Reflections on Practice

Multiple Approaches to Problem Solving in Geometry

Year Group: 3rd Year Higher Level

For the lesson on 25th February 2016
At Carndonagh Community School, Carndonagh, Co.Donegal
Teacher: Siobhan Coll
Lesson plan developed by: Sharon Coyle & Siobhan Coll

Index of Appendices:

Title of the Lesson: Multiple Approaches to Problem Solving in Geometry

Discovering Geometry and Trigonometry

Brief description of the lesson

Developing student problem solving by using a geometrical problem which has multiple solutions. Solutions will include angle theory, algebraic expressions and visualising geometric shapes. This lesson is a revision lesson with third year higher level students.

Aims of the Lesson:

- I'd like to foster my students to become independent learners.
- I'd like my students to become more creative when devising approaches and methods to solve problems.
- I'd like to emphasise to students that a problem can have several equally valid solutions.
- I'd like my students to connect and review previously visited concepts.

Learning Outcomes:

On completion of this lesson, students will be comfortable with the following concepts:

- Improving on geometrical literacy
- Vertically opposite angles are equal in measure
- If a transversal makes equal alternate angles on two lines then the lines are parallel (and converse)
- The angles in any triangle add up to 180 degrees
- Two lines are parallel if and only if, for any transversal, the corresponding angles are equal
- If two triangles are similar, then their sides are proportional, in order (and converse)
- Solve first degree equations in one or two variables, with coefficients elements of Z and solutions also elements of Z
- Recall the axioms and use them in the solution of problems
- Use the terms: theorem, proof, axiom, corollary, converse and implies.
- Apply the results of all theorems, converses and corollaries to solve problems

Background and Rationale

Students need to be able to recognise different angles and apply their knowledge of geometry to solve the problem. Basic algebra is required to find the values of missing angles. The angles in this problem will be unknown to students so it will be important for students to recall the fundamental principles involved in solving linear equations e.g. if <1 + 30° = 180°. Students must know that <1 is the unknown term and use properties of solving linear equations to rearrange the problem to find the value of <1.Students struggle to apply their knowledge to find missing angles when faced with a problem containing many different missing angles. They struggle to recognise the basic vertically opposite, corresponding angles etc. especially when transversals and triangles are included. It is an important skill to be able to problem solve and come up with numerous answers. Students benefit from using different methods to solve the same problem. They can compare and contrast approaches. They can also decide on their preference based on both ease of use and efficiency as a result of exposure to multiple approaches. This is a form of differentiation as students can view problems from different perspectives and it also allows for assessment for learning. For example a student who studies technical graphics may take a different approach from a student who does not solve problems visually. This type of question encourages group work and peer learning which is highly valued by all sectors in society. Students will be working in groups for the duration of the class starting with a matching exercise and then solving the problem.

Research

I posed the question to different teachers who have mathematics in their subject and asked them to discuss how they would solve the problem. This ranged from a variety of subjects. An informative discussion on which methods students might produce as solutions to the questions. This is important as a few methods were very complex and also seeing the variety of methods used;

- www.projectmaths.ie
- Reading of international online articles relating to teaching Geometry.
- Geometry question came from 2013 higher level paper 2 question 9
- Active Maths 1 & 2
- Teacher handbook for Junior Cycle
- Teaching & Learning Plan – Introduction on Synthetic Geometry

About the Unit and the Lesson

Students will understand the concepts of problem solving using a variety of techniques. Through simple examples a clear understanding will develop. As the students see the different types of angles they develop a clearer understanding of approaching a problem. Making connections with how the theory can be applied to the problem.
The numerous basic example of solving angles will build up student's confidence.
Visual learners will respond positively to the problem.
Kinaesthetic learners will enjoy working with the matching exercise and seeing the connections.

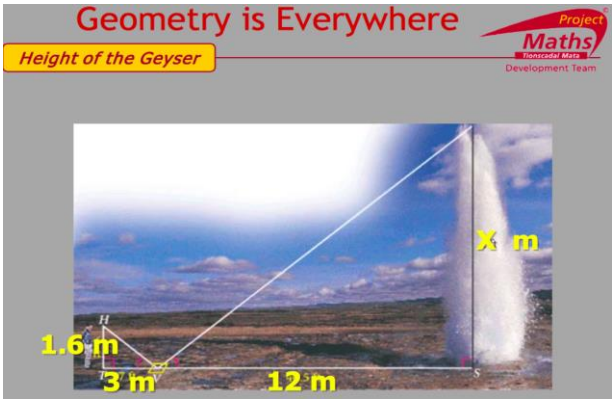
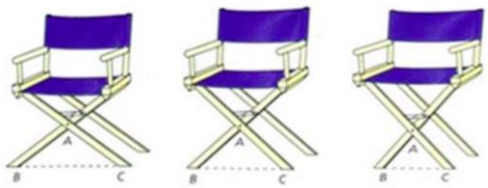
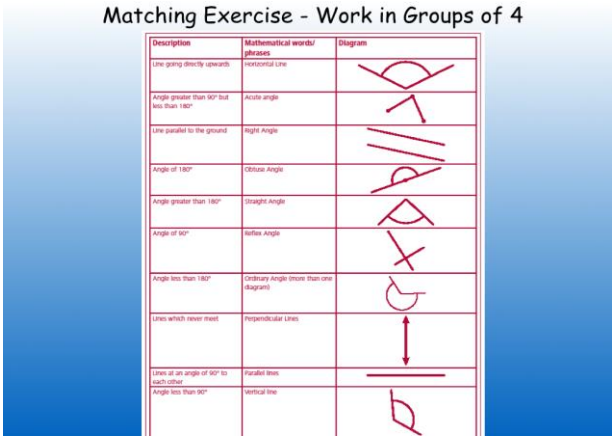
In this lesson, the focus is not just about being able to solve the question but also define each angle and understand different approaches to problem solving.

