Reflections on Practice 2015

Fostering Independent Learning of Algebra, specifically quadratic equations, through the use of GeoGebra

Teachers Invoved:
Margaret Bonner
Donna Martin
Hayleigh Ferry

Regional Development Officer :
Kieran Sweeney

Reflections on Practice

1. **Title of Lesson:** Fostering Independent Learning of Algebra, specifically quadratic equations, through the use of GeoGebra.

2. **Brief description of the lesson:**
   To help students in the revision of solving linear and quadratic equations both algebraically and graphically.

3. **Aims of the Lesson**
   From the teacher's perspective:
   I would like my students to appreciate that algebra is a tool for making sense of certain situations.
   I would like to foster my students to become independent learners.
   I would like my students to connect and review the concepts that we have studied already.

   For students to understand:
   How to solve linear and quadratic equations with real number solutions.
   The link between solving equations algebraically and graphically.

4. **Learning Outcomes**
   As a result of this lesson, students will be able to:
   - Solve linear equations algebraically and use GeoGebra to solve graphically.
   - Solve quadratic equations algebraically and use GeoGebra to solve graphically.
   - Extension: Forming quadratic equations from graphs in GeoGebra files.

5. **Background and Rationale**
   4.2 Solving Equations
   Students should be able to……
   - Select and use suitable strategies (graphic, numeric, algebraic, mental) for finding solutions of the form:
     - $f(x) = g(x)$ with $f(x)=ax+b$, $g(x)=cx+d$, where $a,b,c,d \in Q$
     - $f(x) = k$ with $f(x) = ax^2 + bx + c$ (not necessarily factorisable), $a, b, c \in Q$ and interpret the result
   - Extension: Form quadratic equations given whole number roots.

6. **Research**

Active Maths 3 – Book 1
GeoGebra files
Syllabus
www.projectmaths.ie
Discussion with colleagues and Project Math Regional Development Officer
Teacher Handbook for Senior Cycle – Ordinary Level (LCOL.16)
Teaching and Learning Plan-Equations

7. **About the Unit and the Lesson**

This lesson encourages the student to discover the link that a linear equation has one solution where the line cuts the x-axis. Furthermore, it aids the students learning with regards to discovering the link that two solutions of a quadratic equation are demonstrated by the curve intersecting the x-axis in two places. As an extension students will be encouraged to form quadratic equations from graphs where whole number roots can be observed.

8. **Flow of the Unit**

Lesson	Topic	# of lessons periods
1	Solving linear equations	1
2	Factorising quadratics	2
3	Solving quadratic equations by factorizing	2
4	Solving quadratic equations by formula	1
5	Fostering Independent Learning of Algebra, specifically quadratic equations, through the use of GeoGebra	1
6	Forming quadratic equations given the roots	1

9. Flow of the Lesson

Teaching Activity	Points of Consideration
1. Introduction	In previous lessons, students can: - Solve linear equations algebraically - Solve quadratic equations by factorizing - Solve quadratic equation by formula
2. Posing the task	From solutions found algebraically in the previous lesson, students will be asked to check their solutions using GeoGebra. They will be given a number of graphs and they will have to match the graphs with the linear and quadratic equations they have solved algebraically. As an extension, students will be given separate graphs and asked to form the corresponding quadratic equation.
3.Anticipated student responses	I expect a lot of questions from students in getting started with GeoGebra. To deal with this I made out an instruction sheet in working with GeoGebra (I have attached this instruction sheet in the appendix). In addition there may be a lot of queries about the connection between the solutions found algebraically and what the graphs are telling them.
4.Comparing and discussing	Students will compare their results found algebraically with the results found from the graphs. In addition it is anticipated that students will compare their results with each other in order to correct their work from the previous lesson as well as discuss each other's approach.
5. Summing up	(a) Students will be asked to take out their work from the previous lesson where they solved linear equations and quadratic equations algebraically.
	(b) Students will be introduced to the applications of GeoGebra. They will be asked to check their solutions of the previously solved linear and quadratic equations using the graphs they produce in GeoGebra.

