Lesson Plan for Second year Maths

Topic: Algebra, Multiplying two binomial expressions

Date of Lesson: 3rd and 4th March 2015, at St. Joseph's Secondary School, Foxford, Co. Mayo

Teacher: Paul Philbin

Class: 2A

Observing Teacher: Kevin Flynn

1. **Title of the Lesson**: Multiplication of binomial algebraic expressions using the array model
2. **Brief description of the lesson:** To help students realise that when multiplying two binomial expressions, where each expression has two or more terms, that each term in the first expression is multiplied by each term in the second expression and why this is the case.
3. **Aims of the Lesson:** Long range goals
    a. I'd like to foster my students to become independent learners.
    b. I'd like my students to become more creative when devising approaches and methods to solve problems.
    c. I'd like my students to appreciate that algebra is a tool for making sense of certain situations.

    Short term goals

    a. I'd like students to develop an understanding of the distributive law for multiplication of numbers and to apply this law to the multiplication of binomial expressions in algebra.
4. **Learning Outcomes**: As a result of studying this topic pupils will be able to
    a. Construct array models to simplify the multiplication of natural numbers.
    b. Construct array models suitable for multiplying algebraic expressions.
5. **Background and rationale:**
    a. **Background** (What the students need to learn): Students need to be able to decompose natural numbers and hence to be able to multiply e.g. $(12)(14) = (10 + 2)(10 + 4)$. They must apply the same reasoning to the multiplication of binomial algebraic expressions e.g. $(x + 2)(x + 4)$.

    Later, when factorising quadratic expressions, they will be able to check their answers by multiplying out the brackets. The array model will also help them to deepen their understanding of factorisation. By making these connections they will become more independent learners.

b. **Rationale:** The results of the Maths competency test showed that not all students had made the connection that when multiplying two expressions that all terms in the brackets must be multiplied by each other. The array model may act as a guide to the students' learning and help with scaffolding this topic. It is hoped that the knowledge of the array model method will help students later on when it comes to factorising and dividing for example a linear expression into a quadratic expression.

6. Research:

Twenty five students in the class took the Project Maths: Maths Competency Test and the following difficulties with the multiplication of expressions were identified:

Question 6: which of the following is equivalent to $3x + 3x$?

(a) $6x^2$ (b) $9x$ (c) $9x^2$ (d) $6x$

24% of students answered correctly

16 chose (a), 1 chose (b), 2 chose (c), 6 chose (d).

Question 7: which of the following is equivalent to $2(2x - 5)$?

(a) $4x - 5$ (b) $4x - 10$ (c) $14x$ (d) $2x - 10$

92% of students answered correctly.

1 chose (a), 23 chose (b), 0 chose (c), 0 chose (d) 1 left answer blank

Question 9: Which of the following is equivalent to $(x + 3)^2$?

(a) $x^2 + 3^2$ (b) $x^2 + 6$ (c) $x^2 + 6x + 6$ (d) $x^2 + 6x + 9$ (e) $x^2 + 9$

4% of students answered correctly.

6 chose (a), 3 chose (b), 0 chose (c), 1 chose (d) 11 chose (e)

4 left answer blank

Question 17: A length of wood is 6 metres long. It is cut into two pieces. One of the pieces is x metres long. How long is the other piece?

(a) $(x - 6)$ metres (b) 3 metres (c) I can't say as I don't know what x is. (d) $(6 - x)$ metres

28% of students answered correctly.

0 chose (a), 4 chose (b), 10 chose (c), 7 chose (d) 4 left answer blank

The students also took a multiplication test of integers and algebraic terms and similar difficulties were noted. Students' written comments on this test indicated a lack of

understanding and familiarity and general confusion in this area. Nine students out of 28 could handle multiplying binomial expressions.
Only two out of 28 got all five questions on multiplying binomial expressions correct.

