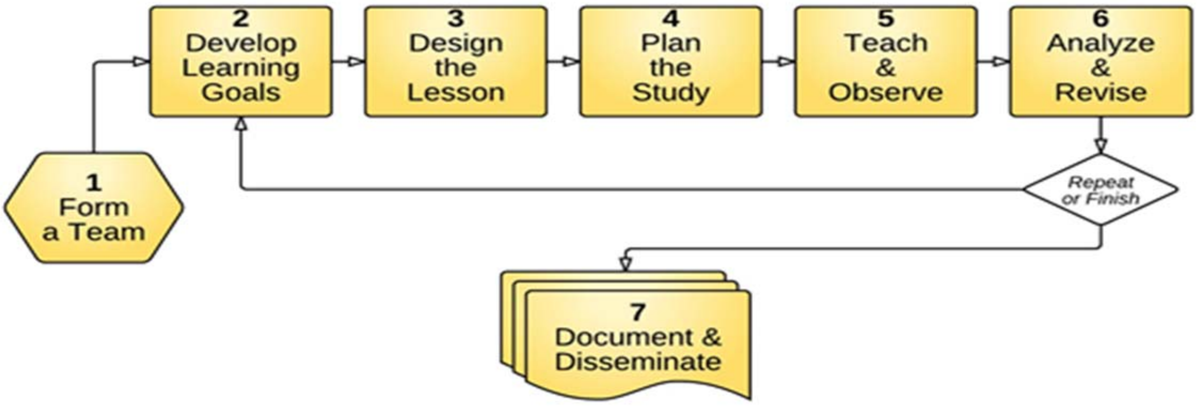
Reflections on Practice



Project Maths Tionscadal Mata Development Team

Lesson Plan for Fifth Year Division of Rational Expressions

For the Lesson On: 18th March 2015
At: Jesus & Mary Secondary School, Salerno, 5th Year Mathematics Class
Teacher: Jane Keenan
Lesson Plan Developed By: Jane Keenan

1. **Title of the Lesson:** Visualising and Making Sense of the "Invert and Multiply" Algorithm for Fraction Division: A Senior Cycle Perspective

2. **Brief Description of the Lesson:** This lesson was devised to help students further understand and visually represent the operation of division by fractions. It was hoped that students would develop their relational understanding of why the "Invert and Multiply" algorithm works to complement their instrumental understanding of the algorithm and apply this to division with rational expressions.

3. **Aims of the Lesson:**

(a) Long-Range/Thematic Goals:

- I'd like my students to make sense of algorithms in mathematics with the help of visualisation aids.
- I'd like my students to be more comfortable with division of numeric fractions and rational expressions.
- I'd like to foster my students to become independent learners and thinkers.
- I'd like my students to become more creative when devising approaches and methods to thinking about problems.
- I'd like to build my students' enthusiasm for the subject by engaging them with stimulating activities.
- I'd like my students to connect and review the concepts that we have studied already.

(b) Short-Term Goals:

- For students to understand and demonstrate division of numeric fractions using a variety of methods
- For students to understand division of rational expressions
- For students to have a good understanding of operations in rational numbers and rational expressions

(c) Learning Outcomes:

As a result of studying this topic students will be able to:

- Represent division of fractions using diagrams
- Understand why the division of fractions algorithm works
- Explain and justify methods of representation to themselves and each other
- Perform the arithmetic operation of division on rational algebraic expressions
- Find and correct mistakes in questions with division of rational algebraic expressions

4. Background and Rationale

Senior cycle higher level mathematics students are required, as stated on their syllabus, to: perform the arithmetic operations of addition, subtraction, multiplication and division on polynomials and rational algebraic expressions paying attention to the use of brackets".

In reality, working with rational expressions and equations is fundamental to all aspects of the syllabus.

Students' learning experience of rational expressions begins with simple fractions in primary school. They are not taught division of a fraction by a fraction at primary level, only division of a whole number by a fraction. Division of a fraction by a fraction is now taught in 1st year at secondary level. During their junior cycle years, students need to be able to "investigate models to help think about the operations of addition, subtraction, multiplication and division of rational numbers".

In theory, they should be close to mastering these skills upon entering their senior cycle years. However, fractions themselves are a difficult topic and often not well understood by the general population. By extension, rational algebraic expressions are poorly understood and it is often the case that the learning of the algorithms of "what to do" takes precedence over why the "doing" makes sense.

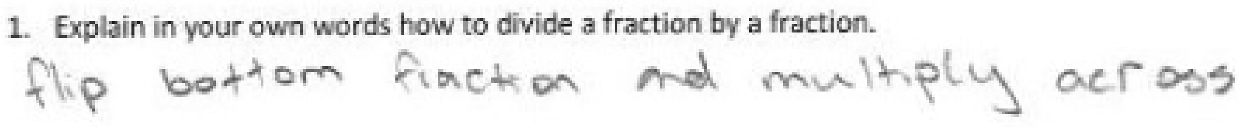
At primary level, students engage with various visual aids and hands on materials to make sense of fractions. The premise of this lesson was to try to reconnect with the basic fundamental concepts of what a fraction is, ***what it looks like*** and how to work with fractions. It was hoped that this connection with the familiar would help students to gain a relational understanding (the why) of the algorithm for division of fractions and division of rational algebraic expressions.

Pre Assessment & Student Difficulties

Prior to the lesson, students were given a pre-assessment worksheet (*see Appendix 1*). The focus of the lesson had to be quite narrow to have any hope of being accomplished. Hence, the worksheet questions were few and focused also. Students were asked to:

“Explain in your own words how to divide a fraction by a fraction.”

All students in the class either said to “multiply by its reciprocal” or to “flip the bottom and multiply”. E.g.

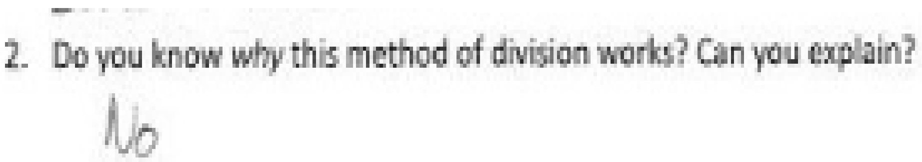

1. Explain in your own words how to divide a fraction by a fraction.

flip bottom fraction and multiply across

Students were then asked:

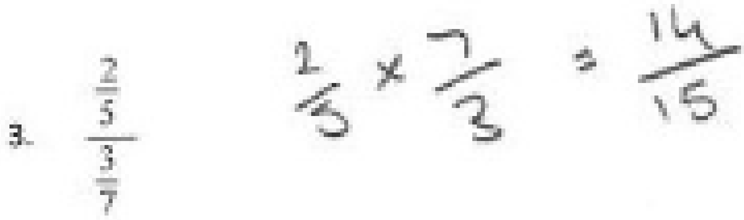
> “Do you know why this method of division works? Can you explain?”

