Introducing Algebra with Snöoker

For the lesson on 25th February 2016
At St Michael's College, Listowel, 1st Year class
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QUIS UT DEUS

Project Maths Tionscadal Mata Development Team

1. Title of the Lesson:

Introducing Algebra with Snöoker

2. Brief description of the lesson:

Using snöoker students will be introduced to the variable, algebraic expressions and substitution. Snöoker is an imaginary game (played in Belgium!) similar to snooker but the value of the balls is unknown to the students.

3. Aims of the Lesson:

I'd like students to:

- Understand that it can be useful to use letters in mathematics to stand for a "value" or "number". Variables are the central idea in algebra. So it is very important for students to feel at ease around them in order to be successful in algebra. Students need to both have a concrete understanding of what a variable is and be comfortable working with variables.
- Use generalised algebraic expressions in context and appreciate that mathematics can be used to communicate thinking effectively.
- Become independent learners.
- Become more creative when devising approaches and methods to solve problems.
- Understand that a problem may have several equally valid approaches and expressions of the solution.
- Experience meaningful mathematics i.e. that they see a need for what they are studying.
- Gain enthusiasm for the subject by engaging them with stimulating activities.
- Connect and review the concepts that we have studied already.
- Experience that numbers can be written in many equivalent ways and build on this understanding to be able to write algebraic expressions in expanded, simplified and factorised forms.
- Apply substitution to evaluate expressions.
- Be introduced to some key definitions and phrases because it is very important that students understand the maths terms that are used in an Algebra class.

4. Learning Outcomes:

As a result of studying this topic students will be able to:

- Use variables to generate equivalent generalised algebraic expressions in expanded, simplified and factorised form.
- Add like terms.
- Understand and apply substitution to evaluate expressions.

5. Background and Rationale

This lesson was designed to:

- Meet syllabus needs as students and teachers engage with the Maths syllabus in a more hands on interactive way that might increase student's enthusiasm and understanding.
- Approach algebra in an active, practical and visual way to engage all learners in a mixed ability class.
- Engage students with a problem to be solved and foster independent algebraic reasoning rather than beginning with a procedural approach.
- Overcome conceptual difficulties involved in adding like terms and evaluation by substitution by promoting a "value of" understanding of the variable.
- Build on students understanding of number to introduce equivalent forms of algebraic expressions.
- Support mathematical literacy through the highlighting key terms.

6. Research

- Several Junior Cycle Maths textbooks.
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- NCCA (2010). *Common Introductory Course for Junior Cycle Mathematics*. [ONLINE] Available at:http://www.ncca.ie/en/Curriculum_and_Assessment/Post-Primary_Education/Project_Maths/Project_Maths_syllabuses/Revised_Common_Introductory_Course_Feb_10.pdf. [Accessed 01 April 16].
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- Boaler, J. (2009). *The elephant in the classroom: Helping children to learn and love maths.* London: Souvenir Press.
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- Ontario Ministry of Education (2013). *Paying Attention to Algebraic Reasoning.* [ONLINE] Available at: https://www.edu.gov.on.ca/eng/literacynumeracy/PayingAttentiontoAlgebra.pdf. [Accessed 01 April 16].

7. About the Unit and the Lesson

The content of this lesson relates to *Topic 4.6 Expressions* from *Strand 4: Algebra* in the current Junior Certificate Syllabus. This syllabus (p. 26-27, as below) describes the content knowledge and learning outcomes required of our students in this area.

In the course of studying this strand the learner will

- make use of letter symbols for numeric quantities
- emphasise relationship-based algebra
- connect graphical and symbolic representations of algebraic concepts
- use real life problems as vehicles to motivate the use of algebra and algebraic thinking

Topic	Description of topic Students learn about	Learning outcomes Students should be able to
4.3 Finding formulae	Ways to express a general relationship arising from a pattern or context.	– find the underlying formula written in words from which the data is derived (linear relations) **– find the underlying formula algebraically from which the data is derived (linear, quadratic relations)**
4.6 Expressions	Using letters to represent quantities that are variable. Arithmetic operations on expressions; applications to real life contexts. Transformational activities: collecting like terms, simplifying expressions, substituting, expanding and factoring.	– evaluate expressions of the form • $ax + by$ • $a(x + y)$

Junior Cycle Syllabus, Strand 4

But critically alongside knowledge content the syllabus also emphasises the importance of developing in our students "synthesis and problem solving skills" requiring students to "explain" their thinking, "communicate" effectively, "apply knowledge and skills", "analyse" information in context and "devise" techniques (p.29, see below).

