Exploration of properties of basic geometric shapes

Lesson Plan for 6D, Geometry (Revision and Application of Basic Geometric Properties, Interaction between Geometry, Trigonometry, Coordinate Geometry and Measure)

This lesson is based on a sixth year ordinary level banded mixed ability class group. They are considered the upper band as per school policy. The class consists of 27 pupils. This "Reflections on Practice" project has been compiled by a group of six teachers and the local Project Maths Regional Development Officer Ciarán Sweeney. The class group has a substantial knowledge of Geometry, Measure & Coordinate Geometry but have a limited knowledge of Trigonometry from the start of their Leaving Cert Cycle.

Exploration of properties of basic geometric shapes

Brief Description

This lesson is a progression from basic knowledge of geometric concepts to their application. A number of methods involved requires higher order thinking. Also students will have the opportunity to apply approaches from the various elements of the Geometry Syllabus (Synthetic Geometry, Coordinate Geometry, Trigonometry, Transformation Geometry and Measure).

Aims of lesson

- **I would like my students to appreciate that mathematics can be used to solve real world problems**
- **I'd like my students to appreciate that mathematics can be used to communicate thinking effectively**
- **I'd like to foster students to become independent learners**
- **I'd like students to become more creative when devising approaches and methods to solve problems**
- **I would like my students to realise and understand that a problem can have several equally valid solutions**
- **I'd like my students to experience meaningful mathematics i.e. that they see a need for what they are studying**

- **I'd like to build my students' enthusiasm for the subject by engaging them with stimulating activities**
- **I also want my students to connect and review the concepts that we have studied already**
- **I want to ensure that students reflect on the differences between these processes, along with the underlying reasons.**
- **I'd like my students to see the benefit of using various problem solving techniques to generalise situations with a view to exploring new problems and challenges**
- **Leaving Certificate Mathematics aims to develop mathematical knowledge, skills and understanding needed for continuing education, life and work. By teaching mathematics in contexts that allow learners to see connections within mathematics, between mathematics and other subjects, and between mathematics and its applications to real life, it is envisaged that learners will develop a flexible, disciplined way of thinking and the enthusiasm to search for creative solutions.**

Objectives

The objectives of Leaving Certificate Mathematics are that learners develop mathematical proficiency, characterised as;

- **conceptual understanding—comprehension of mathematical concepts, operations, and relations**
- **procedural fluency—skill in carrying out procedures flexibly, accurately, efficiently, and appropriately**
- **strategic competence—ability to formulate, represent, and solve mathematical problems in both familiar and unfamiliar contexts**
- **adaptive reasoning—capacity for logical thought, reflection, explanation, justification and communication**
- **productive disposition—habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and perseverance.**

Learning Outcomes

At each syllabus level students should be able to;

- **explore patterns and formulate conjectures**
- **explain findings**
- **justify conclusions**
- **communicate mathematics verbally and in written form**
- **apply their knowledge and skills to solve problems in familiar and unfamiliar contexts**
- **analyse information presented verbally and translate it into mathematical form**
- **devise, select and use appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions.**

In a mathematical problem-solving environment it is recognised that there are three principal things learners need to do:

- **make sense of the problem**
- **make sense of the mathematics they can learn and use when doing the problem**
- **arrive at a correct solution to the problem.**

Learning Outcomes for lesson

Students should be able to

- **solve problems involving the length of sides and area of triangles, squares and rectangles, along with combinations of these**
- **use instruments to perform constructions with precision**
- **perform constructions and apply constructions to problems**
- **use the theorem of Pythagoras to solve problems involving right-angled triangles**
- **calculate the length of a line segment and area of a triangle using coordinate geometry formulae**
- **use trigonometry to calculate the area of a triangle**
- **explore links between various approaches to solving problems**
- **devise appropriate mathematical models and use appropriate techniques to analyse problems**
- **work effectively as a group**
- **communicate mathematical work effectively.**

Background and Rationale

According to the syllabus the student should be making connections and use problem solving skills to fully understand each part of Strand 2. Three key aspects relate to this lesson. Firstly, students should be able to explore patterns; secondly, apply their knowledge and skills to solve problems in familiar and unfamiliar contexts; and finally, devise, select and use appropriate mathematical models, formulae or techniques to process information and draw conclusions. Students need be familiar with the basics of geometry, angles and triangles and their properties, the area of triangles and quadrilaterals. Having an understanding of coordinate geometry is useful but not essential for this task. From trigonometry they should be familiar with the theorem of Pythagoras and ratios to solve right-angled triangles.