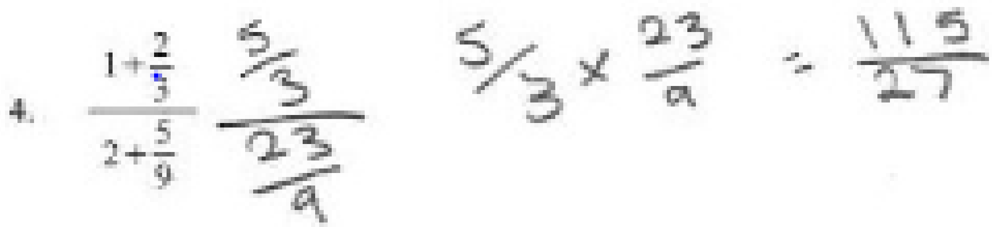
Most students left this **blank**. The *better* able students in the class gave the following response:


2. Do you know why this method of division works? Can you explain?

No

They had little difficulty in solving the division questions with numbers:


3. $\dfrac{\frac{2}{5}}{\frac{3}{7}}$ $\quad \frac{2}{5} \times \frac{7}{3} = \frac{14}{15}$


4. $\dfrac{1+\frac{2}{3}}{2+\frac{5}{9}}$ $\quad \dfrac{\frac{5}{3}}{\frac{23}{9}} \quad \frac{5}{3} \times \frac{23}{9} = \frac{115}{27}$

Interestingly to note, most students added the $1 + 2/3$ first to get $5/3$ before multiplying by the reciprocal of the bottom line; yet they did not use this approach with algebraic fractions.

The last two questions were algebraic. Again, most students were able to correctly finish the less complicated algebraic fraction.

5. $\dfrac{\frac{x}{3}}{\frac{x^2}{5}}$ $\quad \frac{x}{3} \times \frac{5}{x^2} = \frac{5x}{3x^2} \Rightarrow \frac{5}{3x}$

However, almost one third of the class had issues with the final question. Some students had difficulty in applying the algorithm correctly, showing a lack of conceptual understanding of the algorithm.

This student "flipped" each part of the bottom line separately.

6. $\dfrac{x-1-\frac{6}{x}}{x-\frac{4}{x}}$ $\quad \left(\frac{x}{1} - \frac{1}{1} - \frac{6}{x}\right) \cdot \left(\frac{1}{x} - \frac{x}{4}\right)$

This student just "flipped" the fraction term in the denominator.

6. $\dfrac{x-1-\frac{6}{x}}{x-\frac{4}{x}}$ $\quad x - 1 - \frac{6}{x}\left(x - \frac{x}{4}\right)$

Other students struggled with the addition of fractions:

6. $\dfrac{x-1-\frac{6}{x}}{x-\frac{4}{x}}$ $\quad \dfrac{\frac{x-1-6}{x}}{\frac{x-4}{x}} \quad \frac{x-7}{x} \times \frac{x}{x-4}$

Finally, other students could not simplify the fraction due to a lack of relational understanding of fractions and numbers.

Not all of the mistakes identified could be addressed in one lesson. The focus of the lesson was the algorithm for division by a fraction and thus only the mistakes relating to this were targeted. If the approach proved successful, a similar approach could be adopted to address the other mistakes.

Ideally, this lesson would have been given when this topic was first taught and ideally, this would have been part of a series of lessons on a similar vein to address all aspects of rational algebraic expressions. Instead, this lesson was given in isolation as a research test lesson. It is acknowledged that better results may have been achieved had this lesson been taught in a different context.

5. Research

The following materials were used to research this area:-

- Junior Certificate Mathematics Syllabus
- Leaving Certificate Mathematics Syllabus
- The following websites:
  http://fawnnguyen.com/fraction-division-via-rectangles/
  https://blogs.otago.ac.nz/tac/our-suggestions/fractions-and-algebra/
  http://nzmaths.co.nz/sites/default/files/Numeracy/2008numPDFs/NumBk8.pdf
  http://map.mathshell.org/materials/lessons.php?taskid=551

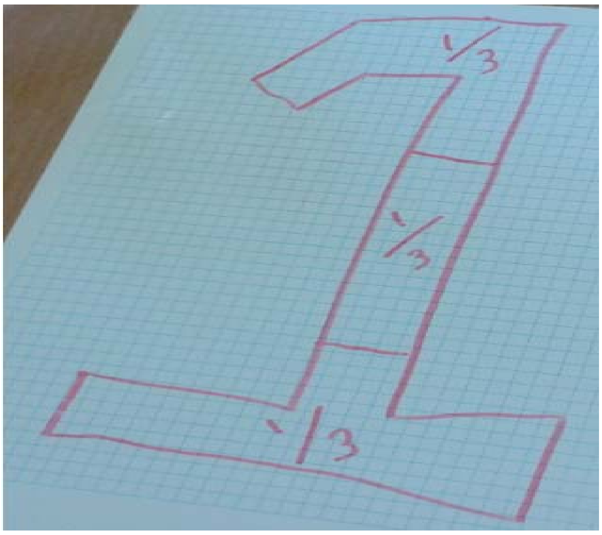
6. About the Unit and the Lesson

This lesson is part of a unit on rational expressions. Prior to this lesson, pupils should have covered addition, subtraction, multiplication and division of rational expressions (Strand 4: Algebra 4.1 Expressions) both at the junior cycle and senior cycle.

7. Flow of the Unit:

Sequence	Title: Rational Algebraic Expressions	
1	• Overview of Rational Algebraic Expressions	
2	• Algebraic Fraction Equivalence	
3	• Adding and Subtracting Rational Expressions	
4	• Multiplying Rational Expressions	
5	• Factorising Rational Expressions	
6	• Division of Rational Expressions	**1 Research Lesson**
7	• Simplifying Mixed Rational Expressions Using Factorising, Multiplication and Division.	