Students should learn about	Students should be able to
4.8 Synthesis and problem-solving skills	– explore patterns and formulate conjectures – explain findings – justify conclusions – communicate mathematics verbally and in written form – apply their knowledge and skills to solve problems in familiar and unfamiliar contexts – analyse information presented verbally and translate it into mathematical form – devise, select and use appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions.

The Common Introductory Course describes the topics and content of the Junior Certificate Syllabus that should be covered by all students in first year. Though this course clearly outlines that "repeating patterns" should be used to promote algebraic reasoning we understand that it does not exclude also using other approaches such as the one used in this research lesson. The general learning outcomes for Strand 4 (as below) allows that teachers may "extend the learning sub-topics or explore the ones listed to a greater depth" requires that "students examine relations derived from some kind of context – familiar, everyday situations, imaginary contexts."

Strand 4: 4.1 Generating arithmetic expressions from repeating patterns Students examine patterns and the rules that govern them and so construct an understanding of a relationship as that which involves a set of inputs, a set of outputs and a correspondence from each input to each output.	– use tables to represent a repeating-pattern situation – generalise and explain patterns and relationships in words and numbers – write arithmetic expressions for particular terms in a sequence – use simple graphs as a tool for analysing relations – develop and use their own mathematical strategies and ideas and consider those of others – present and interpret solutions, explaining and justifying methods, inferences and reasoning
4.2 Representing situations with tables diagrams and graphs Students examine relations derived from some kind of context – familiar, everyday situations, imaginary contexts or arrangements of tiles or blocks.They look at various patterns and make predictions about what comes next.	–

8. Flow of the Unit:

This lesson was designed to introduce Algebra to first years and be the first in a series of *Algebra: Strand 4* lessons taught over (perhaps) a three-week period. Subsequent lessons are required to deepen understanding and develop mathematical skills, such as:

- Generalising a context using mathematical expressions (particularly relating to patterns).
- The idea that the same variable can have different values.
- Writing algebraic expressions in equivalent forms by: adding like terms, expanding brackets and simple factorisation.
- Evaluating through substitution.
- Solving simple equations.
- Graphing linear functions

9. Flow of the Lesson

Introduction	
Teaching Activity	**Points of Consideration**
Welcome the students and take roll call. Materials of 'wipe and write' boards and markers were distributed. Expectations re independent work and board-work are explained to students.	Arrange seating to facilitate independent work.

Task 1 – Numeric Expressions	
Teaching Activity	**Points of Consideration**
Posing Task 1 Students will be shown a video clip of a snooker player potting three red and three black balls and they will be asked to work out, in at least three different ways, how much the player scored.	It is expected that students will seek to clarify the value of the red and black balls. On the board the teacher writes up "1 is the value of the red ball and 7 is the value of the black ball." Teacher circulates the room encouraging and prompting students who get stuck, or students who finish early. Ask, "is there another method?" Don't be too helpful! Allow 5-7 minutes.
Anticipated Student Responses 1. Simple expanded approach 1+7+1+7+1+7 = 24 2. Group the same values 1+1+1 +7+7+7 = 3(1) + 3(7) = 3+21 = 24 Factorised approach (1+7)+(1+7)+(1+7) = 3(1+7) = 3(8) = 24	It is not expected that every student would use all these approaches but most students should be able to complete the task using at least one method and students should be able to explain their thinking. Note all students' methods and prepare selected students to be ready to come to the board. Weaker students might be selected during this easier task.