Students have problems looking at the question and not being satisfied with the limited amount of information provided. Students may not realise that their ability to investigate and use a required formula may be necessary in this case. From finding the area of a quadrilateral, students will be familiar with doing these problems with much more knowledge than this lesson requires. Their problem solving ability is questioned and tested. Students main difficulty in this area is deciphering the question and developing at least one method to solve it.

As students progress through this lesson and other lessons with multiple solutions, our aim is that they build confidence in tackling a problem with as many different methods as possible and to make connections. The basic rules that surround, in this case, geometry, trigonometry, area, shapes, etc. can be easily translated into other questions in other topics.

Students must develop confidence and certainty in the basic geometric operations applied to simple situations. They must explore variety in approaches and methodologies. Furthermore, students need an understanding of the basis of these procedures, particularly how various elements of Strand 2 connect with each other. If students cannot refer back to these links between elements that may have been introduced to them separately, then they will struggle to apply geometric rules consistently correctly. They will also miss an essential purpose of geometry.

Research

Resources used:

- www.projectmaths.ie
- **Project Maths Workshops 2, 3, 6, 9, 10.**
- **NCCA J.C 2016, L.C 2015 Syllabuses.**
- **Project Maths Teacher Handbooks.**
- **Project Maths Teaching and Learning Plans.**
- **ICT - Geogebra and Autograph.**
- **Construction instruments.**
- **Geostrips.**

About the Unit and the Lesson

- **This review lesson will help students understand the logic and rigour underpinning geometry.**
- **The central point of the lesson is to reinforce the correct application of geometric procedures and the connections between various approaches to solving geometric problems.**
- **Students will be organised into pairs or groups of four, to facilitate discussion and reflection on issues arising.**
- **We may use a number of methodologies and strategies to illustrate these connections, to promote a willingness to explore problems and to instill confidence in students when dealing with unfamiliar scenarios.**

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Flow of the Unit:

Lesson	Title: Exploration of geometric shapes	Number of Previous Lessons (37 mins)
1-2	Area of square, rectangle, triangle, trapezium	2
3-4	Pythagoras' theorem and applications	2
5-9	Coordinate geometry of line - length, midpoint, area of triangle, slope	5
10-11	Basic geometric problem, along with extension activities that may arise (THIS LESSON IS LESSON 1 OF THESE TWO)	2

Flow of the Lesson:

Teaching Activity	Points of Consideration
INTRODUCTION • Correct homework from previous class. (5 mins) • State Learning Outcome for this class - "students will devise solutions and use appropriate techniques to analyse a problem involving geometric shapes". (6 mins) **POSING THE TASK** • Divide students into groups of four. Each group is given one A3 sheet with coordinated grid to work on and another on which to write solution. Each student is given an A4 sheet with the problem and should have appropriate drawing equipment. • Present problem on whiteboard (using Autograph or Geogebra diagram). Students	To see if students understand task, ask students relevant questions, eg "is |PQ| = 3?"