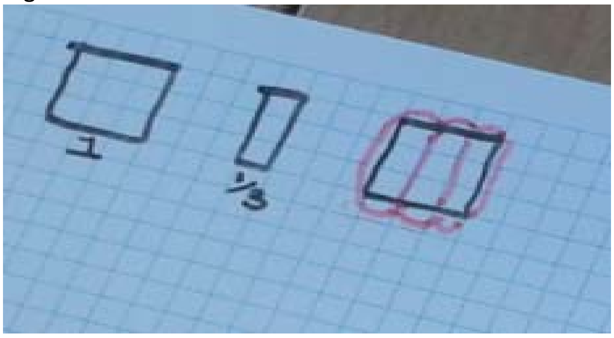
8. Flow of the Lesson

Teaching Activity	Points of Consideration
1. Introduction Begin the lesson with a discussion about what division is. Pose the question: "What does 6 ÷ 3 mean?" asking for an answer in words. Pose the question: "What does 7 ÷ 2 mean?" asking for an answer in words. Pose the question: "What does *a* ÷ *b* mean?" asking for an answer in words. Put students into pre-chosen groups of 2, with one mini-whiteboard and two different coloured markers per group.	Expected responses: "$\frac{1}{3}$ of 6" "How many 3's are in 6?" ***Explain to students that we will work with the idea of how many groups of size 3 are in a group of size 6.*** Expected responses: "How many 2's are in 7?" Expected responses: "*how many groups of size b are in a group of size a*."
2. Activities *Proceed with tasks in the order given:* **Task 1 Represent** $1 \div \frac{1}{3}$ in a diagram. **Student Show & Talk:** Have students present and explain their work. **Key Task:** Choose an ***inappropriate* or *inadequate*** model first in order to evaluate the usefulness of the representation. e.g. 	Expected responses: Students simply write "1 ÷ 1/3". Students use rectangles or pie charts to model units and fractions. Ask questions that challenge the accuracy of their model. E.g. • Is that shape split exactly into thirds? • Can you explain why? • Could you improve it or try a different model that would be more accurate? By showing the better representations after an inappropriate model, students can learn from each other.

Then choose a more appropriate model for students to explain and discuss.

For those who found the first task difficult to start, they will then have better ideas on how to approach the next task.

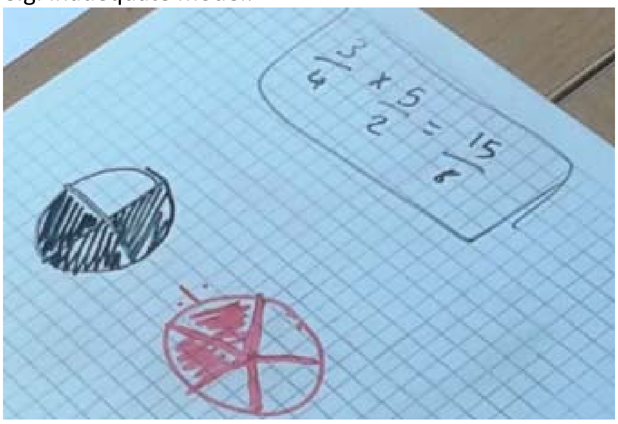
e.g.



Task 2 Represent $\frac{3}{4} \div \frac{2}{5}$ in a diagram.
Fraction divided by a fraction.

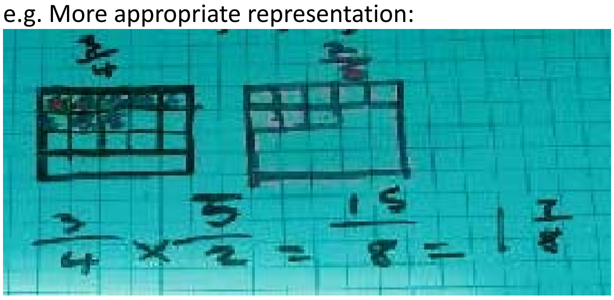
Student Show & Talk: As before.

e.g. inadequate model:



During the process, hopefully students will discover that using rectangles lends itself to an easier and better representation of the problem.

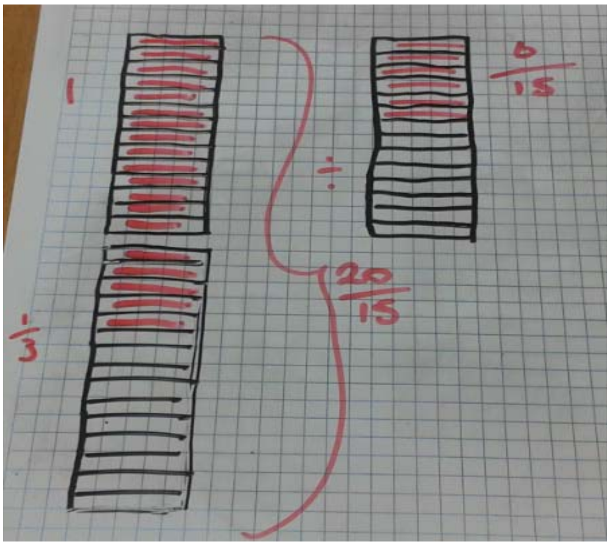
e.g. More appropriate representation:



It will also lead to discovering that the rectangles need to be divided into a number of parts corresponding to the common denominator.

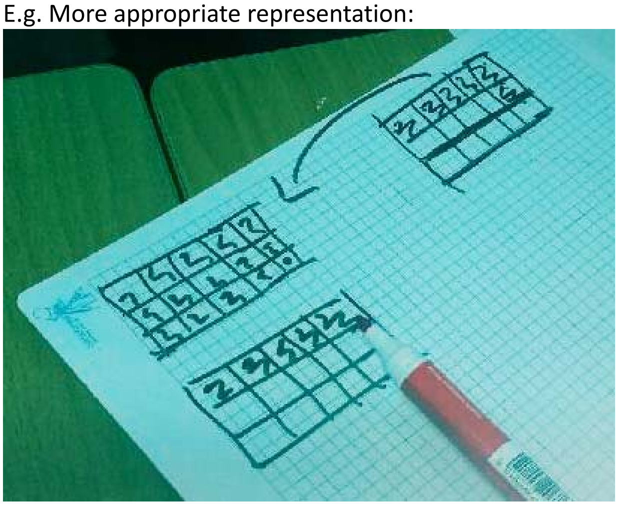
Task 3 Represent $1\frac{1}{3} \div \frac{2}{5}$ in a diagram.
Multiple units with a fraction divided by a fraction to ensure students add the parts of the mixed fraction.

Student Show & Talk: As before.
e.g. Less appropriate representation:



When the denominator becomes sufficiently large, students should hopefully realise that using “bar division” only of rectangles is less efficient than rows and columns.

E.g. More appropriate representation:



These class discussions and tasks should help students realise that they accomplish division of fractions by re-writing the fractions with a common denominator and then diving the numerators.

Key Learning Point
Emphasise to students and write on the board that the 1 and $\frac{1}{3}$ had to be added together first, before counting how many $\frac{2}{5}$ were in $1\frac{1}{3}$.

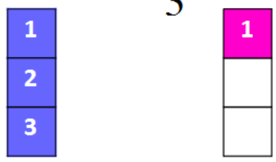
Class Discussion With Overhead Presentation

Review the class examples together to re-enforce their learning and understanding in context.