Comparing and Discussing The teacher asks the first student to come to the board to write out and explain his (expanded) "adding the numbers one at a time" solution. The teacher asks the class if this is the answer they got and the method they used. The teacher asks a second student to come to the board and to present his (simplified) "bringing the same numbers together" method to finding the player's score. The teacher asks a third student to come to the board and show the class his (factorised) "groups of the same value" solution. The teacher asks the class: "Which method do you think is the best and why?" "What can we learn from these different solutions?" The teacher introduces the terms "expanded", "simplified" and "factorised".	Remember to get the students to sign their work at the board. Do students offer different ways of finding the solution? Are students comfortable explaining their thinking? It is important that students are given time to explain their approach to the problem. Which methods have most students used? Which methods do they think are more efficient? Do students express a preference for a particular method and why? Do students recognise that there may be many equivalent ways to express a group of numbers?

Task 2 – Word Expressions	
Teaching Activity	**Points of Consideration**
Posing the Task Students will be shown a second video clip of a "snöoker" player potting three red and three purple balls. After watching they will be asked to work out the score. The teacher will explain that this is snöoker, which they play in Belgium, and we'll have to look up the values of the red and purple balls later but at the moment we don't know their values. But in the meantime, "describe in as much detail as you can, in words, three different ways that you could work out how much the player scored."	Students will want to know "how many points is the purple ball worth?" The success of this task requires that students build on methods used during the first task. The clarity of previous board-work is an essential element.
Anticipated Student Responses 1. Expanded approach - Adding up the values one at a time. "Add the value of the red ball, to the value of the purple ball, to the value of the red ball, to the value of the purple ball, to the value of the red ball, to the value of the purple ball." 2. Simplified approach: "Add three times the value of the red ball to three times the value of the purple ball." 3. Factorised approach: "Add the value of the red and purple balls and multiply your answer by three."	Teacher circulates the room encouraging students to verbalise as many methods as they can. Allow 7-10 minutes. This task may seem long-winded and time consuming but it: allows students to communicate their algebraic thinking and lays the groundwork for seeing the usefulness of the variable in context.

Comparing and Discussing The teacher asks three different students to present to the class expanded, simplified and factorised approaches to the problem. To save time the teacher may have already written out the text of the methods on paper that can be stuck up and signed by the student who has used a similar method. The class may be questioned about the approaches: Which did they use? Which do they think is easiest? Which is most efficient?	Do students offer different ways of finding the solution? Is there a dominant method being used? Has the dominant approach changed since task 1? Are students comfortable explaining their thinking? Do students consider any method(s) to be better than others and why? Are students using the terminology from task 1 "expanded", "simplified" and factorised?"

Task 3 – Algebraic Expressions	
Teaching Activity	**Points of Consideration**
Posing the Task Students will be asked: "Writing out these methods in words is slow and inefficient. Can you think of a shorter method that mathematicians might use to work out this snöoker score? Write down your ideas." Have students discuss their ideas leading them to agree: "Let R be the value of the red ball and P be the value of the purple ball." This is displayed on the board. Explain the term variable. Now ask the students if they could "use the letters R and P to write out the score in the snöoker match?"	Allow 5 minutes. We require students to come up with the idea that symbols or letters could be used to represent the "value of" the balls. Allow 5-7 minutes.
Anticipated Student Responses 1. Expanded method: R + P + R + P + R + P = score 2. Simplified method: 3(R) + 3(P) = score 3. Factorised method: 3(R + P) = score	Teacher circulates the room encouraging students and noting solutions.

Comparing and Discussing The teacher asks three different students to present to the class expanded, simplified and factorised approaches to the problem. The class may be questioned about the approaches: Which did they use? Which do they think is easiest? Which is most efficient? How does these compare with previous solutions on the board?	Do students offer different ways of finding the solution? Are students comfortable explaining their thinking? Have all students remained on task? What misconceptions are evident? Are students building on learning gained in previous tasks?

Task 4 – Evaluating Expressions	
Teaching Activity	**Points of Consideration**
Posing the Task The teacher asks the students to calculate how many points the snooker player scores if the value of the red ball is 10 points and the value of the purple ball is 20 points. Can you work out the score in three different ways? The teacher asks students to explain how they would calculate the score.	Can students correctly use substitution to solve the problem without being shown any example by the teacher? Allow 5 minutes
Anticipated Student Responses 1. Expanded method: 10 + 20 + 10 + 20 + 10 + 20 = 90 2. Simplified method: 3(10) + 3(20) = 90 3. Factorised method: 3(10 + 20) = 90	Teacher circulates the room encouraging students and noting methods being used.
Comparing and Discussing The teacher asks particular students to describe how they got their answer. The teacher prompts the students to come up with the term "substitution".	Have students correctly evaluated the expression? Do students offer different ways of evaluating the expression? Do they find this task easy? Are students comfortable explaining their thinking? Has this changed during the lesson? It is important that students are given time to explain their approach to the problem.

