Teaching the Distance Formula through Problem Solving

SECOND YEARS

Lesson Plan for [Second Year]
For the lesson on [29/02/2016]
At [St. Mary's Academy], [Ms. Cronin's] class
Teacher: [Ms. Cronin]
Lesson plan developed by: [C. Cronin & A. Geoghegan]

1. **Title of the Lesson:** Teaching the distance formula through problem-solving and mapping

2. **Brief description of the lesson:** To help students make the connection between Pythagoras' Theorem and the distance formula for co-ordinate geometry.

3. **Aims of the Lesson:**

Long-range/thematic goals

I'd like my students to recognise the importance of their own thinking in maths.
I'd like my students to appreciate that there are many ways to view the same problem.
I'd like my students to appreciate that mathematics can be used to solve real world problems.
I'd like my students to appreciate that mathematics can be used to communicate thinking effectively.
I'd like to encourage my students to become independent learners.
I'd like my students to be collaborative learners.
I'd like my students to become more creative when devising approaches and methods to solve problems.
I'd like my students to experience meaningful mathematics i.e. that they see a need for what they are studying.
I'd like to build my students' enthusiasm for the subject by engaging them with stimulating activities.

Short-term goals (content goals specific to the lesson)

I'd like my students to gain an insight into the language of geometry.
I'd like my students to recognise the connection between roads and points of intersection.
I'd like my students to understand how to use co-ordinates to mark points of reference on a map.
I'd like my students to recognise the usefulness of Pythagoras' Theorem and co-ordinate geometry to solve a real-life problem.
I'd like my students to recognise the connection between Pythagoras' Theorem and distance formula.

4. **Learning Outcomes:**

As a result of studying this topic students will be able to:

- recognise the co-ordinate plane on a street map
- discuss and determine what is the most direct route between two points
- probe the meaning of the most direct route
- forge a link between the shortest route, a right-angled triangle and Pythagoras' Theorem

5. **Background and Rationale**

In a typical lesson to measure the distance on a co-ordinate plane the lesson often starts with a teacher presenting the distance formula and showing students how to 'plug in the numbers'.

However, this limits the thought process of the student in terms of where the formula comes from and the link between trigonometry and co-ordinate geometry. Thus, it is not quite clear whether or not students have an understanding of the distance formula. Students are not always encouraged to develop their ability to investigate a relationship.

If the formula is introduced at the beginning of the lesson, students are denied the opportunity to develop a mechanism to solve a problem in a way that leads to a formula.

Using a map, students should discover that the shortest distance between two points is a straight line. On a co-ordinate plane this straight line can be constructed as the hypotenuse of a right-angled triangle. Students should be able to draw out the co-ordinate geometry formula through their prior knowledge of Pythagoras' Theorem.

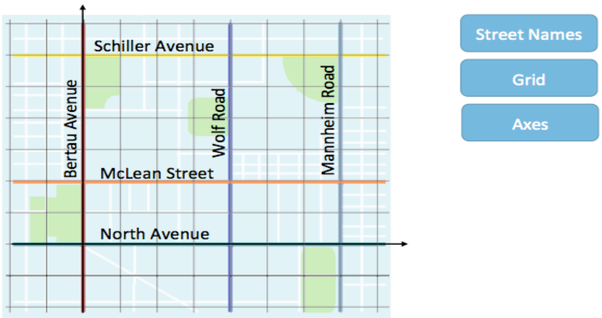
6. **Research**

- Junior Certificate Syllabus
- Second Year Teacher Handbook Strand 2
- Active Maths 1 textbook.

Initial idea for lesson came from Dara O'Briain's 'School of Hard Sums' and further research on cnx.org and iblog.dearbornschools.org

7. About the Unit and the Lesson

Help students to recognise the link between Pythagoras' Theorem, distance and the co-ordinate geometry formula for distance. Distibute a street map and ask students to mark a variety of locations on the map. Using these locations pose some questions about distance. Students' investigations shoud lead them to conclusions about the most direct route, the shortest route and their relationship to a right-angled triangle. The real-life problem involving a street map will make it easier for some students to capture the relationship between distance and the co-ordinate geometry formula more visually.



Learning Outcomes from Syllabus

Topic 2.2 Co-ordinate geometry (pg.20)
The syllabus outlines that students should be able to explore the properties of points, lines and line segments including distance.

Topic 2.3 Trigonometry (pg.20)
The syllabus outlines that students should be able to apply the theorem of Pythagoras to solve right-angled triangle problems of a simple nature involving heights and distances.

The second year Teacher Handbook suggests that Pythagoras' Theorem should precede co-ordinate geometry and our lesson was designed as a bridge between these two topics (pg. 19 & 20).