	(c) Students will be given a sheet with 8 graphs and they will match these graphs with their previously solved linear and quadratic equations (10 solved, so two equations do not have a matching graph).
	(d) As an extension, students will be given a sheet showing one example how to form a quadratic equation from a graph. They will be given other graphs to form the corresponding quadratic equations.

10. Evaluation

As part of my plan for evaluating the lesson I will invite the Regional Development Officer and one of my colleagues to observe the students. They will observe three students each and record the students' correct work and misconceptions on the observation template sheet. They will take photos of the students' work with prior permission as discussed before the lesson. They will also observe discussions between students of the work, ask the students questions to assess understanding, and acknowledge any issues with varying levels of ICT skills including all of these on the template sheet also.

I will also get students to fill in an evaluation sheet at the end of the lesson. This will include multiple choice questions to assess how useful they found the lesson, specifically what part of the lesson they found most useful, and if they would recommend the lesson for use with other students. I will also leave an open ended question for suggested improvements.

11. Board Plan

(i) Solving Linear Equations

Solve

$4(x - 4) = 12$

$4x - 16 = 12$

$4x = 12 + 16$

$4x = 28$

$x = 7$

This method involves solving the linear equation using algebra. Can we check our result using another method????????

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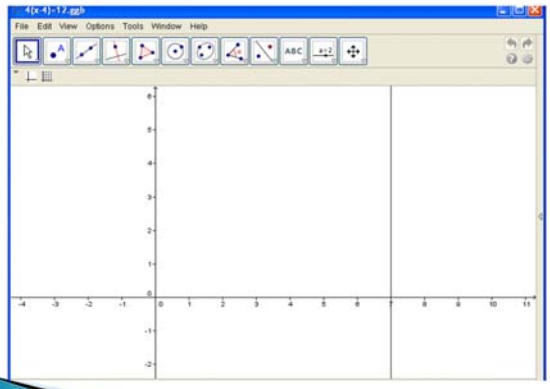
Geogebra file to solve $4(x - 4) = 12$



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(ii) Solving Quadratic Equations

Solve

$x^2 - 100 = 0$

$(x - 10)(x + 10) = 0$

$x-10=0$, or $x+10=0$

$x=10$, or $x=-10$

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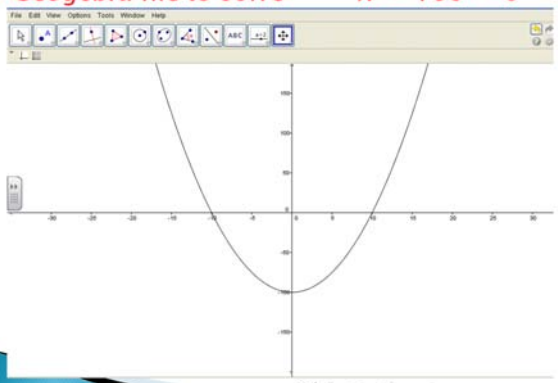
(ii) Solving Quadratic Equations

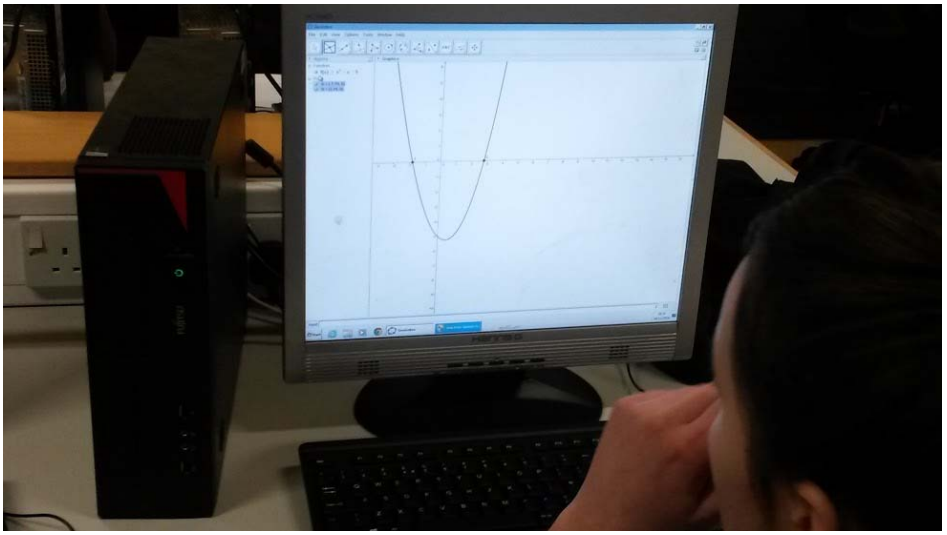
- Geogebra file to solve $x^2 - 100 = 0$