Students should now see that this is 3/3 divided by 1/3. The denominators are equal hence it is the equivalent to 3 divided by 1 which is 3.

Static Screenshot of Classwork Task 1 Slide

$$1 \div \frac{1}{3}$$



How many groups of size 1/3's are in 1 unit?

How many pinks fit in the blue?

$$1 \div \frac{1}{3} = 3$$

Show and review task 2.

Static Screenshot of Classwork Task 2 Slide

$$\frac{3}{4} \div \frac{2}{5}$$

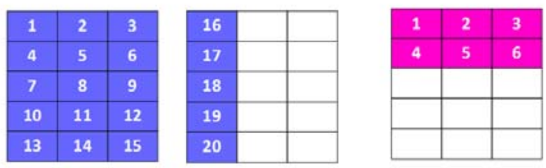
How many pinks fit in the blue?

$$\frac{3}{4} \div \frac{2}{5} = \frac{15}{8}$$

Show and review task 3.
Remark to students that the divisor becomes the new unit. You have 3 and 1/3 of the new unit (which is 2/5) in 4/3.

Static Screenshot of Classwork Task 3 Slide

$$(1 + \frac{1}{3}) \div (\frac{2}{5})$$

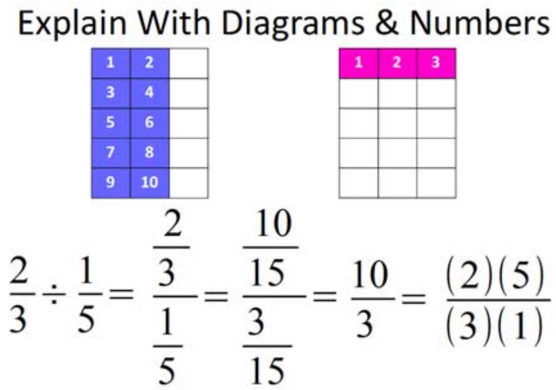


How many pinks fit in the blue?

$$\left(\frac{20}{15}\right) \div \left(\frac{6}{15}\right) = \frac{20}{6} = \frac{10}{3}$$

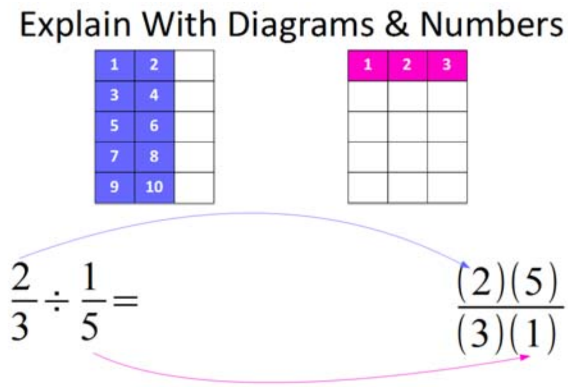
Algorithm Decomposition
Show the detailed example of partitioning using the common denominator and counting.

Static Screenshot of “Flip & Multiply” Slide Part 1



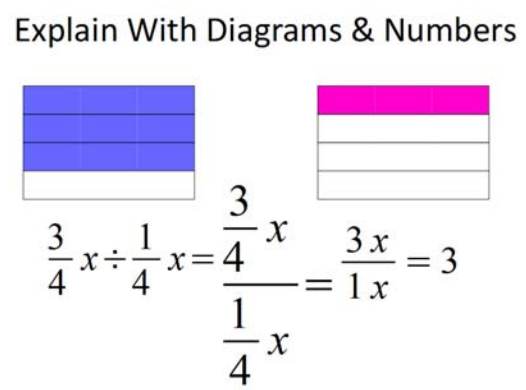
Get students to explain in words why the “flip & multiply” works using the presentation slides to assist.

Static Screenshot of “Flip & Multiply” Slide Part 2



Show and discuss the slide with simple algebraic fraction division. Recall the area model with students letting x = width of the rectangle with length 1. Then area =x. We are looking first at ¾ of x.

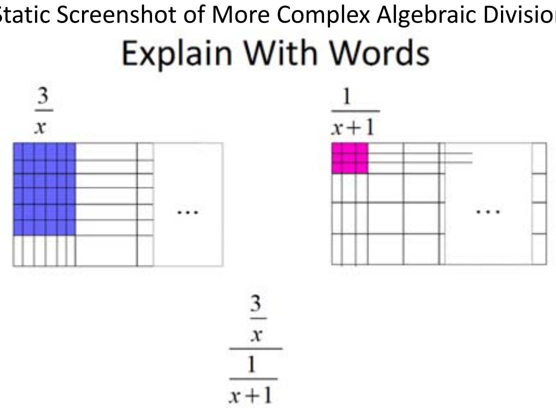
Static Screenshot of Simple Algebraic Division

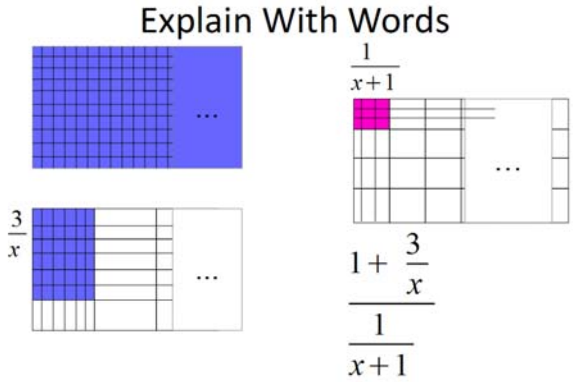
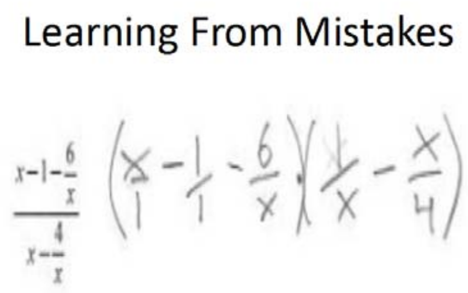


Show and discuss the slide with more complicated algebraic fraction division.

Simplify the expression algebraically using the common denominator method on the board.

Static Screenshot of More Complex Algebraic Division



Show and discuss the slide with more complicated algebraic fraction division. Simplify the expression algebraically using the common denominator method on the board. **Key Prior Learning Use** Specifically refer back to the reference on the board to the fact that 1 and $\frac{1}{3}$ had to be added together first, before simplifying. Draw out from the students themselves that the same method applies here. Simplify the expression algebraically using the common denominator method on the board. **Learning From Mistakes** Show slide with student mistakes and ask students for: • the errors • how to simplify correctly	Static Screenshot of More Complex Algebraic Division  Static Screenshot of Learning From Mistakes 
3. Summing up Use the presentation to recap the main ideas and examples. Provide the homework activity sheet to review and consolidate their learning.	

