of the pattern? Can students explain their reasoning behind the patterns they created? Do students work together effectively in completing the task.
3. Anticipated Student Responses Most students should be able to complete the missing stages of the pattern. Students may find it difficult to explain their reasoning / thinking.	It is important that the teacher supports students in developing their reasoning through suitable questioning.
4. Comparing and Discussing The teacher brings the class back together as a group and asks a student to come to the board to draw in the next stage of the pattern. The teacher asks the class if this is the pattern they got. The teacher asks different students to explain how they reasoned that this must be the next stage of the pattern. The teacher asks another student to come to the board and to complete the next stage of the	Do students offer different ways of looking at the pattern? Are students comfortable explaining their thinking? It is important that students are given time to explain their approach to the problem.

pattern. The teacher asks the class if this is the pattern they got. The teacher asks different students to explain their thinking. The teacher reminds the class of the fact that patterns are predictable in some way.	
5. Posing the Task The teacher asks the students to examine a new pattern and to use their tiles to construct the next two stages of the pattern. The teacher asks students to explain their thinking in constructing the two missing stages of the pattern.	Can students complete the two missing stages of the pattern? Can students explain their reasoning behind the patterns they created? Do students work together effectively in completing the task.
6. Anticipated Student Responses Most students should be able to complete the missing stages of the pattern. Students may find it easier to explain their reasoning this time, having gotten some practice with the previous task.	It is important that the teacher supports students in developing their reasoning through suitable questioning.
7. Comparing and Discussing The teacher brings the class back together as a group and asks a student to come to the board to draw in the next stage of the pattern. The teacher asks the class if this is the pattern they got. The teacher asks different students to explain how they reasoned that this must be the next stage of the pattern. The teacher asks another student to come to the board and to complete the next stage of the pattern. The teacher asks the class if this is the pattern they got. The teacher asks different students to explain their thinking. The teacher reminds the class of the fact that patterns are predictable in some way. The teacher asks students to think about the pattern using the total number of tiles in each stage. The teacher asks students to write in the number of tiles under each stage of the pattern and asks students if they notice anything about the numbers. The teacher emphasizes the fact that this geometrical problem may also be thought of in terms of a number pattern. The teacher asks students to use their traffic-	Do students offer different ways of looking at the pattern? Are students comfortable explaining their thinking? Do students recognize that the numbers associated with the pattern themselves form a numeric pattern? Can students describe the key features of the numeric pattern? It is important that students are given time to explain their approach to the problem.

light boards to show how well they understood what they just did.	
8. Posing the Task The teacher asks students to work together to predict how many tiles are needed to build the 10^{th} stage of the pattern. The teacher asks students to explain the thinking behind their solution.	Can students apply their understanding to work out the number of tiles in the 10^{th} stage? Do students have to make the 10^{th} stage to answer the question? Can students exploit the number pattern they just discovered to predict the number of tiles in the 10^{th} stage? Can students explain how they arrived at their answer?
9. Anticipated Student Responses Some students may construct the 10^{th} stage using their tiles. Some students may use the numeric pattern to predict the number of tiles on the 10^{th} stage. Some students may use both approaches, with the second approach providing a check of the first. Some students may skip count to the solution. Some students may think about the pattern as groups of two.	It is important that the teacher supports students in developing their reasoning through suitable questioning. For students who simply make the pattern, the teacher should encourage them to think numerically about the problem. For students who skip count (additive thinking) the teacher should encourage them to think in groups of two (multiplicative thinking).
10. Comparing and Discussing The teacher brings the class back together as a group projects the solution to the problem on the board. The teacher asks the class if this is the pattern they got. The teacher asks different students to explain how they reasoned that this must be the next stage of the pattern. The teacher asks students - which is a better approach – to construct the pattern using tiles or to predict the pattern using numbers. The teacher emphasizes the power of a numerical approach to pattern prediction. The teacher asks students to use their traffic-light boards to show how well they understood what they just did.	Do students offer different ways of looking at the pattern? Do students think of the pattern numerically? Do students display evidence of additive thinking? Do students display evidence of multiplicative thinking? Are students comfortable explaining their thinking? Can students identify the usefulness of numeric analysis over simply constructing the pattern? It is important that students are given time to explain their approach to the problem.
11. Posing the Task The teacher asks the students to work together to calculate how many tiles will be in the 100^{th} stage of the pattern. The teacher asks students to explain how they reasoned to construct their patterns.	Can students use numeric analysis to solve the problem? Do students recognize that drawing out the pattern is not practical here? Can students reason what the number of tiles must be?
12. Anticipated Student Responses Some students may start by trying to construct the pattern. Some students may think of the pattern in terms of the number of tiles in each arm. Some students may think about the pattern in terms of jumps of two.	It is important that the teacher supports students in developing their reasoning through suitable questioning.