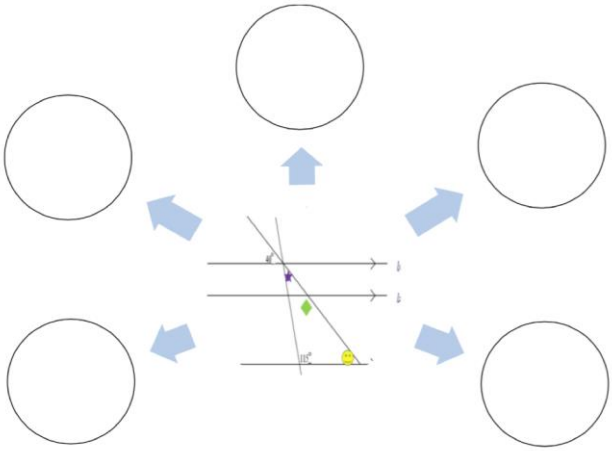
Flow of the Unit:

Teacher Handbook - Third Year pages 21 - 25 was used to devise this lesson plan

Lesson		**# of lesson periods**
1	Lines & Angle Theorems Parallel Lines	40 min
2	Triangle Theorems & Similar Triangles	40 min
3	Parallelograms & Quadrilaterals	40 min
4	**Revision class on applying concepts using various problem solving methods**	**40 min**

Flow of the Lesson:

Teaching Activity	Points of Consideration
1. Introduction 9.40 - 9.50	Aim of today's lesson: Solve the problem using a variety of methods. Review importance of geometry through visual examples i.e. the review of prior knowledge    ***Which chair is most stable? Why?*** Matching exercise to assess students' understanding of the topic 

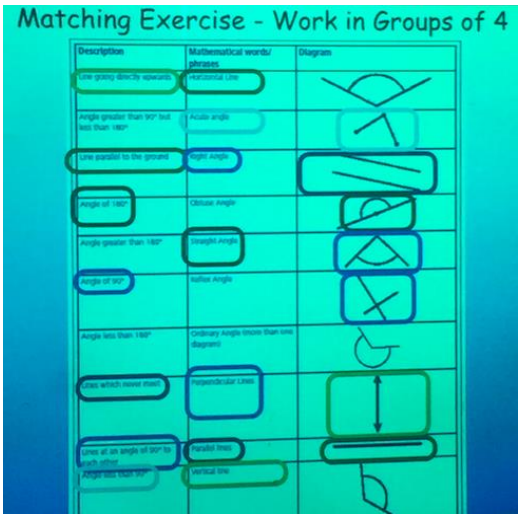
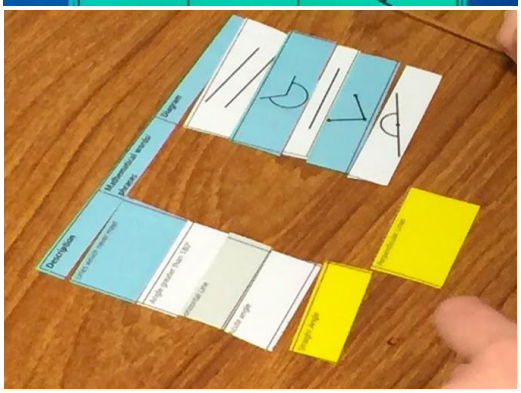
2. Posing the Task 9.50 - 10.20	Students will be recording 5 different ways of answering the problem in the circles  Students will then solve the problem using a variety of methods on the worksheet. Students will be able to clearly mark the value of the angles which they are looking for. I will ensure understanding through the use of effective questioning, group work and in built tasks within the lesson. I will give the students an opportunity to develop their own methods of solving the problem and time to feedback to the other students. I will conclude the lesson by checking understanding through the use of a peer learning. This will be carried out by students displaying the different solutions on the board.
3. Anticipated Student Responses	Expect the following terminology from the students: vertically opposite angles, corresponding angles, alternate angles, straight angle, similar triangle. The groups may find it difficult in discovering 5 solutions

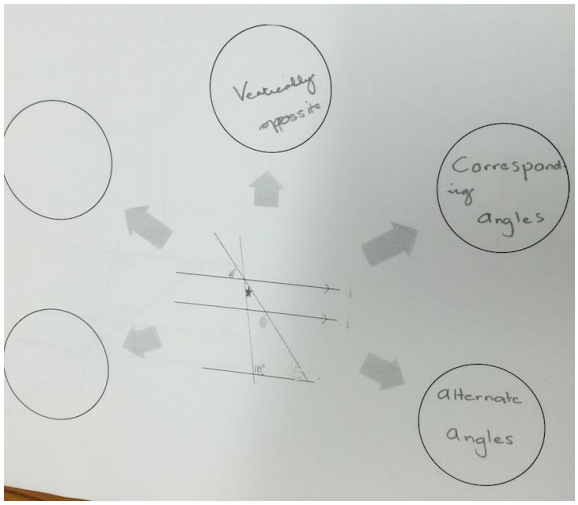
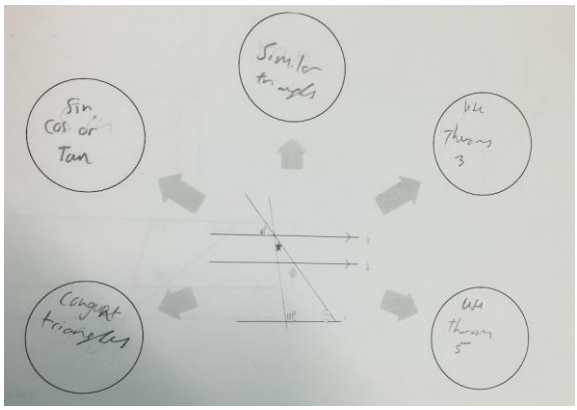
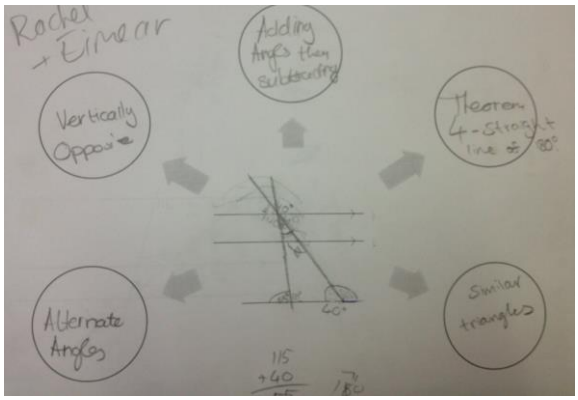
4. Comparing and Discussing	Focus placed on the variety of methods which can be used and encouraging students to explore and contribute all ideas Students will help one another in the groups to understand a different solution, this will show peer learning Students will record any methods that were previously unknown to them
5. Summing up	Students will present their finding by volunteering answers from each group. Students will explain how they arrived at their solution to the rest of the class. Emphasis on gathering the different methods present, no repetitive solutions on the board Answers will be left displayed on the board Groups will then record three things that they learned from the class on the worksheet. By doing this students work will be assessed and we will be encouraging student reflection

Evaluation

Students will be evaluated using an observation sheet and pictures / videos will be used to record their progress. The local Regional Developmental Officer, along with Ms. Sharon Coyle, will observe. They will take photograph of student's workings and of the board to show the development of the different methods. Parental consent in relation to photographs being taken has been obtained. Prior to class, teacher sent permission note home. Any interesting points noted by the students will be recorded on the observation page. Especially the unexpected outcomes generated by students. The observers will not interfere in the flow of the lesson in any way. Prior to the lesson, the observers studied the lesson plan in great detail and have planned their observation in accordance with the timing in the lesson. They have also been given the observation template to note down key observations throughout the lesson. The observers will not be helping the students or interacting with them.