The full set of slides used in the lesson is contained in Appendix 2.

The homework sheet used in the lesson is given in Appendix 3 & Appendix 4.

9. Evaluation

What is your plan for observing students?

One teacher will deliver the lesson and two other teachers will observe the students. Before the lesson, the teacher delivering the lesson will explain the context and learning outcomes of the lesson to the other teachers. The observing teachers can then look for evidence of the learning outcomes being achieved. One teacher is assigned to each side of the room.

Discuss logistical issues such as who will observe, what will be observed, how to record data, etc.

The teachers observing the class will each bring in a camera to take photographs of the tasks in progress. They will also make note of evidence of learning given by student comments during group work and class responses during discussion. They will also watch for student engagement or lack thereof at each stage of the lesson.

What types of student thinking and behaviour will observers focus on?

Can students articulate and explain their models and thinking?
Are students engaged in the tasks?
Are students engaged in the class discussions?
Are students able to answer the class questions posed?

What additional kinds of evidence will be collected (e.g., student work and performance related to the learning goal)?

Completed homework worksheets after the lesson

10. Suggested Board Use

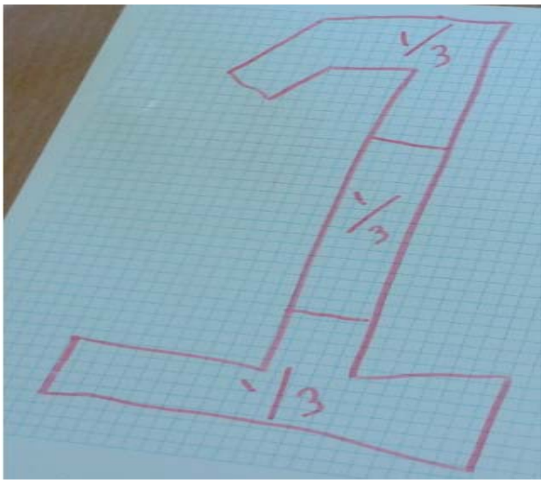
- Leave sufficient blank space in the middle of the board for the overhead presentation during the class.
- On one side of the board, at the appropriate time, write the key concept of "$a \div b$" meaning how many groups of size *b* are in a group of size *a.*
- Write key words on the other side of the board such as Common Denominator, Equivalent Fractions.
- After task 3, write on the board the main point that the 1 and 1/3 had to be added. Use this to refer back to when tackling the algebraic fraction.

11. Post-lesson reflection

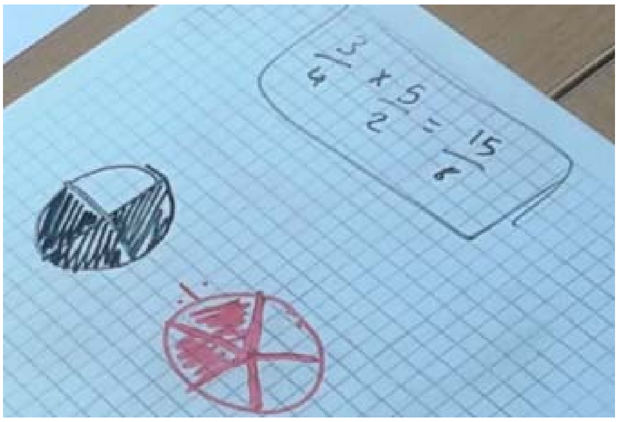
What are the major patterns and tendencies in the evidence?

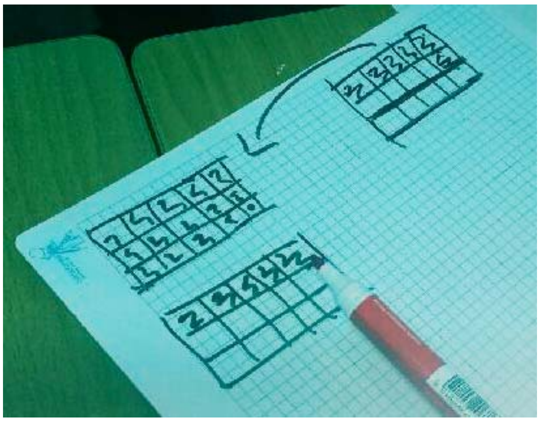
Students were slow and uncomfortable to get started. The visualisation of fraction problems was not very developed initially and as the visualisation improved, their conceptual knowledge improved as a result.

E.g. Initial Work 1 divided by 1/3



Students moved away from using circles to represent fractions once they realised that rectangles and partitioned rectangles were better models for fractions with larger common denominators.



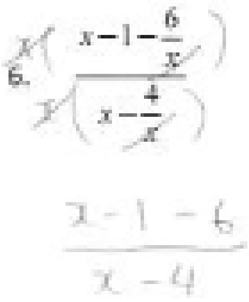


The choice of tasks was also key. Students worked up to the third task of: $(1+\frac{1}{3})\div(\frac{2}{5})$

Letting students make sense of a compound number example with comfortable, familiar numbers bridged a gap in their understanding of similar compound algebraic fractions when they met them later.

Students who had other difficulties with rational expressions did not improve in these areas. The work below of student 'R' indicates that they do not see the denominator of a fraction as one number which may be decomposed into parts so if the denominator is to be multiplied by a number, then all parts of the denominator must be multiplied by the same number. It seems that the student did not readily use the strategy of trying out specific numeric examples when unsure with algebraic expressions.

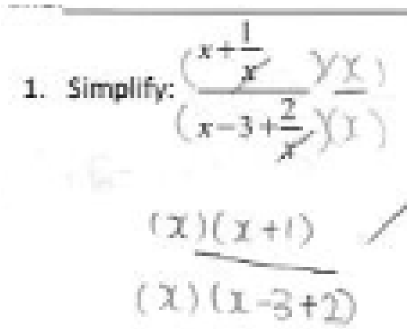
Student 'R's work before the lesson



6. $\dfrac{x\left(x-1-\frac{6}{x}\right)}{x\left(x-\frac{4}{x}\right)}$

$\dfrac{x-1-6}{x-4}$

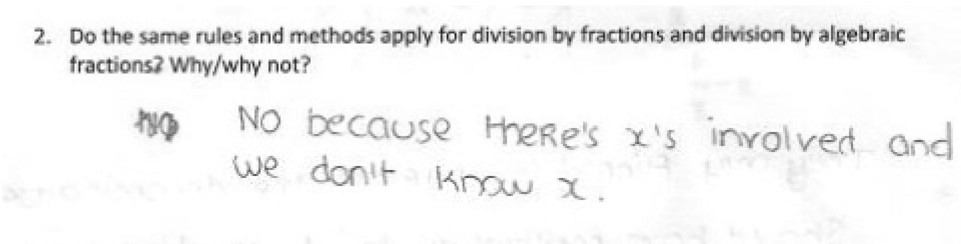
Student 'R's work after the lesson



1. Simplify: $\dfrac{\left(x+\frac{1}{x}\right)(x)}{\left(x-3+\frac{2}{x}\right)(x)}$

$\dfrac{(x)(x+1)}{(x)(x-3+2)}$

This same student was the **only** student in the class to reply "no" to the following homework question. Clearly, the student fails to make any connection between her sense of numbers and her sense of algebra. This is a part of a wider problem that could not be dealt with during this lesson.

