# Using Linear Patterns to Develop Algebraic Reasoning 

FIRST YEARS

1. Title of the Lesson: Using linear patterns to develop algebraic reasoning.
2. Brief description of the lesson: To introduce students to algebra through guided discovery of linear patterns. Students will be able to describe in their own words a growing linear pattern.

## 3. Aims of the Lesson:

## Long-range/thematic goals

I'd like my students to appreciate that there are many ways to view the same problem.
I'd like to build my students' enthusiasm for the subject by engaging them with stimulating activities.
I'd like my students to appreciate the value of working witht their peers to solve problems.

## Short-term goals (content goals specific to the lesson)

I'd like my students to give students an appreciation of linear patterns.
I'd like my students to develop rules to describe the relationship between two variables.
I'd like my students to understand the power of such rules.

## 4. Learning Outcomes:

As a result of studying this topic students will be able to:

- Work with their peers to share their ideas and generate with their own understanding of the pattern.
- Analyse a growing linear pattern, create the next two terms and verbalise the general rule of the pattern.


## 5. Background and Rationale

The study of patterns is a key part of algebraic thinking. It is important that students are able to recognise and analyse patterns and make generalisations about them. Patterns can be visual or spatial and can be created from shapes, colours or numbers. They can be repeating (B B A B B A B B A) or they can be growing ( $1,1,2,3,5,8 \ldots)$. Students will have met patterns in their every day life from as early as their primitive childhood years. It may have been as students, or as children matching the shape/pattern of the block to the cut out or continuing a simple pattern of circle, square, triangle, circle, square etc.
In the past there has been no link made between patterns and algebra and how these two sections of maths can be interlinked to boost student knowledge and understanding. A study conducted by National Council of Teacher of Mathematics (2008) found three main reasons to support the idea that patterns were the foundations on which to build algebraic reasoning. The first reason was that patterns were presented visually and could assist spatial reasoning, they could "be used as an informal introduction to the concept of a variable, and three they can be used to generate equivalent expressions".

## 6. Research

Patterns are a crucial foundation for the topic of algebra as working with patterns allows the students to develop logical reasoning skills, make conjectures, and test their ideas about them (Roebuck, 2005). In the INTO Publication (2005, p.30, Table 9) entitled "Maths in the Primary Schools" it was reported by teachers that patterns posed more of a problem to primary students than probability or problem solving. As a result students may have negative attitudes towards this topic or may not be as proficient in the area as expected. Students have met algebra from junior infants to sixth class in primary school. They have progressed from studying simple patterns; to rules and properties, equations and variables in algebra. "The child should be enabled to explore the concept of a variable in the context of simple patterns." (Primary school curriculum, 1999). In addition they will need to understand natural numbers, integers and fractions. They should also be able to manipulate fractions and recognise patterns. Students have already studied topics such as sets, natural numbers, the fundamental principle of counting, integers, rational numbers, decimals, percentages and probability in their study at the junior cycle level. The topics the students have previously met in both primary and secondary school will aid their understanding of algebra and patterns. Students will be able to apply previous knowledge learnt, in this topic.

In the design of the lesson, the teachers brainstormed possible solutions of the pattern and how the students could view and/or describe the pattern. The teachers also prepared an extra worksheet to guide those who struggled to build/describe the pattern.

## 7. About the Unit and the Lesson

The Syllabus requires students to 'explore patterns and formulate conjectures' (NCCA, 2015). This initial introduction to patterns aims to lay the groundwork for 'progression to symbolic representation, equations and formulae.' (NCCA, 2015)

The syllabus requires students to use a table to represtent a repeating pattern. (NCCA, 2015, p 27). This could be an option to aid students when describing the growing pattern. The syllabus also aims to enable students to generalise and explain patterns and relationships (NCCA, 2015, p 27). This was the main goal of this lesson and was achieved through the use of using mathcsticks to develop and grow the pattern as well working as part of a pair and then a larger group to share and brainstorm ideas. The students also worked within a marketplace activity where one student was the teacher and the other student was a student. The teacher remained in their seats while the other student moved around the classroom speaking to and sharing ideas with other teachers. The goal of the following lessons will aim to achieve the final outcome set by the NCCA, that is to write an arithmetic expression for the general term in the pattern. (NCCA, 2015, p 27).

## 8. Flow of the Unit:

Handbooks would be useful here

|  |  | periods |
| :---: | :---: | :---: |
|  | Title Patterns | $4 \times 30 \mathrm{~min}$. |
| 1 | $\bullet$ Introduction to Patterns: Linear and Quadratic Patterns | $3 \times 30 \mathrm{~min}$. |
| 2 | $\bullet$ Algebraic Expressions | $3 \times 30 \mathrm{~min}$. <br> $(\# 1=$ research <br> lesson $)$ |
| 3 | $\bullet$ Simple Linear Equations |  |

## 9. Flow of the Lesson

| Teaching Activity |
| :--- |
| 1. Introduction |
| The students will be introduced to the learning |
| outcomes of the lesson. |
| Introduce new terms to the students such as |
| pattern, term and limb. |

## 2. Posing the Task

Introduction of ted, when he was born, on his first, second and third birthday. The students are given a box of matchsticks and in pairs they must make ted on his first, second and third birthday.
The students then make the fourth and fifth term of the pattern.
The teacher then questions the students about how many matchsticks they would need to make ted on his $20^{\text {th }}$ and $100^{\text {th }}$ birthday.
The students are encouraged to generate a variety of ideas to generate the number of matchsticks for ted's $20^{\text {th }}$ and $100^{\text {th }}$ birthday.