Incorrect methods will be identified and tracked to see if an understanding of the problem through the lesson develops.
Student's worksheets will be collected at the end of the lesson which includes reflections on the lesson.

  	Prior knowledge was assessed and revised through the use of the matching exercise. This was excellent in refreshing students' prior knowledge for this problem. It assisted students in solving the problem. However due to the short amount of time available to revise prior knowledge, there was too many concepts covered in the exercise. Some of the more important concepts like alternate and corresponding angles were being identified but not understood. Approximately 32% were unsure of alternate angles and 26% unsure of corresponding angles. Also not enough time was spent correcting the matching exercise. A new matching exercise has been developed for this lesson focusing on the most relatable terminology

	This is an example of students work to initiate the problem solving process. It is evident to see from this example the students in the group are very aware of the key terms in relation to geometry but there is no further information given, i.e. they are aware that they can solve this question using vertically opposite angles, corresponding angles and/or alternate angles but they fail to recognise where they can be used to solve the problem. Also this group of students were limited to three methods of solving the problems, their answers were very restricted and some struggles to properly use key geometrical terms.
	This group came up 5 methods to solve the question however they failed to recognise that no right angle triangles was present and suggested the method of trigonometric ratios to solve the problem. This highlighted an important misconception. This group of students viewed the problem as a triangular question rather than an angle question.
	In comparison to other groups this group worked backwards by firstly finding the angles and then exploring the alternative methods to solve the problem.

Expected Solutions

(Ranked in order of difficulty)

Solution 1:

Solved using alternate angles, vertically opposite and straight angles

< 1 = 115° ... Alternate Angles
<1 = < 2 = 115° … Vertically Opposite Angles
< 3 = 180° - (40° + 115°) … Straight Angle Adds to 180°
< 3 = 180° - 135°
< 3 = 25°

 = < 3 = 25°

→  = 25°
<4 = 40° … Vertically Opposite Angles
<4 = <5 = 40° … Alternate Angles
 = 180° - <5 … Straight Angle
= 180° - 40°

→  = 140°
<6 = 115° … Alternate Angles
<7 = 180° - 115° … Straight Angle
<7 = 65°

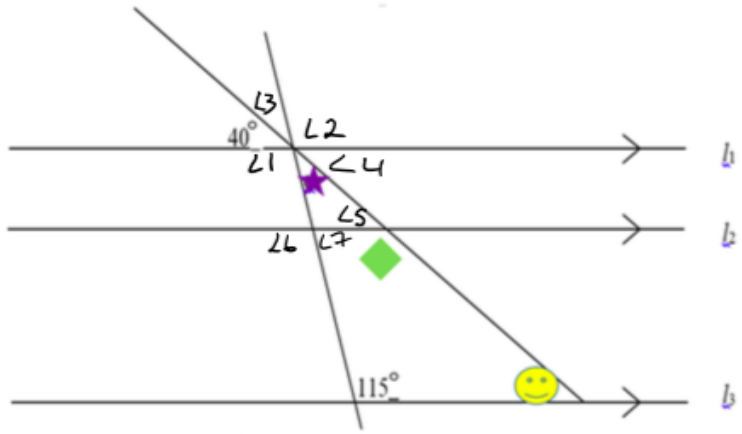
<7 + 115° +  +  = 360° … Angles in a Quadrilateral Add to 360°

65° + 115° +140° +  = 360°

320° +  = 360°

 = 360° - 320°

→  = 40°



- $\angle 1 = 115^\circ$ ... alternate angles
  $\angle 1 = \angle 2 = 115^\circ$ ... vertically opposite
  $\angle 3 = 180^\circ - (40^\circ + \angle 2)$ ... straight angle
  $180^\circ - (40 + 115^\circ)$
  $180^\circ - (155^\circ)$
  $\angle 3 = 25^\circ$
  ★ $= \angle 3$ ... vertically opposite
  ★ $= 25^\circ$
- $\angle 4 = 40^\circ$ ... vertically opposite
  $\angle 4 = \angle 5 = 40^\circ$ ... alternate angles
  ◆ $= 180^\circ - \angle 5$ ... straight angle
  $= 180^\circ - 40^\circ$
  ◆ $= 140^\circ$
- $\angle 6 = 115^\circ$ ... alternate angles
  $\angle 7 = 180 - 115^\circ$ ... straight angle
  $\angle 7 = 65^\circ$
  $\angle 7 + 115^\circ +$ ◆ $+$ ☺ $= 360^\circ$ ... angles in a quadrilateral
  $65^\circ + 115^\circ + 140^\circ +$ ☺ $= 360^\circ$
  $320 +$ ☺ $= 360^\circ$
  ☺ $= 360^\circ - 320^\circ$

☺ $= 40^\circ$

Solution 3:

Solved using vertically opposite, corresponding and three angles in a triangle add up to 180 degrees