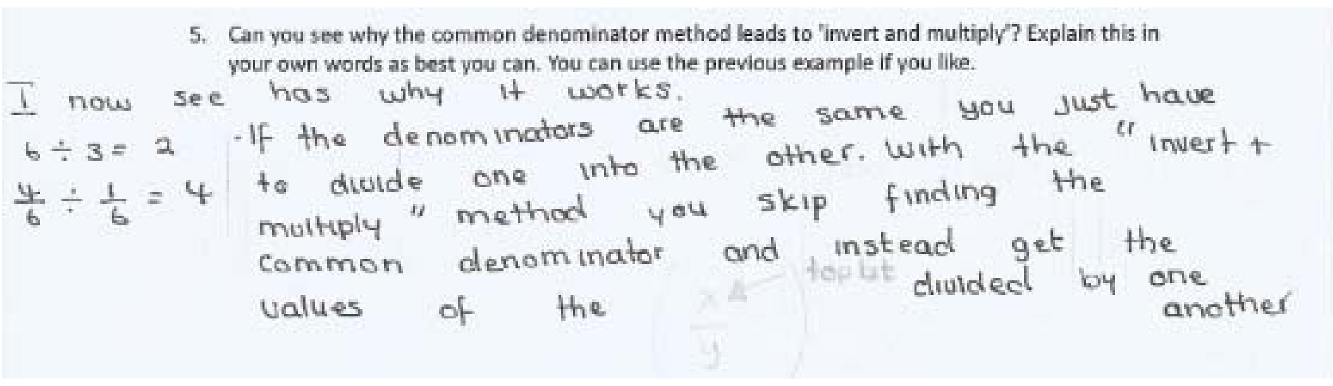


2. Do the same rules and methods apply for division by fractions and division by algebraic fractions? Why/why not?

No because there's x's involved and we don't know x.

In what ways did students achieve or not achieve the learning goals?

Students were able to explain the division algorithm in their own words after the lesson, whereas **none** of the students could explain the algorithm before the lesson.



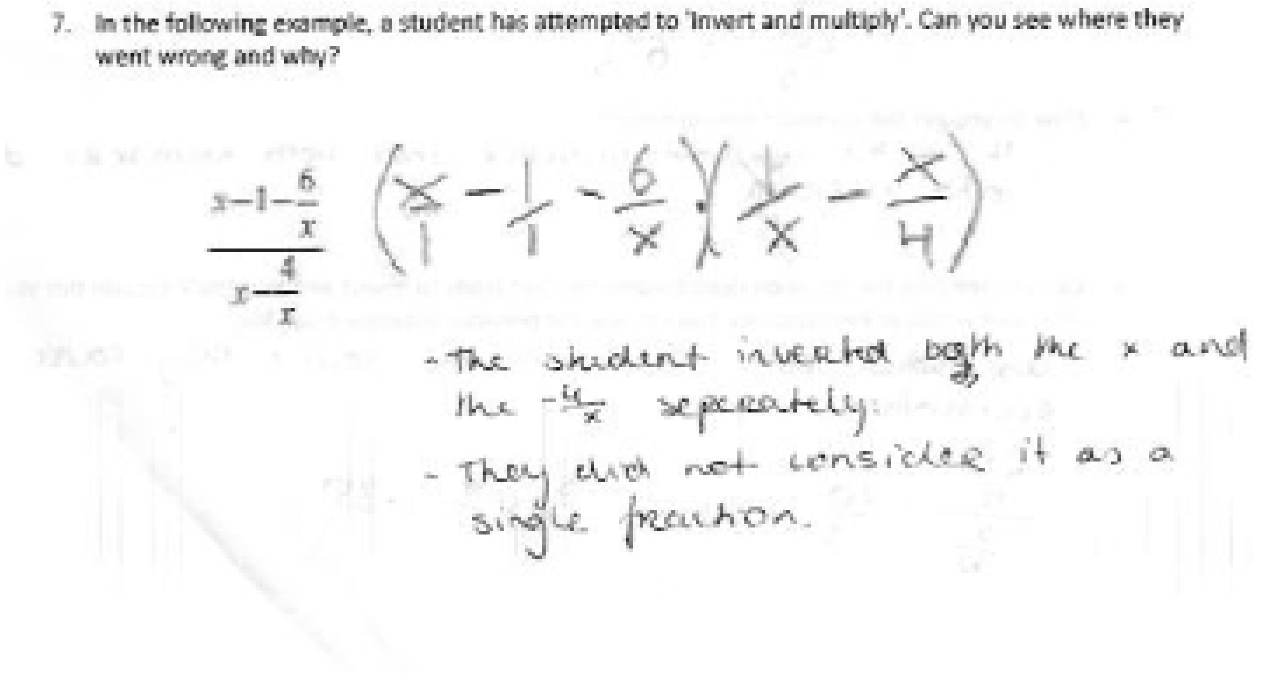
5. Can you see why the common denominator method leads to 'invert and multiply'? Explain this in your own words as best you can. You can use the previous example if you like.

I now see has why it works.

$6 \div 3 = 2$ - If the denominators are the same you just have to divide one into the other. With the "invert + multiply" method you skip finding the common denominator and instead get the values of the top bit divided by one another

$\frac{4}{6} \div \frac{1}{6} = 4$

After the lesson students could identify and correct mistakes in applying the division of fractions algorithm.



7. In the following example, a student has attempted to 'Invert and multiply'. Can you see where they went wrong and why?

$$\frac{x-1-\frac{6}{x}}{x-\frac{4}{x}} \quad \left(\frac{x}{1}-\frac{1}{1}-\frac{6}{x}\right)\cdot\left(\frac{1}{x}-\frac{x}{4}\right)$$

- The student inverted both the x and the $-\frac{4}{x}$ seperately.
- They did not consider it as a single fraction.