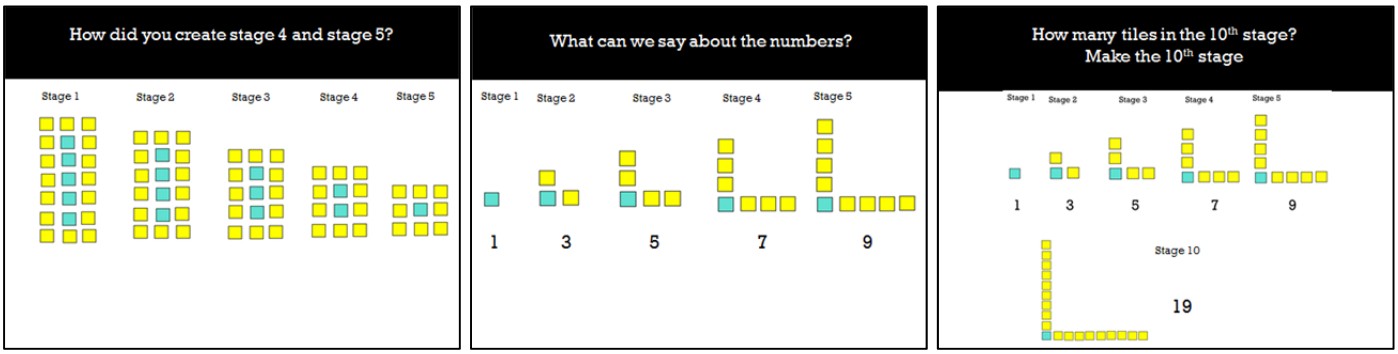
13. Comparing and Discussing The teacher brings the class back together as a group and asks students to write down their answers on their show-me boards. The teacher writes the answer on the board. The teacher asks students if drawing out the 100^{th} stage is practical. The teacher asks students to describe how they got their answer.	Have students gotten the correct answer to the problem? Do students offer different ways of looking at the pattern? Are students comfortable explaining their thinking? Do students think of the pattern in numeric terms? Do students use multiplicative thinking to get to their answer? It is important that students are given time to explain their approach to the problem.
14. Summing up The teacher asks the students to summarise what they have learned in the lesson. The teacher emphasizes the fact that students have solved problems based around patterns. The teacher emphasizes that there are many ways to solve such pattern problems and that often one approach is as valid as another. The teacher emphasizes the fact that the students solved these problems themselves, with very little help from the teacher. The teacher presents students with homework and explains what they are expected to do.	Do students recognize that they themselves did most of the maths in the class? Do students recognize that maths is doable with a little effort and thinking? Do students recognize that different approaches to solving a problem are acceptable and that there is more than one way to solve a problem?

10. Evaluation

Students learning will be assessed through:

- Verbal responses
- Student questioning
- Observation of student group work
- Successful completion of tasks
- Completion of homework worksheet

11. Board Plan



12. Post-lesson reflection

All observers felt that the lesson was a great success. Here are our main findings:

- The timing of the lesson was very good. Students were challenged throughout but not so rushed as to find the concepts being investigated difficult to absorb.