## 3. Anticipated Student Responses

The students may make a table to observe the number of matchsticks needed for each birthday. The students may describe how each limb has the same number of matchsticks as ted's birthday. The head and the body are a constant. The students may describe that it is four times ted's birthday and add 5 for the head and body. The students may describe it as repeated addition of each limb (the birthday) and add five for the head and body.

## 4. Comparing and Discussing

The students will participate in a 'Stray and Stay' activity where they number themselves 1 and 2. Number 1's are the teacher and remain in

Points of Consideration
Students understand the terminology used in learning outcome statement.
"Be able to describe the growth pattern in your own words and use this understanding to identify how many matchsticks are needed for any term in the pattern."

How do we know if students understand the task?
The teacher moves around the room and monitors the group's progress. The teacher questions the students on what they are doing to create each term.

The teacher observes the group's progress and questions students on what they are doing to make the fourth and fifth term.
The teacher will ask the students to use their words to describe how they found the $20^{\text {th }}$ and the $100^{\text {th }}$ term. They are probed to be able to describe their rule in general for any of ted's birthdays.
The teacher will have an extra worksheet for those who struggled with the pattern. This will guide those students through a method, which should enable them to describe what happens to ted each birthday.
The students should see a need to come up with a new way of calculating the number of matchsticks needed instead of using the matchsticks to depict ted.
Those who finish early will be prompted to investigate different ways of describing how the pattern grows. They will also be encouraged to verbalize the rule so that it could work in general for any term number.

That there are a variety of ways to generate the same solution. Students will engage in the marketplace activity where they will be able

| their seats. Number 2's are the students and <br> move from teacher to teacher gathering the <br> different ideas from each group. At the end of <br> this activity the students share their ideas at each <br> table. The students put their group's idea onto <br> paper and then share their ideas on the board in <br> a designated section. | to appreciate and develop an understanding <br> for other students thinking. The student then <br> returns to own partnership and group to share <br> the different methods they uncovered during <br> the discussion. |
| :--- | :--- |
| 5. Summing up <br> The students work as part of group and put their <br> best ideas into writing. They then come to the <br> board and share this idea with the class. The <br> teacher reads through these ideas and questions <br> the group and class about the idea. | The teacher will allow groups to share ideas <br> that they generate and that they observed <br> during the 'stray and stay' activity. The <br> teacher will make the students aware that <br> there is several different correct ways of <br> viewing the same pattern. |

## Evaluation

- What is your plan for observing students?
- 3 observors and the teacher were present during the class.
- Discuss logistical issues such as who will observe, what will be observed, how to record data, etc.
- Cathy Craddock (Developer of the lesson), Stephen Gammell (Project Maths Team) and Sinead Kenny (Maths Teacher) will observe the lesson. Stephen and Cathy were designated to record the data through picture taking, recording of the different student solutions and through the use of questioning the students.
- What observational strategies will you use (e.g., notes related to lesson plan, questions they ask,)?
- What remains the same for each birthday?
- What changes for each birthday?
- Can you explain what's happening each year in your own words?
- Explain the growth pattern.
- Is there a relationship between each birthday and the number of matchsticks?
- How many matchsticks do you need for his $20^{\text {th }}$ Birthday?
- How many matchsticks do you need for his $100^{\text {th }}$ Birthday?
- What did you do to find the number of matchsticks?
- How could being able to describe the pattern in general be useful to you?
- Why do we need to be able to describe the pattern?
- What types of student thinking and behaviour will observers focus on?
- The variety of solutions the students will generate in their groups.
- The conversation the students will engage in with their partners/groups to generate ideas to describe the pattern.
- The enjoyment and engagement students demonstrate throughout the lesson.
- What additional kinds of evidence will be collected (e.g., student work and performance related to the learning goal)?
- Samples of students work will be collected.
- Images of students during the lesson.
- Student feedback (student questionnaire)


## 10. Board Plan



## 11. Post-lesson reflection

- What are the major patterns and tendencies in the evidence? Discuss
- Students were extremely engaged throughout the lesson. They enjoyed the physical making of the pattern along with the social interaction which they were able to engage in throughout the lesson.
- What are the key observations or representative examples of student learning and thinking?
- All students were able to work with their partners to verbalise the rule of the pattern. Students who found this difficult were given the opportunity to be either a teacher or a student and engage in a verbal interaction with a new student to discuss the pattern and their own solutions.
- What does the evidence suggest about student thinking such as their misconceptions, difficulties, confusion, insights, surprising ideas, etc.?
- Students found it difficult to relate the patterns to their own life and to find use of such a skill.
- Students found it difficult to move into representing the general term using an algebraic expression.
- Some students found this pattern quiet easy and move through the task quickly.
- In what ways did students achieve or not achieve the learning goals?
- Students achieved the learning outcomes by being able to describe what was happening to ted for each birthday. They used a variety of methods to enable them to calculate the number of matchsticks needed to make ted on his $20^{\text {th }}$ and $100^{\text {th }}$ birthday.
- Based on your analysis, how would you change or revise the lesson?
- I would encourage students to write their general rule out and instead of using full words could they use a letter to represent a word.
- What are the implications for teaching in your field?
- Students work better in an environment where social interaction and innovative activities are used.