<1 = 40° … Vertically Opposite Angles

<1 = <2 = 40° … Corresponding Angles

<2 =  = 40° … Alternate Angles

→  = 40°

 = 180° - <2 … Straight Angle

= 180° - 40°

= 140°

→  = 140°

<3 = 115° … Alternate Angles

<4 = 180° -  … Straight Angle

= 180° - 140°

= 40°

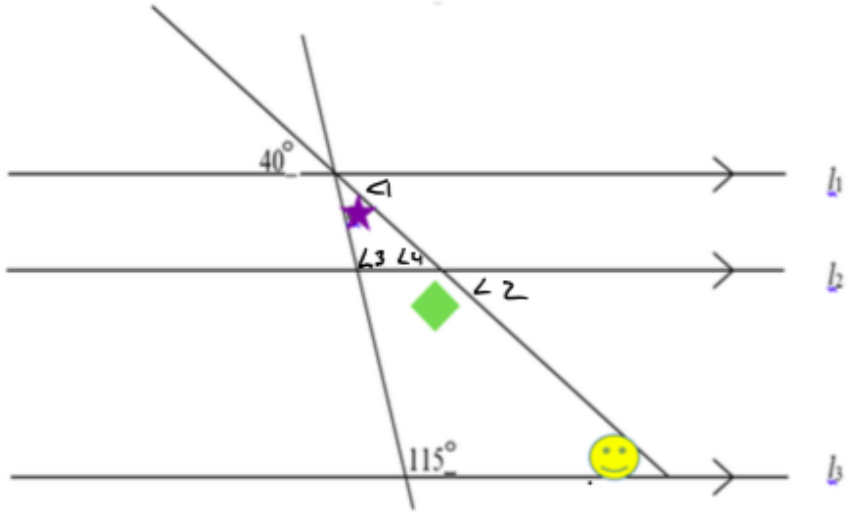
 + <3 + <4 = 180° … 3 Angles in a Triangle add up to 180°

 + 115° + 40° = 180°

 + 155° = 180°

 =180° - 155°

→  =25°



- $\angle 1 = 40^\circ$ ... vertically opposite
- $\angle 1 = \angle 2 = 40^\circ$ ... corresponding angles
- $\angle 2 =$ ☺ $= 40^\circ$ ... alternate angles

☺ $= 40^\circ$

- ◆ $= 180^\circ - \angle 2$ ... straight angle
  $= 180^\circ - 40^\circ$

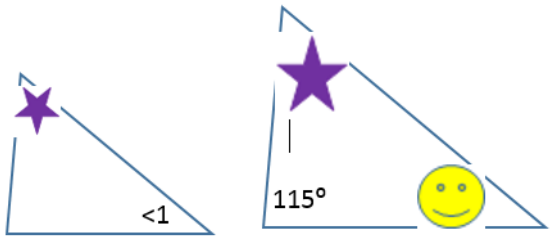
◆ $= 140^\circ$

- $\angle 3 = 115^\circ$ ... alternate angles
- $\angle 4 = 180^\circ -$ ◆ ... straight angle
  $= 180^\circ - 140^\circ$
  $= 40^\circ$
- ★ $+ \angle 3 + \angle 4 = 180^\circ$ ... 3 angle in a $\Delta = 180^\circ$
  ★ $+ 115^\circ + 40^\circ = 180^\circ$
  ★ $+ 155^\circ = 180^\circ$
  ★ $= 180^\circ - 155^\circ$

★ $= 25^\circ$

Solution 4:

Solved using Similar Triangles



<1 = 40° … Corresponding

<1 = ☺ = 40° … Similar Triangles

→ ☺ = 40°

★ = 180° - (115° + ☺) … Three Angles in a Triangle add up to 180°

= 180° - (115° + 40°)

= 180° - (155°)

= 25°

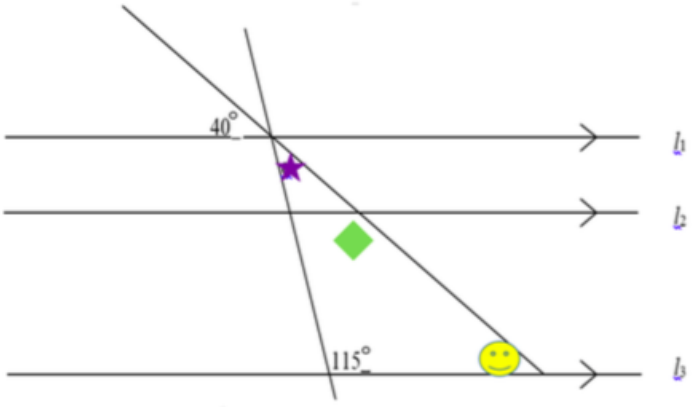
→ ★ = 25°

◆ = 180° - <1 … Straight Angles add up to 180°

= 180° - 40°

= 140°

--> ◆ = 140°



- $\angle 1 = 40^\circ$ ... corresponding
  $\angle 1 =$ ☺ $= 40^\circ$ ... similar Δ's
- ★ $= 180 - (115^\circ +$ ☺$)$ ... 3 angles in a Δ
  $= 180 - (115^\circ + 40^\circ)$
  $= 180^\circ - (155^\circ)$
  ★ $= 25^\circ$
- ◆ $= 180^\circ - \angle 1$ ... straight angle
  $= 180^\circ - 40^\circ$
  ◆ $= 140^\circ$

Solution 5:

Solved by using how the property that all angles at the centre of a circle adds up to 360°

<1 = 40° … Vertically Opposite Angles
<4 = 115° … Alternate Angles
<2 = <4 = 40° … Vertically Opposite Angles

<1 + <2 + <3 + <4 + 40° + ★ = 360° .... All degrees at centre of a circle add to 360°

40°+40°+<3+115° +40° + ★ = 360°

310° + <3 + ★ = 360°

<3 + ★ = 360° - 310°

<3 + ★ = 50° … BUT, <3 = ★ ... Vertically opposite

→ ★ =25°

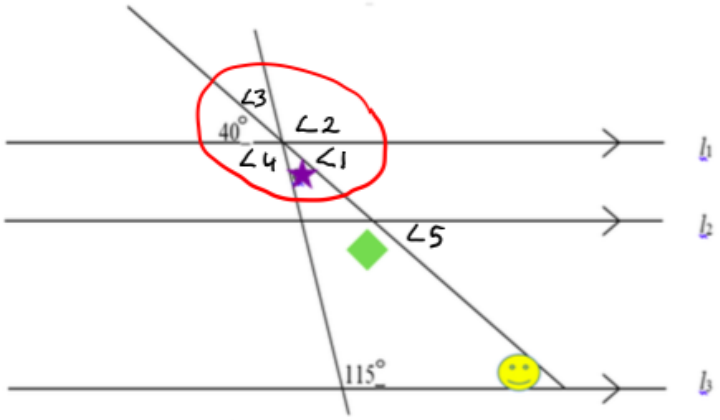
☺ = 180° - (115° + ★) … 3 Angles in a Triangle Sum to 180°
= 180° - (115° +25°)
= 180° - (140°)