Conclusion	
Teaching Activity	**Points of Consideration**
The teacher asks the students to write a reflection on "what did I learn today?" and "what would be a good name for this lesson?" The teacher presents students with homework and explains what they are expected to do.	Do students recognise that different approaches to solving a problem are acceptable and that there is more than one way to solve a problem? What misconceptions are evident?

10. Homework task

In a game of snöoker a player potted the following balls.
Can you express his score in three different ways?

If we knew that the value of the yellow ball was 5 and the value of the green ball was 4 can you work out the score in three different ways?

11. Evaluation

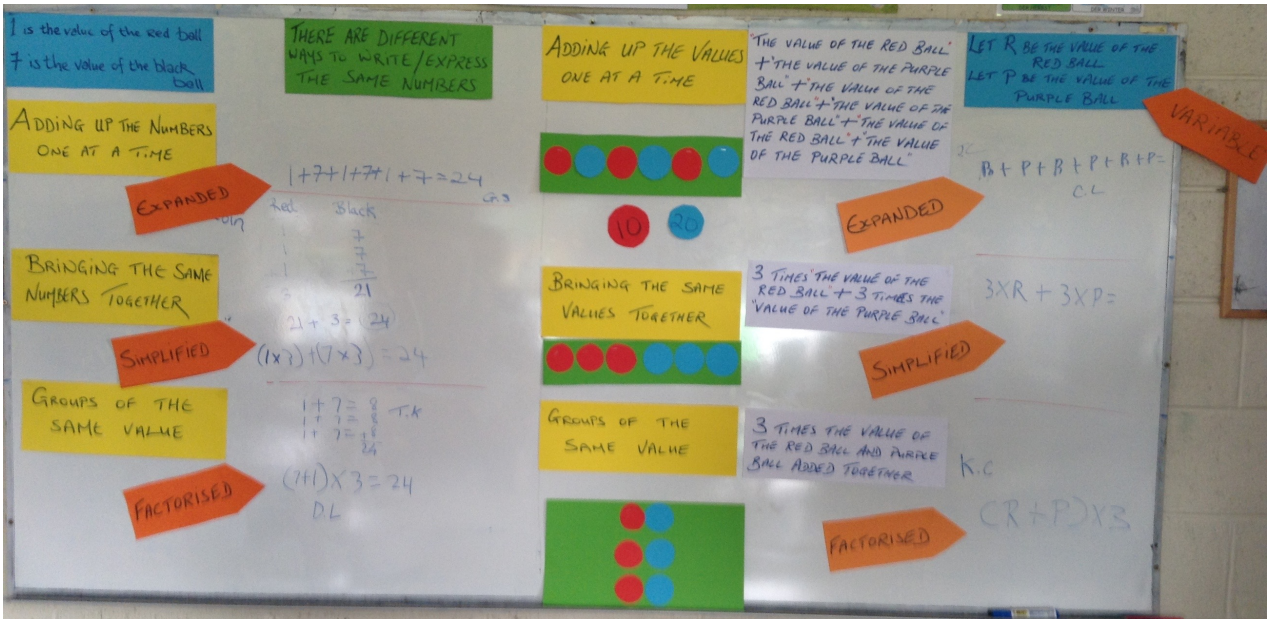
There were five observers in the lesson along with the teacher. The teacher interacted with students during activities and used questioning strategies to find out about students' thinking. The students also used *show-me boards*, which helped identify which students were able to complete the various activities and which students were having difficulties. Students also had opportunities to explain their ideas in front of the class and present them at the board. The students were also required to write a post lesson reflection on their learning.