- Students understood the content of the lesson. This was observed through their responses to questions, their questioning and their work as a group.
- Students were engaged with the lesson. They enjoyed the lesson. More lessons like this would be useful.
- Students were challenged by the tasks in the lesson, particularly the more difficult task. There was a noticeable rise in the levels of “noise” in the classroom when students were challenged. This “noise” was a result of students increased efforts to tackle the problem. Students willingly discussed maths.
- Students used the tile resources well. This helped them engage with understanding the pattern.
- The language used during the lesson by the teacher was of the highest order. Students were constantly reminded of the fact that they were dealing with a pattern and what this meant.
- The language used by students during the lesson was excellent. Students drew on the language of maths to describe what they saw in front of them. One student commented that the number of tiles in each stage of a pattern were prime numbers. Students showed little hesitation in making connections between the lesson and prior knowledge.
- The range of answers / approaches suggested by students was excellent. With little support many students developed their own reasoning to solve the problem. Most students had no difficulty sharing their reasoning with the teacher and the rest of the class. Some students naturally tended toward a geometrical solution to the tasks while others preferred a purely numerical approach.
- The use of traffic lights to assess student understanding was excellent.
- The learning environment was excellent. Students worked independently, stayed on task, asked questions when needed and shared their thinking with the entire class.
- Questioning was used very effectively throughout the lesson to encourage students to think more deeply about a concept. The use of appropriate wait time was crucial in this.
- Students were comfortable questioning other students’ reasoning and had no problem stating that they didn’t understand where this reasoning came from. The teacher gave plenty of time to discuss the various approaches used by students.

There are a few changes we would make if the lesson was to be taught again. These are:

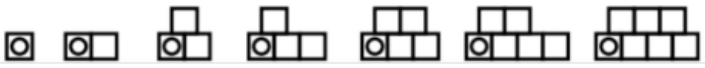
- We thing the Introduction to the lesson should be altered because the students were not sure of what was expected of them for the first few minutes. Setting the scene of the lesson is difficult as we don’t want to use the word “pattern” at the start, rather we want students to come up with this term themselves. We think that presenting students with the first task and saying something simple like “We are going to look at a simple problem to start today’s class. Have a look at the screen and using the tiles in front of you, I want you to make up what goes into the missing spaces”. Having completed the task, the teacher can get students to then come up with the word pattern themselves by asking simple questions such as “How did you know what to make?” and so on.
- To help with this change to the launch of the lesson we would recommend changing the first slide in the PowerPoint to include boxes around the first three stages of the pattern and empty boxes for the missing stages. This might help students understand what they are trying to do a little more easily.

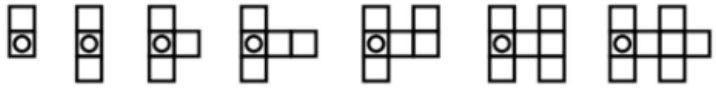
- Students worked well in groups, but in some cases there was a clear leader who seemed to be doing more of the work. This would need to be monitored to ensure full engagement of all students.

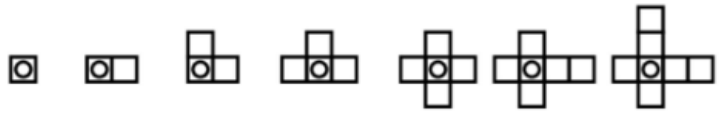
Patterns Practice Sheet

Name:

Draw the next shape in each of these patterns:

a. 

b. 

c. 

Write the numbers that come next and the size of stage 50 in each pattern below:

d. 1, 3, 5, 7, 9, ___, ____, ____ Stage 50 = _____

e. 2, 4, 6, 8, 10, ___, ____, ____ Stage 50 = _____

f. 24, 34, 44, 54, ____, ____, ____ Stage 50 = _____