Resources that will be used: Whiteboard and Students' copies, Student "Show me Boards", pens and erasers, textbook questions, Project Maths problems

7**. About the unit and the lesson**

To help students realise that numbers can be decomposed into smaller numbers, multiplication of for example a pair of two digit numbers, may be carried out using the distributive property,
e.g. $78 \times 56 = (70 + 8) \times (50 + 6) = 70 \times 50 + 70 \times 6 + 8 \times 50 + 8 \times 6.$
Use of the distributive law simplifies the multiplication operation into more manageable steps so that it can be done more easily than the standard long multiplication algorithm from memory or on pen and paper. The greatest benefit of such an approach to the multiplication of natural numbers is that it develops the students' skill of decomposing numbers. This skill in turn helps with students' efforts at estimation, and should help students make sense of problems by breaking them down into smaller steps. It also provides a useful lead in to the multiplication of binomial expressions in algebra.

The distributive law will be applied to the problem of multiplying out binomial expressions in algebra e.g.. $(x + 3)(x + 4) =$

	x	+3
x	x^2	$+3x$
+4	$+4x$	12

The diagram of the array model will make it easier for some students to capture the relationship between the factors to be multiplied and the resulting quadratic expression , visually.

8. Flow of the unit

Algebra 1 – Second year Maths (Higher Level – Chapter 1 – normally done in September)

Each lesson takes one class period, depending on students' ability.

Lesson 1: Simplifying expressions

Lesson 2: Removing brackets (#1 Research lesson)

Lesson 3: Evaluating expressions

Lesson 4: Solving Linear equations

Lesson 5: Solving problems using linear equations

Lesson 6: Inequalities

Lesson 7: Solving Inequalities.

9**. Flow of the lesson**

Teaching Activity	Points of consideration
1. Introduction Lesson title: The array model for multiplication. Ask students to multiply out two binomial expressions (which they have previously learned to do using conventional methods :first term by first term, first by second, etc). Explore the distributive property of multiplication using John's garden example, and how this may be illustrated using the array model. Move on to larger numbers and repeat the method of the array model multiplication with these numbers. Students will appreciate that a large number can sometimes be broken down into factors. These factors can be decomposed into more manageable numbers and then multiplied by each other.	Lesson title: The array model for multiplication. The observing teacher needs to note the students who have difficulty with this problem. Use **"Show me Boards"** to take answers and determine which table of 4 students can do it. Try to ascertain the students who are in the most difficulty. Students may have difficulty understanding the problem posed, particularly those with specific learning difficulties. The observing teacher will monitor these students. Classroom teacher will attempt to uncover the difficulties encountered through questioning. Check if students are drawing and completing the array tables properly (visually by observing teacher/class teacher). Can students handle the multiplication in the array and the subsequent addition of the numbers to find the final answer?
2. Posing the task Using the "back garden" example, x is introduced as a "generalised number". An algebraic expression must be formed for the area of the garden. The "car park" problem leads to the multiplication of $(x+5)(x+2)$ Pose questions as to how we might do this. Present the array model. Q1: How do we get x^2 in top left corner? Q2: How do we get 10 in the bottom right corner? Q3: What should the remaining boxes contain if we multiply the length by the width?	Understanding the problem posed and relating the dimensions in the problem to the array model. By using logic and working within a group, students will be able to make sense of why this is done. Students will copy the diagram from the board and each other. How do we know if students understand? Observation of written work and structured questioning. Observation of group answer to each question via the 'show me' boards. A1: multiply x by x A2: Multiply the factors, 2 by 5. A3: Areas $5x$ and $2x$

Q4: What is the <u>sum</u> of the areas of the remaining boxes? Hence the factors $(x+5)(x+2)$ multiply to give the expression $x^2+7x+10$	A4: $5x + 2x = 7x$ Now set the students work – further exercises via PowerPoint presentation.
3. Anticipated student responses R1 – Students are unsure of where to place the factors outside of the array diagram. R2 – Students are unsure how to calculate the resulting quadratic expression from the array diagram – they do not realize that the diagonal boxes containing the single x terms must be added.	Talk students through the problem with reference to the solution on the board. Emphasize that area =length times width so the factors must be written on the outside of the diagram. Monitor students work and question individual students about their thinking where errors arise as seen when circulating the room. If students finish early (before others) prescribe some additional work or allow them to start homework early.
4. Comparing and discussing Write a correct and incorrect solution to a problem given earlier and ask the students to identify by a show of hands which one is the correct solution and to explain why.	Ideas to focus on: Verifying your answer by multiplying out your answer using the method that you learned in first year.
5. Summing up Short question and answer session on how to multiply out using the distributive law : a. numbers and b. algebraic expressions Observation of students' work Prescribe homework.	Students state verbally where the appropriate terms are placed in the diagram –x^2 , x terms and number (constant) term. Students state which two boxes must be added to give the resulting x term. Complete exercises and start homework if finished.