will be allowed to assume that smaller triangles are congruent. (10 mins) **ANTICIPATED STUDENT RESPONSES** **A key element to start some of these approaches is an appropriate sub-division of the figure** • **R1: Measure with ruler - find length then using (L)(B) or L^2. [Possible problem: scale and accuracy].** • **R2: Counting squares - combining pieces to form the equivalent of a square (used in science).** • **R3: Four halved rectangles with inner square. [Possible issue - sub-dividing the full figure appropriately].** • **R4: Divide figure into four triangles and square - divide the figure appropriately and apply the area of triangle formula ½bh.** • **R5: Pythagoras - making the connection with the area and the side of the square.** • **R6: Pythagoras - Based on squares of lengths of sides (standard use of Pythagoras).** • **R7: Coordinate Geometry - Find the length of one side. This requires development of a coordinated plane. [Possible issue: IPQI=3; why not].** • **R8: Coordinate Geometry - area of a triangle formula. Easiest triangle is PDS, assuming D is the origin. Students can also find areas of other triangle using formula but will have to do translation first.** • **R9: Divide bigger triangles, e.g. DPS, into triangles/rectangles/trapeziums (assume one third/two thirds splits).** • **R10: Trigonometry: area = ½ IABI Sin C. [Why would you want to know area of one triangle?].** • R11: Trigonometry: Tan of A =⅓ ⟹ A = 18.43°, therefore Sin 18.43° = 1/L. L = 1/Sin(18.43°) = 3.16… which is √10. [Issue correct use of calculator where students are not rounding off too early].	Names on top of page. Each group to produce at least one answer per question. If disagreement, how will they decide. (Debate, CHECK, vote?) With any uncertainty or possible error, how will students check answer? [Try to look at other approaches, cross-check answers?] If time, get students to look for other approaches, or to suggest possible extension activities/problems to solve (eg Why is |PQ| not equal to 3?). Ideas to focus on during discussion: • developing certainty on approaches by testing answers. • highlighting connections between various

PRESENTATION OF SOLUTIONS • **Each group debates its work and presents an attempted solution on board, with explanation.** • **This presentation will be done by the Kyouzai-kenkyu model, ie the teacher will place prepared partial solutions (of anticipated responses) on the board as a basis for students' explanations.** • **The teacher will plan an order of presentation of group solutions based on observation of mathematical level of groups' work.** **(30 mins)** **SUMMING UP** • **Finalise discussion on solutions. Can class decide on definite answer?** • **Give homework - preplanned or based on an extension activity that arises in class.** • **Ask students to write briefly (e.g. on post-its) what they thought of lesson - what did you learn?; strong points, difficulties, learning issues, etc. (35 mins)**	approaches and benefits of looking at these. Check are all students clear on homework. Leave all worksheets on desk. Place post-its on desk on way out.

Pre-evaluation and Plan

Inform students in advance about arrangements

- **guest and extra teachers.**
- **observation and possible video (permission from principal and parents).**
- **work in groups of four.**
- **use of diagrams and construction instruments.**

Plan to observe students

- **each teacher will observe one group of four.**
- **each teacher will be given observation templates.**

What to observe

- **how effectively are the groups going about the problem?**
- **what difficulties or confusion are being encountered?**
- **presentation of work?**
- **dynamics of group work - recommendations to make these more effective.**
- **preparation to communicate solution on whiteboard to class - effective?**

Differentiation

- **differentiated level of discussion - level of questioning.**
- **extra work to do.**
- **help for students with difficulties.**