Students were observed to be actively engaged with the material throughout the lesson. Only one of the observing teachers was also a teacher of mathematics. They both remarked how surprised they were that such a seemingly simple set of tasks could hold the students attention. They were further impressed given it was last class of the day and their first day back from a break of three school days for a school tour.

Individual student's application of the algorithm also improved.
Student 'H's homework before the lesson:

6. $\frac{x-1-\frac{6}{x}}{x-\frac{4}{x}}$ $\left(\frac{x}{1}-\frac{1}{1}-\frac{6}{x}\right)\cdot\left(\frac{1}{x}-\frac{x}{4}\right)$

Student 'H's homework after the lesson:

1. Simplify: $\frac{x+\frac{1}{x}}{x-3+\frac{2}{x}}$

$$\frac{\frac{x^2+1}{x}}{\frac{x^2-3x+2}{x}} = \frac{x^2+1}{x}\cdot\frac{x}{x^2-3x+2} = \frac{x^3+x}{x^3-3x^2+2x}$$

$$= \frac{(\cancel{x})(x^2+1)}{(\cancel{x})(x^2-3x+2)}$$

$$\boxed{= \frac{x^2+1}{x^2-3x+2}}$$

Student P's work before the lesson:

6. $\frac{x-1-\frac{6}{x}}{x-\frac{4}{x}}$ $\frac{-6}{x}\times\frac{x}{3} = \frac{-6x}{-3x} = -2$

Student P's work after the lesson:

2. Simplify: $\frac{2-\frac{1}{x}}{2x+3-\frac{2}{x}}$

$$2-\frac{1}{x} = \frac{2}{1}-\frac{1}{x} = \frac{2(x)-1(1)}{x(1)} = \frac{2x-1}{x}$$

$$2x+3-\frac{2}{x} = \frac{2x(x)+3(x)-2(1)(1)}{x(1)(1)} = \frac{2x^2+5x-2}{x}$$

$$\frac{2x-1}{x}\cdot\frac{x}{2x^2+3x-2} = \frac{2x^2-x}{2x^3+3x^2-2x}$$

Student F's work before the lesson:

6. $\dfrac{x-1-\frac{6}{x}}{x-\frac{4}{x}}$

$$x - 1 - \frac{6}{x}\left(x - \frac{x}{4}\right)$$

$$x^2 - \frac{x^2}{4x} - x + \frac{x}{4} - \frac{6x}{x^2} + \frac{6x}{4x^2}$$

Student F's work after the lesson.

1. Simplify: $\dfrac{x+\frac{1}{x}}{x-3+\frac{2}{x}}$

$$\frac{\frac{(x)}{(x)}\frac{(x)}{(1)} + \frac{1}{x}\frac{(1)}{(1)}}{\frac{(x)}{(x)}\frac{(1)}{(1)}\frac{x}{1} - \frac{3(x)}{1(x)} + \frac{2}{x}\frac{(1)}{(1)}} = \frac{\frac{x^2}{x} + \frac{1}{x}}{\frac{x^2}{x} - \frac{3x}{x} + \frac{2}{x}}$$

$$= \frac{x^2+1}{x^2-3x+2} = \frac{(x+1)(x-1)}{(x-1)(x-2)} = \frac{(x+1)}{(x-2)}$$

Based on your analysis, how would you change or revise the lesson?

There are a number of revisions I plan to implement.

Firstly, I would give this lesson as part of a series of lessons at the start of the year on fractions and rational algebraic expressions. The same approach of visualising fractions would be taken to compare, partition, add and subtract, factorise and simplify numeric fractions followed by algebraic fractions. Only after that has been achieved would this lesson take place. Students would then be more confident and comfortable modelling fractions.

Secondly, if possible, the lesson should be extended and spread over a double class. This would give students more time to grasp the concepts at hand and offer an opportunity for the better able students to go further.

Thirdly, to further build relational understanding, examples of fractions in context from words only should be added to the learning experience for the students. E.g.

"A jug holds one third of a litre when full. A smaller cup holds one sixth of a litre when full. How many cups can be filled from the jug?"

What are the implications for teaching in your field?

Fractions are a difficult topic to teach and to learn. It is clear that while many students can perform the algorithms, they do not have a relational understanding of the operations on rational numbers. Teaching with visual representations helps to achieve this. A full depth of understanding of fractions and the algorithms for operations on rational numbers cannot be achieved quickly. It would be a worthwhile endeavour to set aside some time ***each year*** to build this understanding.

12. Appendix 1 Pre-Assessment Worksheet

NAME:___________________________ **NO CALCULATORS PLEASE!!!!**

1. Explain in your own words how to divide a fraction by a fraction.

2. Do you know *why* this method of division works? Can you explain?

Simplify each of the following:

3. $$\frac{\frac{2}{5}}{\frac{3}{7}}$$

4. $$\frac{1+\frac{2}{3}}{2+\frac{5}{9}}$$

5. $$\frac{\frac{x}{3}}{\frac{x^2}{5}}$$

6. $$\frac{x-1-\frac{6}{x}}{x-\frac{4}{x}}$$

13. Appendix 2 Class Presentation Slides

Slide 1

Flip & Multiply: WHY?

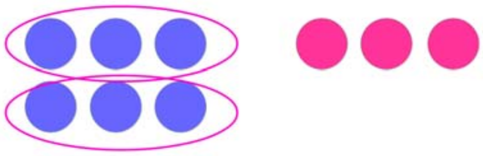
Today, we will:

- Look at and think about division with numbers.
- Use diagrams to help explain and understand division.
- See WHY the 'Flip & Mulitply' works.
- Apply the same approach to algebraic fractions in a similar way.

Slide 2

Explain With Words & Diagrams

$$6 \div 3$$

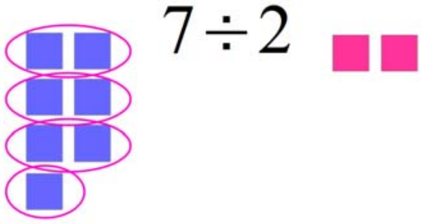


$$\frac{6}{3} = 2$$

Slide 3

Explain With Words & Diagrams

$$7 \div 2$$

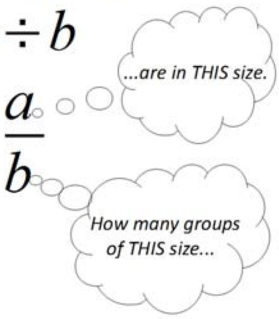


$$\frac{7}{2} = 2\frac{1}{2}$$

Slide 4

Explain With Words

$$a \div b$$



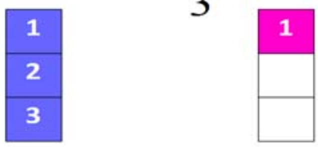
Slide 5 Part 1

$$1 \div \frac{1}{3}$$

How many groups of size 1/3's are in 1 unit?