10. Evaluation

What is your plan for observing students? The teacher will circulate after prescribing work during the class.

Logistical issues: The observing teacher will watch the students to see if they are making the connection between decomposing expressions in the array diagram and then writing out the product as the correct quadratic expression.

Data will be recorded using a tally system. The observing teacher will record the number of groups of students who completed the task successfully on the 1st task, 2nd task, etc. This can be done by using the "Show me Boards" and verified by the observing teacher. The observing teacher will then focus on individual tables to identify the weaker students who are struggling and perhaps being carried by their group. By observing the students he will record the language students are using, their mistakes and misconceptions surrounding this topic. He will also note the questions the students are asking, and will keep an eye on the lesson plan during the class period to see that it is being followed.

Student thinking and behaviour – The observing teacher will check to see, for example, if the students appear puzzled, how effective they are at learning from examples from the board, whether or not the students' questions are connected to the work they have done in their copies, whether or not their understanding of the topic is reflected in their written work, students' level of participation etc.

Student work will be collected on a sheet of squared grid paper for further evaluation.
This will be checked and corrected to help evaluate the lesson.

11. Board Plan – see Microsoft PowerPoint presentation. Teacher board work will be done as the need arises during the lesson as well as being based on planned work and will be built around the lesson presentation.

12. Post lesson reflection

- **What are the major patterns and tendencies in the evidence? Discuss.**

  Students were reluctant to use the array model unless they could see a need for it. About half of the students indicated that they would adopt the array model to multiply binomial expressions as they found it easier. Students who were competent in multiplying binomial expressions using the traditional method were reluctant to adopt the array model.

  Students enjoyed the learning experience, in particular, working in groups. Weaker students in particular, reported that they preferred problem solving in groups to individual work.

  Students had difficulty with formulating expressions from the "car park" problem. Even when given the diagram, students had difficulty writing down the correct expression for the area of the car parking space. Some of those students who did get the correct expression had difficulty multiplying it out correctly.

- **What are the key observations or representative examples of student learning and thinking?**

  The importance of place value when multiplying terms in the array model and adding the results to find the answer

- **What does the evidence suggest about student thinking such as their misconceptions, difficulties, confusion, insights, surprising ideas, etc,**

  Wrong answers given suggest that many of the students had an underlying confusion with the addition and multiplication of algebraic terms. For example:

  $$(x + 2)(x + 5) = 7x^2$$

  Here a student interprets the plus sign as an instruction to add 2 and 5, the x terms combined in the x squared term, and the equals sign treated as an instruction to write the answer as a single term. The student has not grasped the idea that number and x terms should be added separately and that no sign between the brackets indicates multiplication.

  $$(x + 2)(x + 5) = (2x)(5x) = 10x^2$$

  In this example a student combines the number and x terms (incorrectly) in each bracket, and then multiplies the result correctly.

  $$10 + 5x = 15x$$

  Again in this example a student wishes to combine number and x terms into a single answer.

- **In what ways did students achieve or not achieve the learning goals?**

  The students all achieved, to some extent, the short term learning goal of developing an understanding of the distributive law of multiplication of numbers and to apply this law to the multiplication of binomial expressions in algebra.

  The students were divided into groups. The extent of their success at finding correct answers to the problems posed is shown in the tables below for the first and second class periods of the lesson.