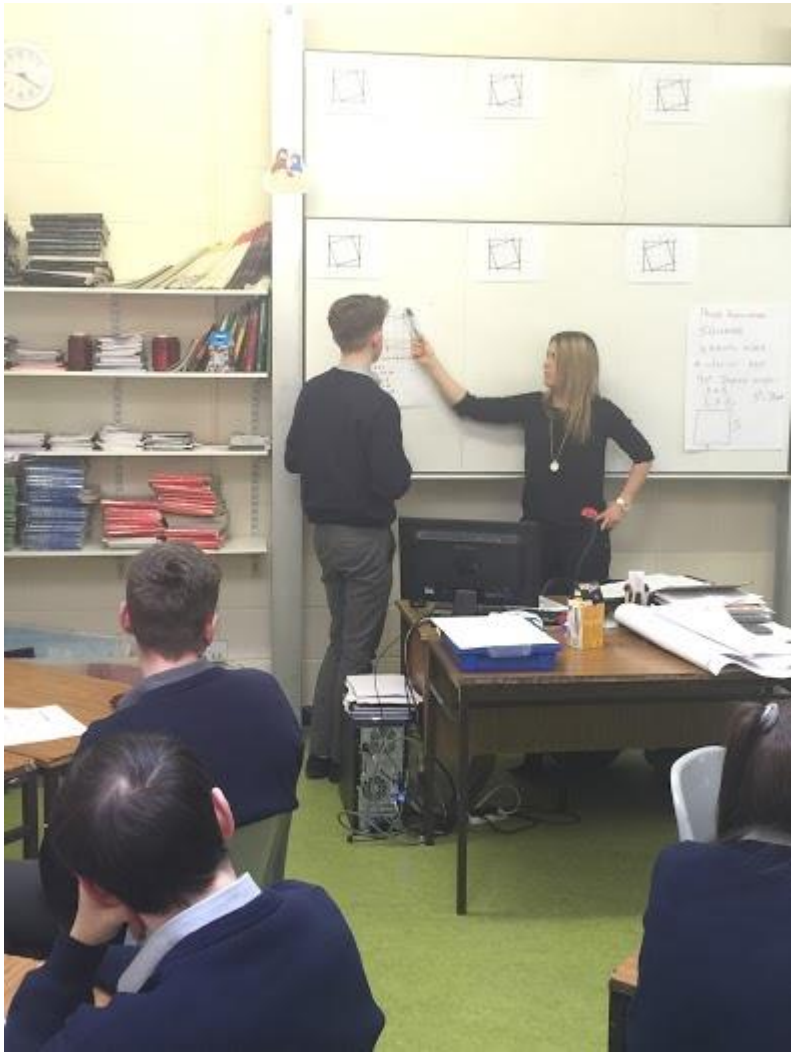
Other issues

- **are the flow, content and methodologies of the lesson effectively achieving the learning outcomes?**

Whiteboard plan

PRE LESSON BOARD PLAN:

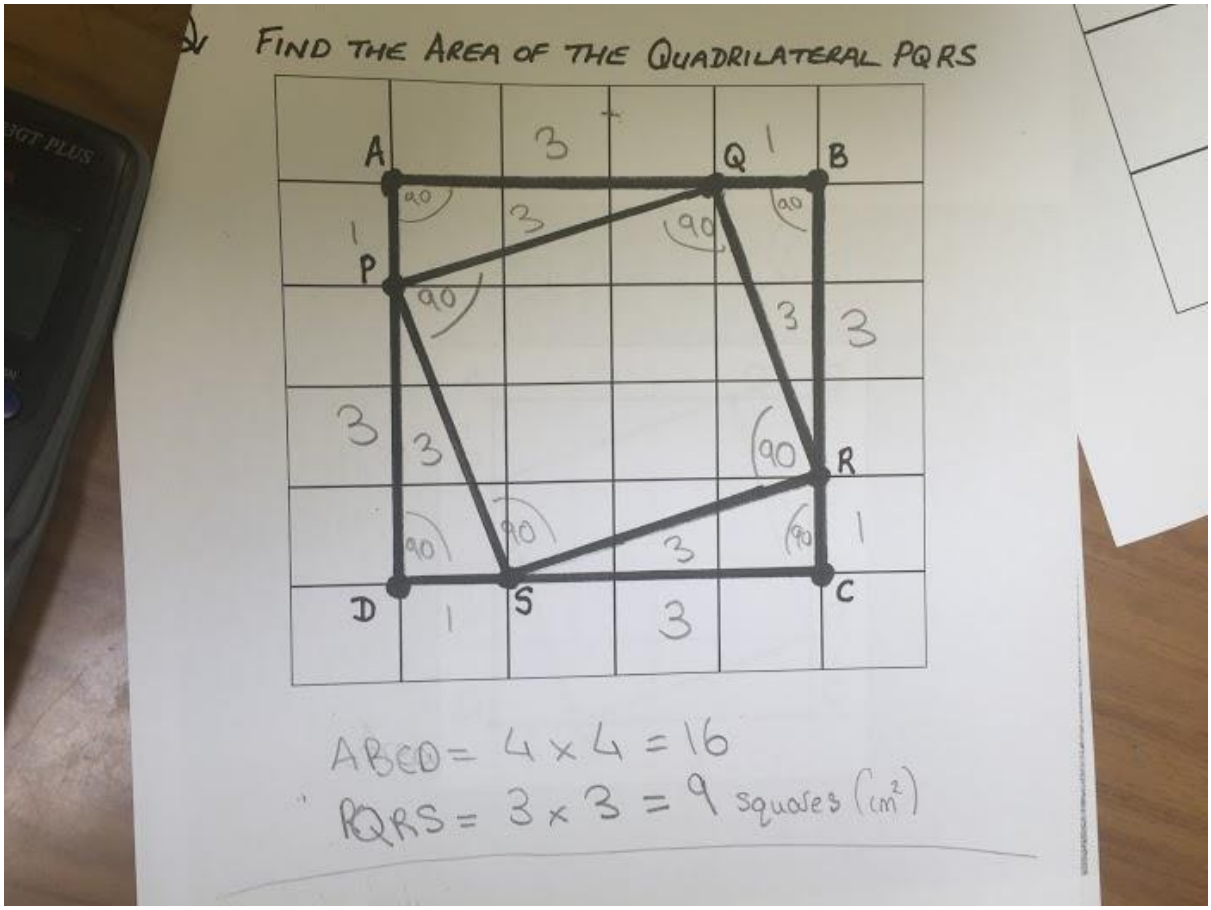
We planned solutions in order of difficulty that we thought the students would offer. The teacher put copies of the diagram on the board and had a large version on an adjacent screen. She also allocated board space for anticipated solutions in advance. The teacher group had produced outline solutions to be placed in the board space for further development by the students.