8. Flow of the Unit:

Lesson		**# of lesson periods**
	Title: Geometry	
1	• Pythagoras' Theorem (Lesson Idea 2.11 of Second Year Teacher Handbook)	1 x 60min.
2	• Reinforcing first year prior knowledge of plotting co-ordinates and introducing the distance formula through problem solving. (Lesson Idea 2.12 and 2.15 of Second Year Teacher Handbook)	2 x 60min. **(#1 = research lesson)**
3	• Slope of a line as a ratio of rise to run • How to generalise from this concept to the slope of a line formula (Lesson Idea 2.14 of Second Year Teacher Handbook	1 x 60 min.
4	• Midpoint of a line segment • How to generalise from this concept to the midpoint formula	1 x 60 min.

9. Flow of the Lesson

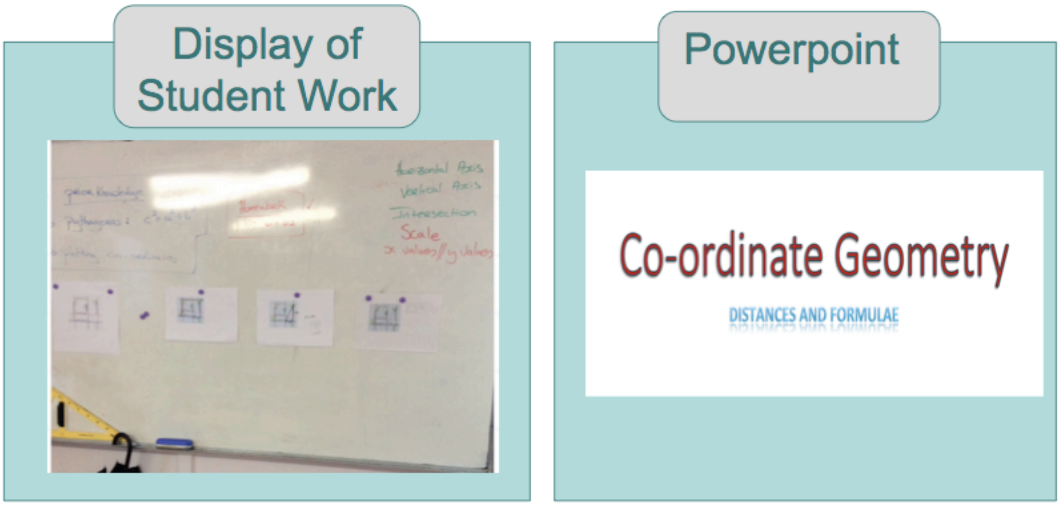
Teaching Activity	Points of Consideration
1. Introduction Outlining prior knowledge of Pythagoras' Theorem. Consolidation of prior knowledge of the co-ordinate plane.	Distribution of white boards, markers and dusters for students to show their work. Question students on the meaning of the words axis, co-ordinate, vertical, horizontal, intersection and origin.
2. Posing the Task Distribute street maps among students. Ask them to mark the location of various landmarks on the map. Pose the question of how to get from the scene of an accident to the hospital, both of which will be marked on the map by the students. Once students have decided on the most direct route, they must come up with a way of measuring the distance.	Circulate the room observing student work. Ask probing questions about intersecting roads and the concept of the most direct route. The teacher should be looking for evidence of axes and/ or grids being drawn on the maps. The teacher should also look for evidence of students including a right-angled triangle in their work, or some reference to Pythagoras' Theorem. Depending on the progression of students/ groups in solving the problem, transparencies of a grid can be handed out at the discretion of the teacher.
3. Anticipated Student Responses Some students may struggle with the concept of the most direct route. Students may struggle with measurement and scale.	Some students may ask whether the means of transport is limited to road. The teacher should encourage students to consider all possibilities in terms of route and transport. Some students may use their ruler for measure. The teacher should encourage students to consider a more general unit of measure, using x and y co-ordinates.
4. Comparing and Discussing Use magnets to display students' maps midway through the tasks to compare routes chosen and to compare their methods for measuring the distance.	Have students explain their reasoning.
5. Summing up Towards the end of the class the teacher should help students formulate a singular method for calculating the distance between two points.	The teacher can use examples of student work as a reference for what might be considered best practice.

Evaluation

Observing teacher(s) could:

- take note of teacher-student interaction
- take note of student discussion and their approach to solving the problems. Are they attempting to use co-ordinate geometry? Are they attepmting to use Pythagoras' Theorem?
- photograph student work
- look for evidence of student understanding/ difficulties
- use Padlet for real-time feedback from observing teacher and from students

10. Board Plan



11. Post-lesson reflection

Obserations

- Students were engaged in the problem that was posed.
- The range of ability in the class was evident from student discussion and the solutions that they offered. Some students found the tasks more challenging than others.
- Student feedback would suggest that they enjoyed working in groups and were invested in the problem as it seemed relevant to real life. Teacher observation corroborates this.
- It took some students time to recognise that the shortest distance was the hypotenuse of the triangle.
- Some students jumped to the usefulness of co-ordinates on street map to problem solve quickly.
- Not all students made the link between the street map and the co-ordinate plane.
- All students used Pythagoras' Theorem to support their problem solving.

Recommendations

- Underestimated the amount of time it would take to complete all four tasks and make connection between problem and distance formula.
- Recommend splitting lesson over 2 classes. Use second lesson to further explore the formula.