First class period							Second class period			
Group	**Test: multiply expression** $x^2 + 7x + 10$	**Q1. Area of garden** $5 \times 12 =$ **60 m²**	**Area of garden and patio.** $10 + 50 =$ **60 m²**	**Q2. Multiplying numbers** $14(5) = 70$ $37(3) = 111$ $59(4) = 236$	**Q3. Expression of area of garden and patio:** $10 + 5x$	**Q4. Multiply** 56×78 $= 4368$	**Group**	**Q5. Multiply** $56 \times 38 =$ **2128**	**Q6. Multiply** $29 \times 35 =$ **1015**	**Q7. Area of car park space** $= (x + 2)(x + 5)$
A	Y						A			
B	N	Y	Y	Y	N	N	B	N		N
C	Y	Y	Y	Y	N	N	C	Y		N
D	Y	Y	N	Y	Y	Y	D	Y	Y	N
E	Y	Y	Y	Y	Y	N	E	Y		Y
F	Y	Y	Y	Y	N	N	F	Y		N
G	y	Y	N	Y	N	Y	G			

After some initial difficulties in the first class period (Q4 above), students were competent in using the array model to multiply natural numbers (Q5 above).

Students' difficulties formulating expressions from a problem/diagram were evident in Q3 and Q7 above. It is thought that this difficulty stems from an underlying confusion between the algebraic (x) and number terms and how they should be combined in an expression to represent an area.

There is a clear pattern in the evidence above that shows the students' lack of understanding as the lesson moved from the familiar territory of natural numbers to the less familiar area of number and algebraic terms. Many students reverted back to typical errors at this stage.
After the lesson homework was given, some students who had difficulty with traditional algebraic multiplication successfully adopted the array model for the multiplication of expressions. Other students who were competent with traditional algebraic multiplication had no need to adopt the array model, but possibly understood the algorithm better.

- **Based on your analysis, how would you change or revise the lesson?**

  The lesson was useful in developing the second years understanding of algebraic multiplication. A lesson on the addition and subtraction of number and algebraic terms using algebraic tiles might have provided a better introduction to the topic with this group. This might have cleared up some of the misconceptions around addition of number and algebraic terms. This could have been followed up with the lesson on the array model.

  The array model lesson itself belongs in first year, and should be included. At the start of first year (Teacher handbook Section 1, Natural Numbers - Strand 3) I would teach students how to multiply natural numbers using the array model.

  Towards the end of first year (Teacher handbook, section 7, patterns and algebra – strand 4) I would adopt the array model for the multiplication of algebraic expressions as suggested in the handbook. I would teach the array method before introducing the traditional method. Use of algebra tiles to explore the differences between numbers and algebraic terms should also be useful here.

  In second year, the array model can be used again when the students meet algebraic multiplication and division.

- **What are the implications for teaching in your field?**

  Much confusion around the addition and multiplication of number and algebraic terms has been identified in Second year Maths. More time should be devoted in First year in particular to ensure that students can add number and algebraic terms correctly.

  The array model is a useful tool for weaker students in particular when multiplying algebraic expressions. Early introduction of the array model in the natural numbers section at the start of first year would mean students are comfortable using it. The subsequent use of the array model for the multiplication of algebraic terms in first and second year should aid student understanding of this topic.

Appendix 1

Scripted Teacher Questions – Lesson on Array Model

1. **(slide 1)** What do two brackets back to back mean, in algebra? **Answer: Multiplication.**
2. **(slide 1)** Can you multiply the following two expressions, with two terms in each bracket?

   $(x + 5)(x + 2)$ (Binomial expression) **Answer:** $\boldsymbol{x^2 + 7x + 10}$
3. **(slide 3)** What is the length of John's patio? (recall) **Answer: 2 m**
4. **(slide 3)** What is the width of John's patio? (recall) **Answer: 5 m**
5. **(slide 3)** How do we find the area of John's patio?

   Write down the calculation. **Answer:** $\boldsymbol{5(2) = 10\ m^2}$
6. **(slide 5)** What are the dimensions of the rest of John's garden?

   Answer: 5 m wide, 10 m long.
7. **(slide 5)** What is the area of the rest of John's garden? **Answer: 5 (10)=50** $\boldsymbol{m^2}$
8. **(slide 7)** Can you split a large number, i.e. a number bigger than 10 into two numbers

   – One is a multiple of 10, the other is a single digit number, i.e. a number from 1 to 9.

   e.g. 75 = 70 + 5.