→ ☺ = 40°

<5 = <1 = 40° … Corresponding Angles

◆ = 180° - <5… Straight Angle Sums to 180°
= 180° - 40°

→ ◆ = 140°



- $\angle 1 = 40^\circ$ ... vertically opposite
- $\angle 4 = 115^\circ$ ... alternate angles
- $\angle 2 = \angle 4 = 115^\circ$ ... vertically opposite
- $\angle 1 + \angle 2 + \angle 3 + \angle 4 + 40^\circ + \bigstar = 360^\circ$ ... degrees in a circle

$40 + 115^\circ + \angle 3 + 115^\circ + 40^\circ + \bigstar = 360^\circ$

$310 + \angle 3 + \bigstar = 360^\circ$, but $\angle 3 = \bigstar$ (vertically opposite)

$\Rightarrow 2\bigstar = 360 - 310^\circ$

$2\bigstar = 50^\circ \div 2$

$\bigstar = 25^\circ$

- $\text{☺} = 180^\circ - (\bigstar + 115^\circ)$ ... 3 angles in a $\Delta = 180^\circ$

$\text{☺} = 180^\circ - (25^\circ + 115^\circ)$

$\text{☺} = 180^\circ - (140^\circ)$

$\text{☺} = 40^\circ$

- $\angle 5 = \angle 1 = 40^\circ$ ... corresponding angles
- $\blacklozenge = 180^\circ - \angle 5$ ... straight angle $= 180^\circ$