Slide 5 Part 2

$$1 \div \frac{1}{3}$$



How many pinks fit in the blue?

$$1 \div \frac{1}{3} = 3$$

Slide 6 Part 1

$$\frac{3}{4} \div \frac{2}{5}$$

How many groups of size $\frac{2}{5}$ are in $\frac{3}{4}$ of a unit?

Slide 6 Part 2

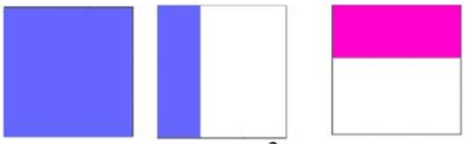
$$\frac{3}{4} \div \frac{2}{5}$$

How many pinks fit in the blue?

$$\frac{3}{4} \div \frac{2}{5} = \frac{15}{8}$$

Slide 7 Part 1

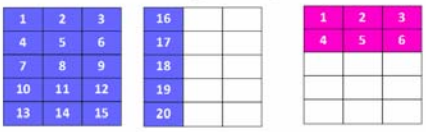
$$\left(1+\frac{1}{3}\right)\div\left(\frac{2}{5}\right)$$



How many groups of size $\frac{2}{5}$ are in 1 and 1/3 units?

Slide 7 Part 2

$$\left(1+\frac{1}{3}\right)\div\left(\frac{2}{5}\right)$$



How many pinks fit in the blue?

$$\left(\frac{20}{15}\right)\div\left(\frac{6}{15}\right)=\frac{20}{6}=\frac{10}{3}$$

Slide 8 Part 1

Explain With Diagrams & Numbers

$$\frac{2}{3}\div\frac{1}{5}=\frac{\frac{2}{3}}{\frac{1}{5}}=\frac{\frac{10}{15}}{\frac{3}{15}}=\frac{10}{3}=\frac{(2)(5)}{(3)(1)}$$

Slide 8 Part 2

Explain With Diagrams & Numbers

$$\frac{2}{3}\div\frac{1}{5}=\frac{(2)(5)}{(3)(1)}$$

Slide 9 Part 1

$$\frac{\frac{5}{4}}{\frac{3}{4}}=\frac{5}{3}$$

Slide 9 Part 2

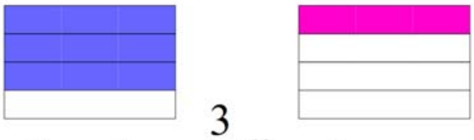
$$\frac{\frac{5}{4}}{\frac{3}{4}}=\frac{5}{3}$$

$$\frac{\frac{11}{7}}{\frac{2}{7}}=\frac{11}{2}$$

$$\frac{\frac{8}{12}}{\frac{5}{12}}=\frac{8}{5}$$

Slide 10

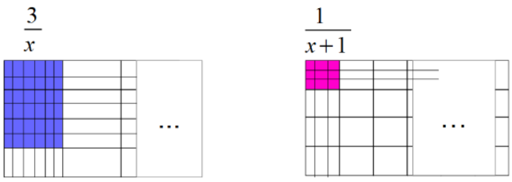
Explain With Diagrams & Numbers



$$\frac{3}{4}x \div \frac{1}{4}x = \frac{\frac{3}{4}x}{\frac{1}{4}x} = \frac{3x}{1x} = 3$$

Slide 11

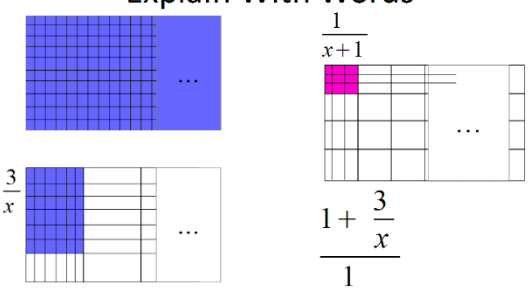
Explain With Words



$$\frac{\frac{3}{x}}{\frac{1}{x+1}}$$

Slide 12

Explain With Words



$$\frac{1+\frac{3}{x}}{\frac{1}{x+1}}$$

Slide 13

Learning From Mistakes

$$\frac{x-1-\frac{6}{x}}{x-\frac{4}{x}} \quad \left(\frac{x}{1}-\frac{1}{1}-\frac{6}{x}\right)\left(\frac{1}{x}-\frac{x}{4}\right)$$

Slide 14

Learning From Mistakes

$$\frac{x-1-\frac{6}{x}}{x-\frac{4}{x}} \quad x-1-\frac{6}{x}\left(x-\frac{x}{4}\right)$$

14. Appendix 3 Post-Assessment Worksheet 1

NAME:______________________

1. Explain in your own words what *a* divided by *b* means.

2. Do the same rules and methods apply for division by fractions and division by algebraic fractions? Why/why not?

3. Can you explain in your own words and numbers, using the common denominators method, how to divide $1+\frac{2}{3}$ by $\frac{1}{4}$?

4. How do you get the common denominator?

5. Can you see why the common denominator method leads to 'invert and multiply'? Explain this in your own words as best you can. You can use the previous example if you like.

6. In the following example, a student has attempted to 'invert and multiply'. Can you see where they went wrong and why?

$$\frac{x-1-\frac{6}{x}}{x-\frac{4}{x}} \qquad x - 1 - \frac{6}{x}\left(x - \frac{x}{4}\right)$$

7. In the following example, a student has attempted to 'invert and multiply'. Can you see where they went wrong and why?

$$\frac{x-1-\frac{6}{x}}{x-\frac{4}{x}} \qquad \left(\frac{x}{1} - \frac{1}{1} - \frac{6}{x}\right)\cdot\left(\frac{1}{x} - \frac{x}{4}\right)$$

15. Appendix 4 Post-Assessment Worksheet 2

NAME:____________________________

1. Simplify: $\dfrac{x+\dfrac{1}{x}}{x-3+\dfrac{2}{x}}$

2. A car travelled an unknown distance *d*, amd then returned to its original location by the same path.
   On the outbound journey, the car travelled at 30km/hr.
   On the return journey, the car travelled at 60 km/hr.

   (a) Find an expression for the time taken for the first part of the journey in terms of d.

   (b) Find an expression for the time taken for the return part of the journey in terms of d.

   (c) Hence find an expression for the the average speed of the entire trip and simplify this term.

3. Simplify: $\dfrac{2-\dfrac{1}{x}}{2\mathrm{x}+3-\dfrac{2}{x}}$

4. Simplify: $\dfrac{x-3-\dfrac{30}{x-2}}{x-1-\dfrac{20}{x-2}}$