The peer observation was planned prior to the class. One teacher moved through the room recording the lesson using Lesson Note App and took multiple photographs of the students' work. Four other teachers, at the side of the room, observed and noted students' interaction with the class with an emphasis on:

- How are they engaged during each task?
- How are they learning/successfully completing tasks?
- Student questions and responses.
- Quality of independent written work.
- How are they explaining when at the board?
- Evidence that the stated learning outcomes and aims of the lesson are being achieved?

Teachers discussed their observations on the students learning during the class at a post-lesson evaluation meeting, which took place one week after the lesson was taught.

12. Board Plan



13. Post-lesson reflection

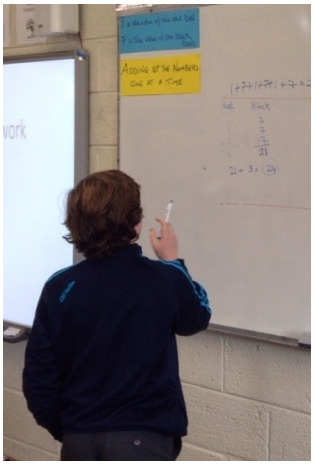
What are the major patterns and tendencies in the evidence?

The observing teachers noted that the students' level of engagement with the task was immediate and persistent throughout the lesson. The students' curiosity seemed to be stimulated by the initial slide with the word "Snöokered" while the use of the short videos to introduce the questions seemed to fully capture their attention. Starting the class well, engaging students from the beginning in a manageable interesting task that has been clearly explained was we feel an essential ingredient for the success of this problem solving lesson. "Tús maith, leath na hoibre." What we noticed was that the students really wanted to succeed at each task. As tasks became more challenging the even the weaker students still persisted in making a very good effort.

Students were fully engaged from the outset

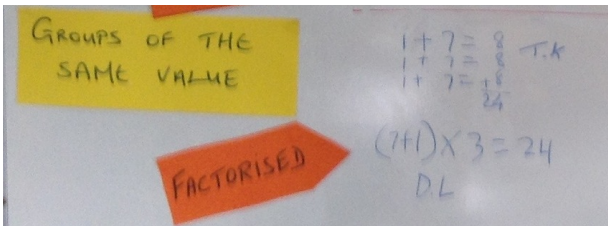
In our discussion afterwards we wondered if using snooker as a gateway into algebra would work as well with another group. The teacher was aware beforehand that some of the students played pool and snooker. It is probably important to choose the task carefully to hold the interest of the your particular class of students.

Beforehand the teachers had concerns and reservations about the process and how the students would respond. Would having six teachers present in the room watching them effect their interactions? Would students be uncomfortable and hesitant to come to the board to explain their thinking? The teacher had prepared the students by making, coming to the board and explaining solutions a normal part of the class. The benefits of this practise were evident looking hat how naturally, confidently and enthusiastically students shared their thinking in front of others.



A confident student at the board, notice the left hand in his pocket.

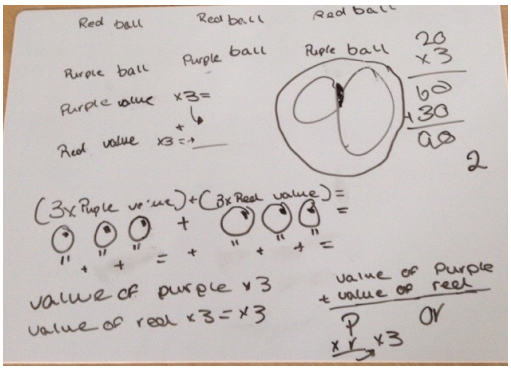
Is it realistic to expect students to come up with all the central questions, anticipated responses and required approaches? Would all of the students find it difficult to fill the time given to independent work in a productive way? The predominant feeling that was discussed in the post-lesson evaluation meeting was regarding the way students went beyond our expectations in achievement. We were surprised and amazed at how students seemed to be saying all the right things, asking the perfect questions and presenting all the solutions that were required to move the learning in the direction planned by the teachers. For instance students themselves came up with phrases such as "the value of…" and with a little prompting "substitution." They even came up with the variable themselves. Students also of their own accord used "expanded", "simplified" and the less obvious "factorised" approaches to each task. The students' response to the lesson and the extent to which they achieved the learning goals of the lesson was the real eye-opening experience for the teachers involved.