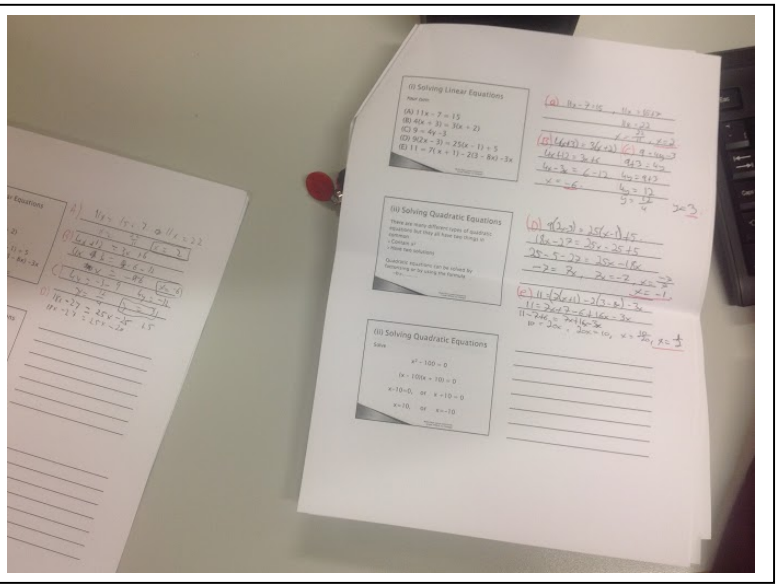
Linear and Quadratic Equations	Solutions	Graph No
(A) $11x - 7 = 15$		
(B) $4(x + 3) = 3(x + 2)$		
(C) $9 = 4y - 3$		
(D) $9(2x - 3) = 25(x - 1) + 5$		
(E) $11 = 7(x + 1) - 2(3 - 8x) - 3x$		
(F) $x^2 - 4x = 0$		
(G) $x^2 - 9 = 0$		
(H) $x^2 - 3x - 20 = 0$		
(I) $2x^2 + 2x - 1 = 0$		
(J) $-2x^2 - 5x + 9 = 0$		

12. Post-Lesson Reflection

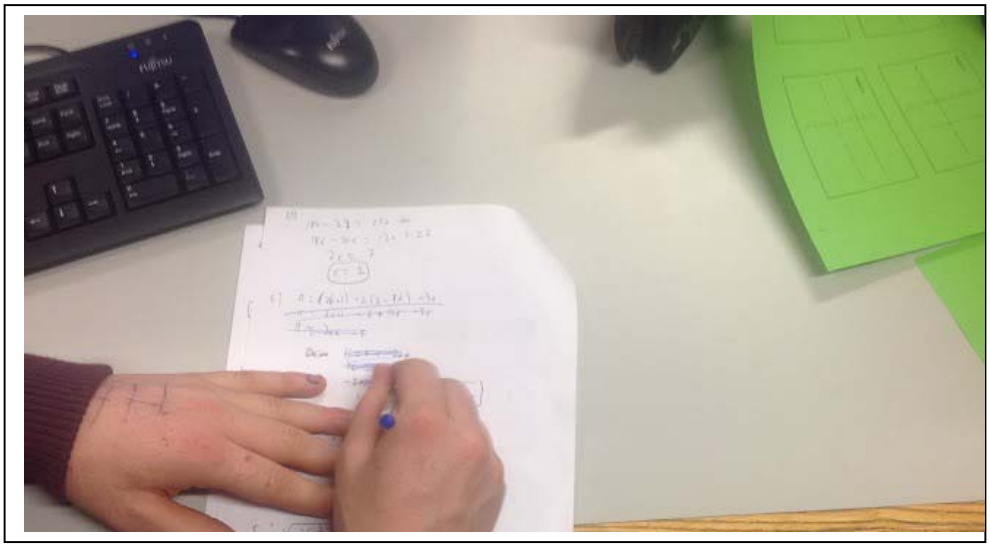
- The beginning of the lesson was a bit frustrating as the students took more time than anticipated to get to grips with the use of GeoGebra, i.e. exploring GeoGebra before listening to instructions. The linear equations could be inputted as they are but the quadratic equations needed f(x) = before they were inputted. However, their skills did improve exponentially with practice. Indeed I was extremely surprised with the students' progress with GeoGebra, i.e. reading off the solution from the left-hand side of the screen. I did not expect them to make such rapid progress with the software.