   Can you split up 34? **Answer: 34 = 30+4**

   This skill is called the decomposition of numbers.

   – You are breaking the bigger number up into two smaller numbers.

   (Can you decompose 34 using any two other numbers?)

   Did you know that when multiplying two numbers we can first decompose the numbers into smaller parts and then multiply the parts?

   We will get the same answer as multiplying the bigger numbers directly.
9. **(slide 8)** Decompose 12 into two numbers and using the distributive law of multiplication, multiply both parts by 5. i.e. $12(5) = (10 + 2)5 = (10)5 + (2)5 = 50 + 10 = 60$
10. **(slide 8)** Decompose 59 into two numbers and use the distributive law to multiply both its parts by 4. **Answer:** $\boldsymbol{(59)4 = (50 + 9)4 = (50)4 + (9)4 = 200 + 36 = 236}$
11. (**slide 8)** What is $37\ (3)$?
    Answer: $\boldsymbol{37(3) = (30 + 7)3 = (30)3 + (7)3 = 90 + 21 = 111}$

    Could you decompose 37 in other ways? If you did would you get the same answer for $\boldsymbol{37(3)}$
12. **(slide 9)** We need to write a general expression for the area of a garden where the actual length of the garden is unknown. Given the dimensions shown, write down the area of the patio and the area of the garden. **Answer: The area of John's Patio is:** $\boldsymbol{2(5) = 10}$ **square metres**
    The area of John's remaining garden = $\boldsymbol{x(5) = 5x}$ **square metres**
    Total area $\boldsymbol{= (10 + 5x)}$ **square metres**

Appendix 1

13. **(slide 11)** Here are three multiplication problems. You need to decompose each number and multiply the(constituent) parts of each number using an array diagram and then add the results to find your answer.

    What size array diagram do we need to multiply two numbers between 11 and 99?

    Answer: A two by two array. i.e. 56 x 78 = 4368. 56 x 38 = 2128. 35 x 29 = 1015.

14. **(slide 14)** In the "parking bay" problem the car has width x metres. So how wide is the car parking space? **Answer: The width is** $(x + 2)$ **metres.**

15. **(slide 14)** How long is the parking space in terms of x? **Answer: The length is**$(x + 5)$ **metres**

16. **(slide 14)** Now how do you write down the expression for the area of the parking space? (length times width). **Answer: The area is** $(x + 2)(x + 5)$.

17. **(slide 14)** How would you go about multiplying out these two expressions?
    Answer : using brackets or the array model.

18. **(slide 16)** Here is a maths problem. Have you seen it before?
    Answer: at the start of the class.

    Has your approach to solving this problem changed? **Answer: Hopefully yes!**

    (slide 17) Here are some other binomial expressions (2 numbers in brackets separated by a plus or minus sign) for you to multiply out and finish for homework ….

19. Have you enjoyed the lesson on the array model? Answer YES/NO by show of hands.

20. Have you learned any new maths skills that you will use in future?
    Rate how useful you find this new information on a scale of 1 to 5.
    Answer by show of hands - i.e. 5 = very useful, 1= not useful.

Appendix 2: Lesson PowerPoint Slides

PROJECT MATHS

Research lesson (Algebra)

Calculators are not allowed during this lesson.

Test: Multiply the following two expressions

$$(x + 5)(x + 2)$$

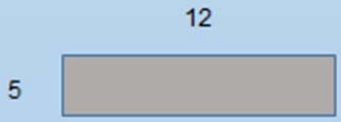
The back garden problem

- John lives on a terraced street. The houses are identical to each other.
- They all have back gardens. There is a windy road behind them.
- Each back garden has the same width, but the gardens are all different lengths.

Appendix 2: Lesson PowerPoint Slides

Area of John's back garden

- John's Garden is 12 metres long and 5 metres wide.
- Find the area of John's garden.



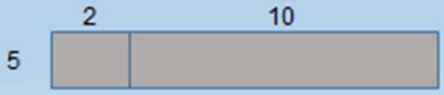
Answer!