Students board work showing two "factorised" approached to Task 1

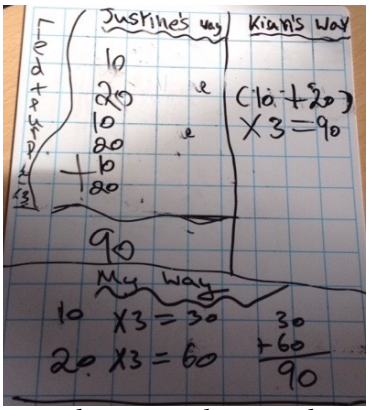
It was obvious that students were able to build on and apply previous knowledge and gained knowledge as they moved through the phases of the class. This was clear by the way students presented work and used highlighted new terminology to describe their approaches. The visual aid of the board-work was an essential element in progressing the students learning.

Giving students time, space to think and a platform to share their mathematical ideas is very interesting, beneficial and worthwhile.



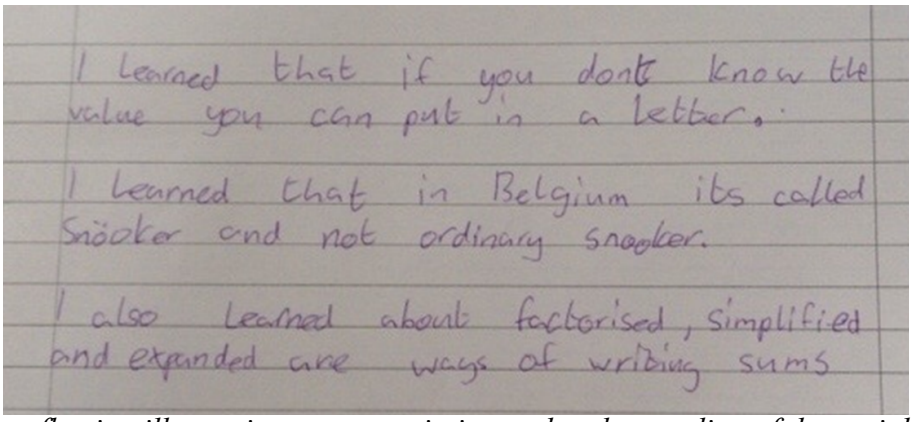
Creative algebraic reasoning

A highpoint was achieved towards the end of the class when asked to evaluate the snöoker score given that 10 is the value of the red ball and 20 is the value of the purple ball it was incredible to hear a very weak student explain "It's 90 because if you add 10 and 20 and multiply your answer by 3 you get 90." The ease at which students could evaluate this expression in multiple ways without any formal examples just illustrates the potential waiting to be untapped in all of our students.



Student's work showing how to evaluate in three different ways

The student reflections were useful. Some misconceptions were identified in the student reflections highlighting their potential as a valuable formative assessment tool. These misconceptions would be addressed in subsequent lessons. If time was short this itself would be a useful homework activity.



Student reflection illustrating an appreciation and understanding of the variable

The observing teachers were satisfied that the learning aims of the class seemed to be comprehensively met by the majority of the students and it was no small bonus that the students themselves expressed how much they both enjoyed and learnt from the lesson.

What are the implications for teaching in your field?

It is interesting that going back to the embryonic stages of the new Project Maths course Kelly et al (2012), summarised in Cosgrove et al (2012), reporting on teachers' experiences in the Pilot Schools noted that:

> *Teachers struggled with the new role of facilitating students as active learners, and reported that it was common to revert to the traditional examination preparation techniques as the State Examinations approached... They also underlined their need for appropriate support and resources to allow them to continue to develop in this new role. Second, some teachers commented positively on the changes in their teaching and collaboration between teachers was viewed as valuable.*

What has changed? The importance and potential of the *Reflection on Practice* initiative is evident from the realisation that teachers are still engaged in this same "struggle". Teachers really do want to facilitate active learning in their classrooms but the constraints persist and the collaborated support isn't always facilitated. Building collegial collaboration through *Reflections on Practice* while working together to design, teach and reflect on a structured problem-solving lesson such as "Introducing Algebra with Snöoker" is potentially a real game-changer for Irish maths classrooms.