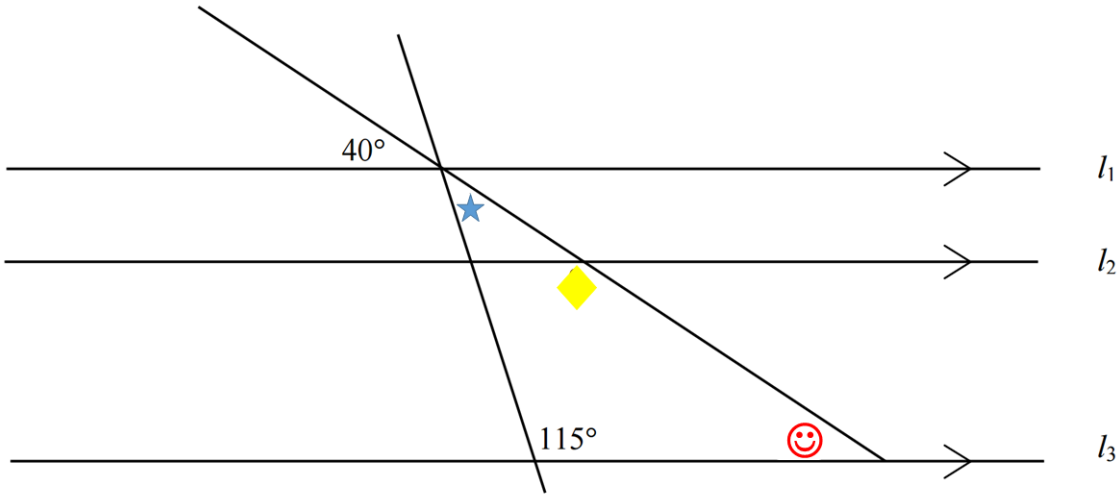
$= 180^\circ - 40^\circ$

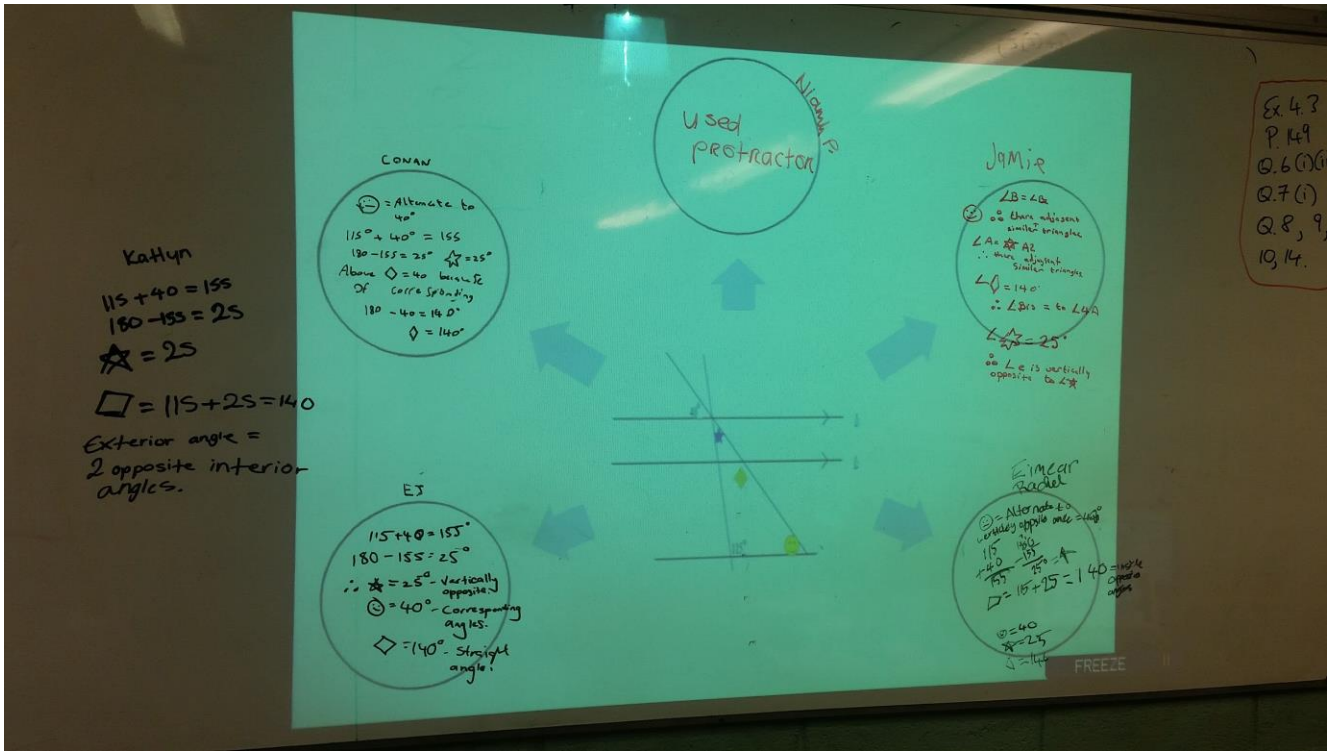
$\blacklozenge = 140^\circ$

Board Plan

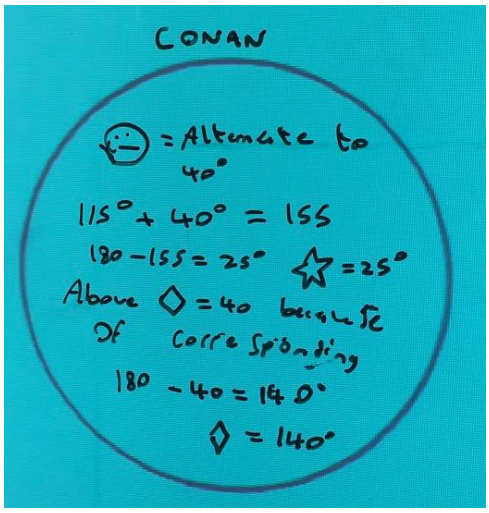
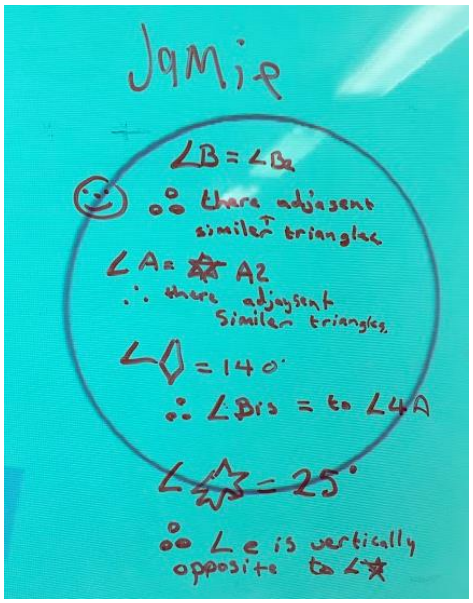
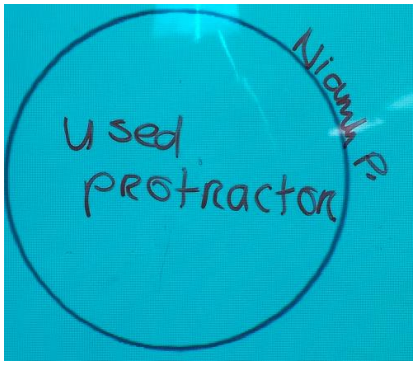
Main Problem Solving Task

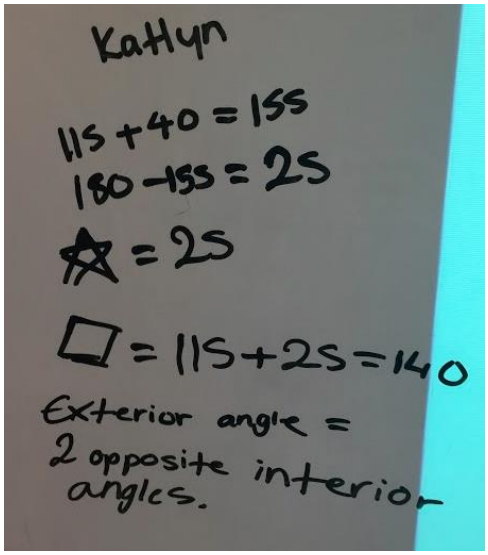
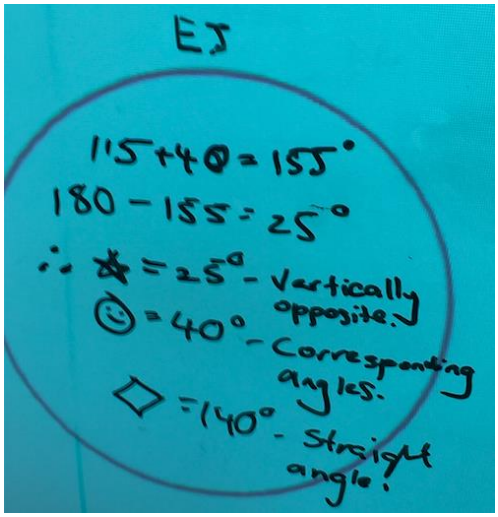
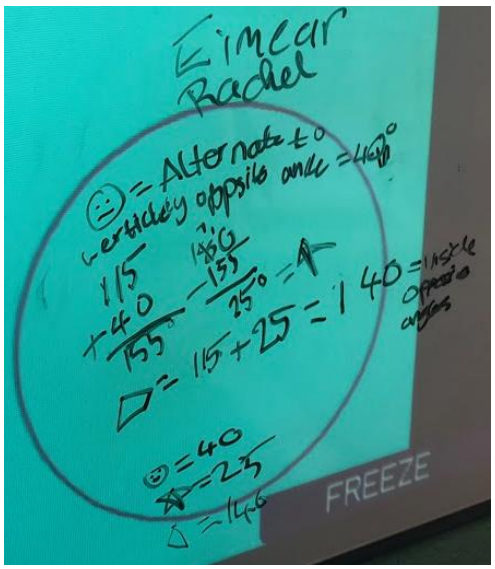
Find the value of each symbol in the following diagram. Give a clear reason to support all your work. The original exam paper question contain 3 greek letters. We replaced these letters with coloured symbols to engage the students with a more visually pleasing problem.





Students Solutions on the Board

	The majority of students solved the problem using this method. As previously mentioned students were aware of the terminology yet mixed up when to use each one
	Only one group applied the knowledge of similar triangles to solve the problem despite many groups mentioning the method initially
	Surprisingly only one group used a protractor to measure the angles. However the groups discussed checking their answers with a protractor. We pointed out the errors of this approach.