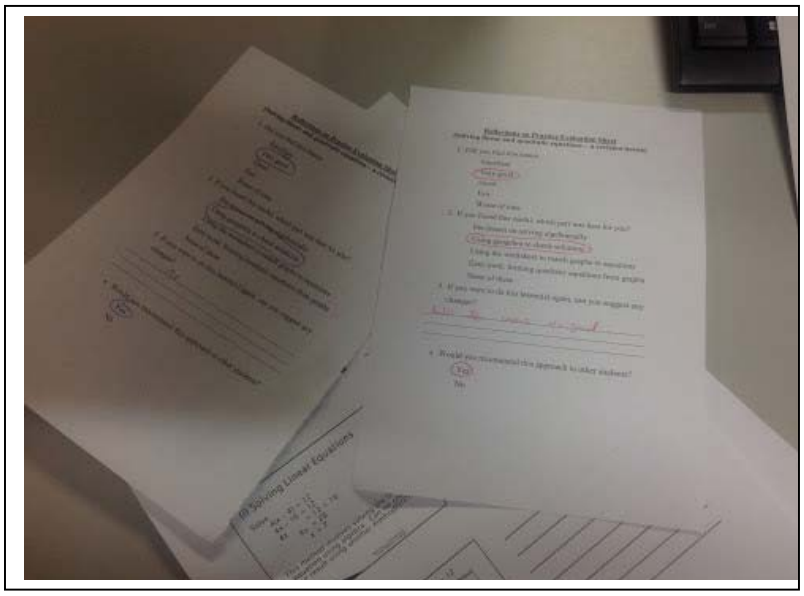
- Some weaker students were somewhat uncomfortable with the freedom of this lesson. They needed to be reminded that the roots of any equation is where the graph cuts the x-axis and reassured that their approach was correct at various stages of the lesson.



- The students worked extremely well independently in checking their solutions with the graphs obtained using GeoGebra. Group work resulted in teams of students comparing their answers with each other and in some cases this led to students self-correcting their work. In addition competition developed among the students as to who could match their solutions in the quickest time.

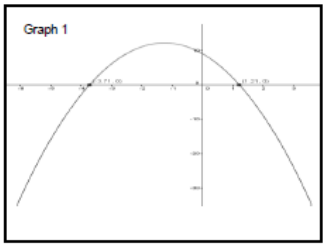


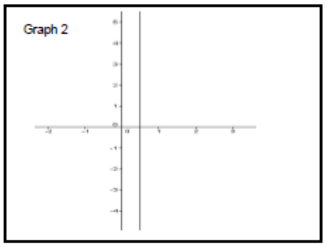
- Such group work was not planned but it was very refreshing to see the outcomes of such engagement.
- In terms of timing, I overestimated how much work I expected to cover in the lesson. To improve on this I should have spent some time in the previous lesson going through the GeoGebra instructions. In the next lesson following this project, I worked on the extension work to help students identify that the two roots of a graph can be used to form a quadratic equation.
- The responses of the students were very positive in the self-evaluation sheets distributed at the end of the lesson. Most students found the lesson very useful, highlighting the use of GeoGebra to check their solutions algebraically of most value. The students said that they would recommend this lesson to other students and looked forward to using this approach again in another area of Mathematics.