- The area of John's Garden is:
- $(12)(5) = 60$ square metres!

Appendix 2: Lesson PowerPoint Slides

John gets a patio.

- The patio dimensions are 2 metres long by 5 metres wide.
- The rest of the garden (10m long) is behind the patio.
- Find the area of the garden and the area of the patio.



Answer!

- The area of John's Patio is: $2(5) = 10$ square metres!
- The area of John's remaining garden is: $10(5) = 50$ square metres!
- Total Area: $10 + 50 = 60$ square metres

Appendix 2: Lesson PowerPoint Slides

Multiplication distributes over addition

- Using the same method, multiply the following numbers.
- (i) 14(5) (ii) 59(4) (iii) 37(3)

Hint :(Split the larger number into two smaller numbers, then multiply these by the other number)

Answer!

14(5)		5		
	10	50		50
	4	20		20
				70

37(3)		3		
	30	90		90
	7	21		21
				111

59(4)		4		
	50	200		200
	9	36		36
				236

Appendix 2: Lesson PowerPoint Slides

General expression for the area of the garden

- The length of each back garden along John's street varies. How do we represent a number which can vary?
- The width of each garden is 5m.
- The length of each patio is 2m behind each house.
- The length of garden behind the patio is unknown.
- Write down an expression for the total area of the garden.

Answer!

- The area of John's Patio is: $2(5) = 10$ square metres!
- The area of John's remaining garden is: $x(5) = 5x$ square metres!
- Total Area: $10 + 5x$ square metres

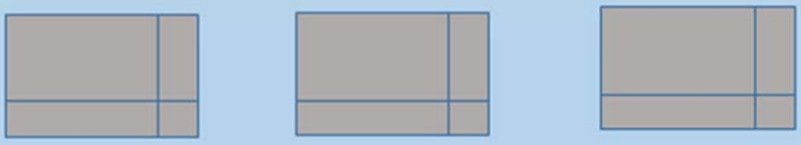
$5(x+2)$			
	5		
x	$5x$		
2	10		

Appendix 2: Lesson PowerPoint Slides

Further multiplication using the array model

- Using the array model, multiply:
- (i) 78(56) (ii) 56(38) (iii) 29(35)

Hint – split both numbers into two smaller numbers and multiply. You will need a bigger array!



Answers to multiplication of numbers using the array model

	50	6	
70	3500	420	3500
8	400	48	400
			420
			48
			4368

	50	6	
30	1500	180	1500
8	400	48	400
			180
			48
			2128

	30	5	
20	600	100	600
9	270	45	270
			100
			45
			1015

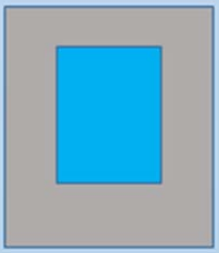
Appendix 2: Lesson PowerPoint Slides

Car Parking Problem

- A car park in a caravan site is being marked out.
- The parking spaces are wider and longer than normal.
- Each space must be suitable to fit a caravan or motor-home and a car by its side.

Car Parking Problem

- A car measures x metres wide, the space must be 2 metres wider than the car and 5 metres longer than the width of the car.
- Write down an expression in x for the area of the parking space



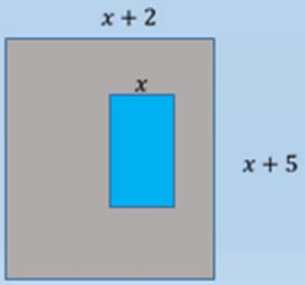
Appendix 2: Lesson PowerPoint Slides

Answer!

- Width of space = $x + 2$ metres
- Length of space = $x + 5$ metres
- Area of space, A

$A = (x + 2)(x + 5)$

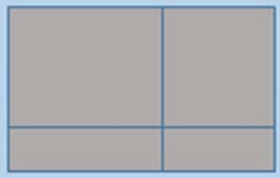
- $A = x^2 + 7x + 10$



Can you do the question now?

- *Test:* Can you multiply the following two expressions?

$(x + 5)(x + 2)$



- Where did you see this before?

Appendix 2: Lesson PowerPoint Slides