	A surprising method of solving the problem which was not anticipated, this group based their solution around the idea of exterior angles.
	Similar to the first method of solving however the answers are found in a different order
	This group focussed on using alternate and vertically opposite angles in order to solve the problem

Post-lesson reflection

Key Findings:

- Students were able to describe a similar triangle and how it can be used to solve a problem.
- $\frac{6}{19}$ students did not fully understand the concept of alternate angles, while $\frac{5}{19}$ were unsure of corresponding angles. Students are fully aware of the key literacy terms yet did not understand the concepts. This is a worrying observation. The emphasis placed on literacy in the modern classroom may be impacting on comprehending the concept.
- Students struggled with filling in page 1 of the worksheets, could not come up with 5 different ways of solving the problem. I had to encourage groups to look at the different methods.
- All groups were able to solve in just way. A few students had only key words and could not explain how they were going to apply these keywords.
- It was observed that students were giving restricted and afraid to write something that may be wrong.

With relation to solving the problem, during the matching exercise students worked well and were eager to match up the correct terms. However, there were too many mathematical terms used (10 in total) I would in future use a shorter version focusing on the key terms in the lesson (make a new matching exercise).Oral questioning proved highly effective as students understood the importance of geometry in everyday life. Pictures emphasized the scope of where geometry is used, “the triangle has a wide base and a low centre of gravity, and therefore is stable”

In this task peer learning was evident through discussion amongst students. “if you still don’t get it think about it like this” was overhead in one of the groups. Students cooperated well in the groups yet again were reluctant to explore additional strategies.
Students were labelling angles which helped to simplify the problem. Students were tracing a Z shape over the problem but unaware why they were doing this.
Initially students were focussed on finding angles and the measure but not giving reasons for their answers. With encouragement from the teacher, students began to explain and show method.

Students gave feedback by volunteering solutions and displaying on the board. Students explained their solutions to the rest of the class this was good evidence of peer learning. Self-correction took place by students recording solutions they did not obtain.
At the end of the lesson students recorded three things they learned from the lesson. Which was good reflective practise.

An example of students' learning outcomes:

Today we have learned:

1. "Communication is important"
2. "There are many methods to calculate 1 math sum"
3. "There are many ways to solve problems"
4. "Some are more complicated than others"
5. "New ways to solve sums"
6. "Sometimes just use a simple thing"
7. "To use alternate angles"
8. "To use corresponding angles"
9. "How to recognise corresponding and vertically opposite angles"
10. "That there are multiple methods for geometry questions through theorems"

Unexpected Outcomes:

- An unexpected outcome was the large proportion of students with misunderstandings of alternate and corresponding angles.
- One group of students used the method of trigonometric ratios to solve the problem without realising there was no right angled triangles.
- No students were able to visualise a full circle being 360 degrees to find the measure of the angles.
- Interestingly one group used the exterior angle theorem to solve the problem while another group used congruent triangles. Neither method was anticipated.
- Students focussed mainly on writing their solution directly onto the diagram instead of using the space around. A larger diagram would be more beneficial as students work got very cramped.
- ⅕ of the expected outcomes were reached these included the use of vertically opposite angles, corresponding angles, alternate angles, straight angles, theorems and similar triangles.

Looking Towards the Future:

If the task was to be repeated, more time would be allocated to the class (possibility of doing this in a double class). Alterations would be made to the matching exercise at the beginning of the class so that only relevant geometry terms for our questions would be included. With a longer class, there would also be more time to discuss the matching exercise in detail, rather than only taking one minute to correct it. With having longer time students might not have misunderstood alternate and corresponding angles like they did. The worksheet students were working on would

also undergo a change in the form that the diagram be enlarged so that students would have more room to work within the problem. Again, with the additional time suggested for the class, a more in depth discussion of the solutions could occur.

Conclusions:

From this lesson various conclusions were sought. Firstly, as literacy has become such a key concept in our classroom, we are placing too much focus on the keywords and not enough on why these are keywords. Across the board, it was observed students are fairly competent with keywords but could not apply the theory behind the keywords to solve the problem. Secondly, students are very reluctant to try new methods, correct or incorrect. This stems from constricted subjects where there can only be definition, one method or one correct answer to a question. Students are in a regiment state where they find it difficult to think outside the box. Thirdly, it was discovered that students sometimes associate shapes with a certain part in maths and easily forget when the theory behind that shape can be used. A clear example for this was using the Z shape to highlight alternate angles. Students were aware of the Z shape and able to draw it in on the diagram but were unable to make any comment on what angles were equal or what the Z shape meant. Secondly in reference to the association of shapes, one particular group identified the triangle in the problem but connected the triangles to trigonometry. The group failed to recognise that the triangles formed in the problems were not right angled triangles and could not be solved by using trigonometric ratios.

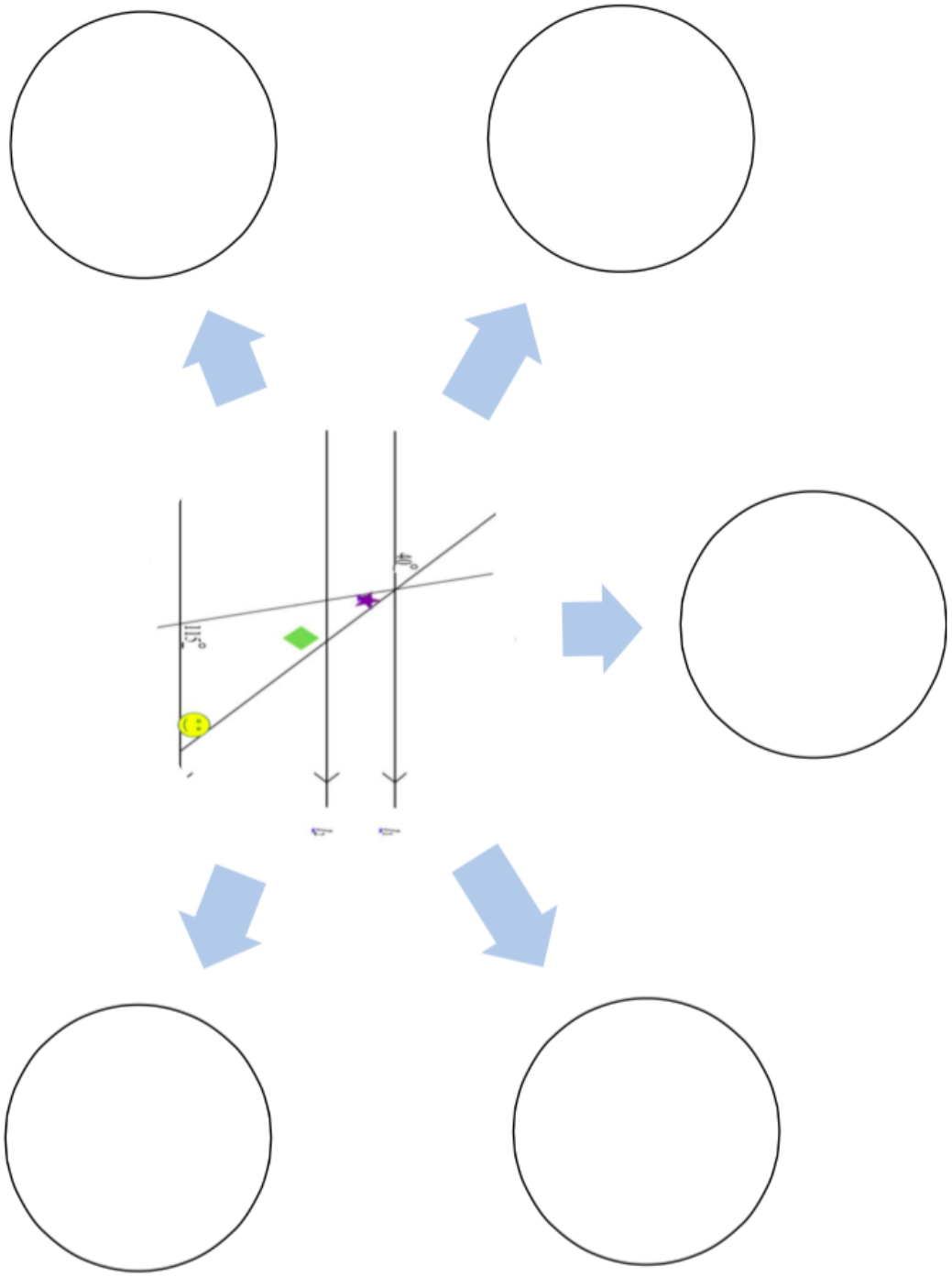
Appendix 1:

Observation Template
School: Carndonagh Community School
Teacher: Siobhán Coll
Title of Lesson: Multiple Approaches Problem Solving in Geometry
Date: 25th February 2016

Prior Knowledge:	
Posing the Problem:	
Problem Solving Section of the lesson:	
Student Feedback:	
Unexpected Outcomes:	
Expected Outcomes:	

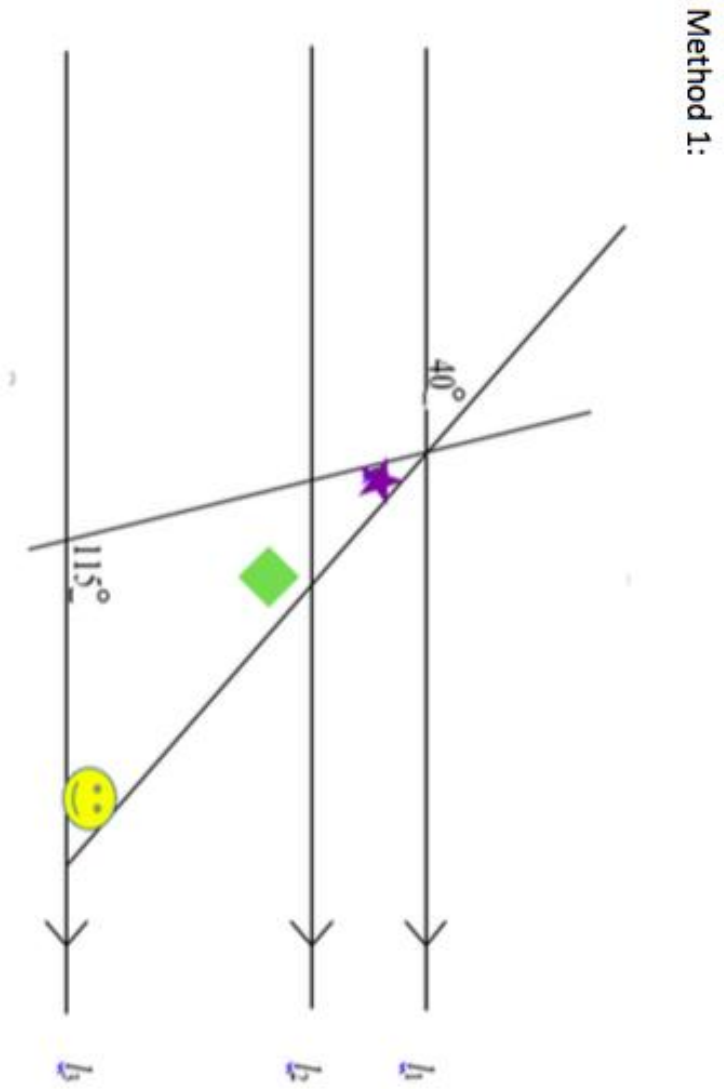
Appendix 2:

First Page of Student Handout



Appendix 3:

Second Page of Student Handout (4 more identical pages)



Appendix 4

(Matching Exercise)

Description	Mathematical words/ phrases	Diagram
Line going directly upwards	Horizontal Line	
Angle greater than 90° but less than 180°	Acute angle	
Line parallel to the ground	Right Angle	
Angle of 180°	Obtuse Angle	
Angle greater than 180°	Straight Angle	
Angle of 90°	Reflex Angle	
Angle less than 180°	Ordinary Angle (more than one diagram)	
Lines which never meet	Perpendicular Lines	
Lines at an angle of 90° to each other	Parallel lines	
Angle less than 90°	Vertical line	

Appendix 5:

Consent Form

21st February 2016

Dear Parent,

Your son/daughter's maths teacher has been involved this year in a national professional development initiative *Reflections on Practice*. This initiative has the support of the Department of Education & Skills.

This national professional development initiative has required your son/ daughter's maths teacher to attend 6 evening sessions after school hours in a local Education Centre. *Reflections on Practice* involves how to enhance students' problem solving skills and promotes individual teacher and school group collaborative planning, classroom observation and analytic reflection on classroom practices and active student engagement in problem solving mathematics.

Central to *Reflections on Practice* is classroom observation. This entails maths teachers from the maths department in a school/across schools visiting your son/daughter's maths class while the class is being delivered.

I trust that you appreciate continuing professional development is a core factor in developing the teaching profession and accordingly you consent to your son/daughter taking an active part in the problem solving lesson being observed by other teachers.

Regards,

Siobhan Coll

Appendix 6

Student responses to "Today I have learned…"