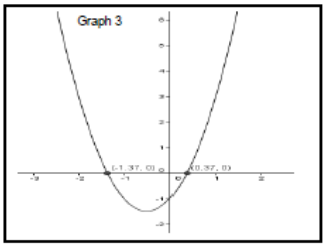


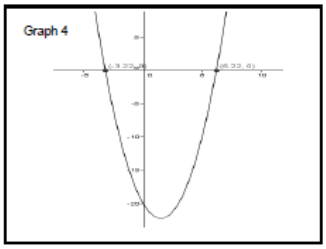
- I would use this approach in developing a similar lesson in exploring the chapter on Functions.

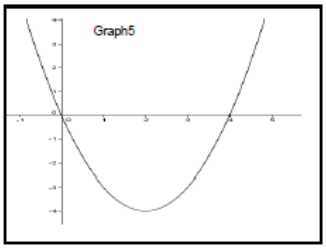
Appendix 1 – Examples from Graph Matching Exercise

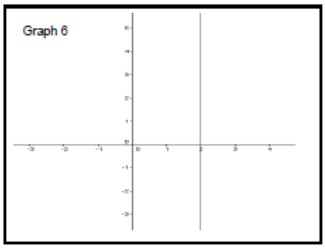








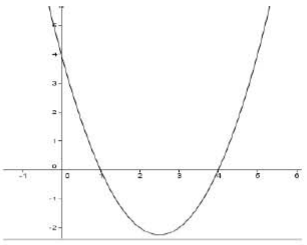




Appendix 2 – Extension Activities

Extension work – Forming the quadratic equations from the graphs

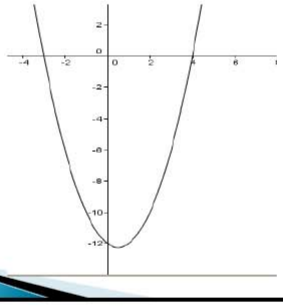
Graph 11- find the quadratic equation



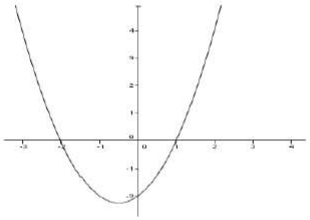
Graph 11

- Roots are x = 1 and x = 4
- $x - 1 = 0$ and $x - 4 = 0$
- $(x - 1)(x - 4) = 0$
- $x^2 - 4x - 1x + 4 = 0$
- $x^2 - 5x + 4 = 0$
- Check your answer by typing inyour answer into geogebra in the form $f(x) = x^2 - 5x + 4$ in the input bar

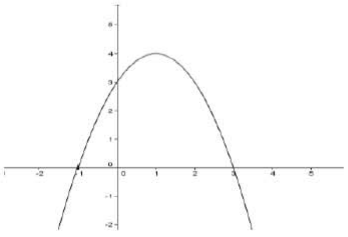
Your turn: Graph 12 – find the quadratic equation



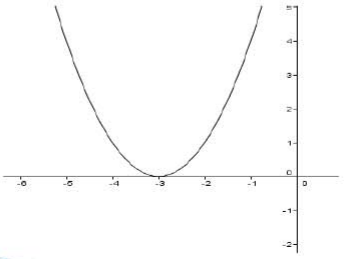
Your turn: Graph 13 – find the quadratic equation



Your turn: Graph 14 – find the quadratic equation



Your turn: Graph 15 – find the quadratic equation



Appendix 3 – Student Evaluation Sheet

(Solving linear and quadratic equations – a revision lesson)

1. Did you find this lesson

   Excellent

   Very good

   Good

   Fair

   Waste of time

2. If you found this useful, which part was best for you?

   Pre-lesson on solving algebraically

   Using GeoGebra to check solutions

   Using the worksheet to match graphs to equations

   Extension work: forming quadratic equations from graphs

   None of these

3. If you were to do this lesson(s) again, can you suggest any changes?

_____

_____

_____

4. Would you recommend this approach to other students?

   Yes

   No