POST LESSON BOARD PLAN (STUDENTS' SOLUTIONS).

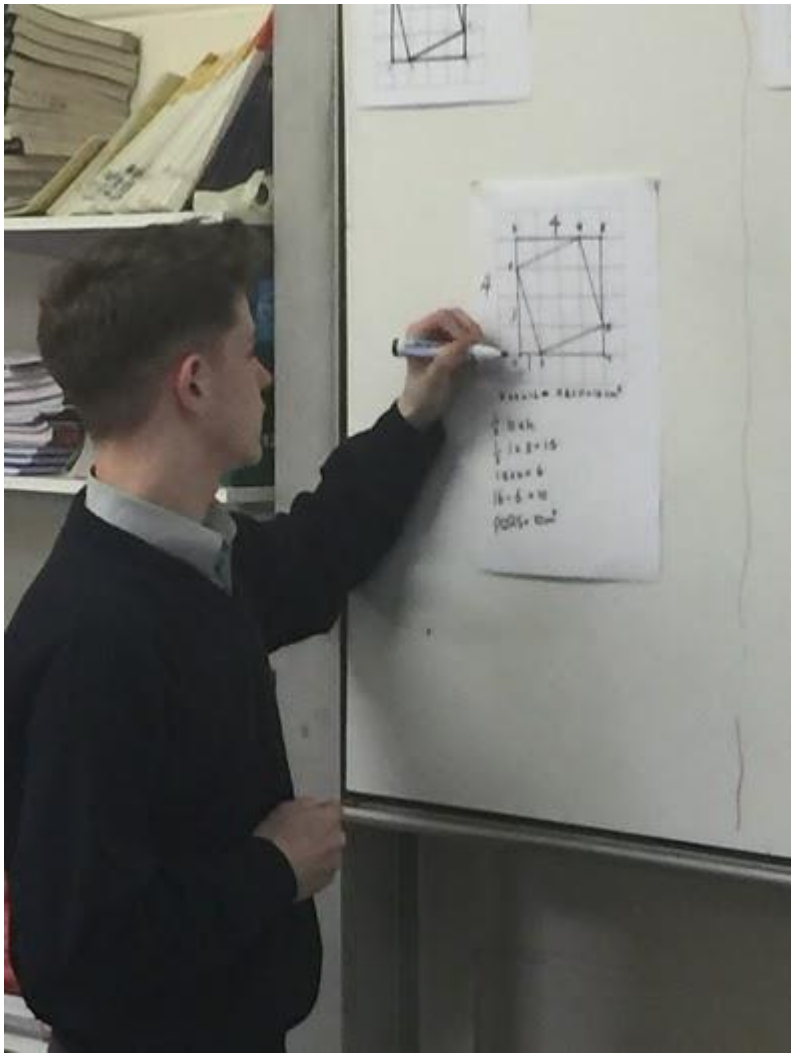
Post-lesson reflection

The class was structured on the basis of think pair share. Each student had an individual worksheet and the group overall had one large sheet on which to present its answer. Observers noted that the individual worksheets were especially helpful in engaging students. For the first few minutes of group work, any group that attempted the problem by finding |PQ| were suggesting |PQ| = 3.

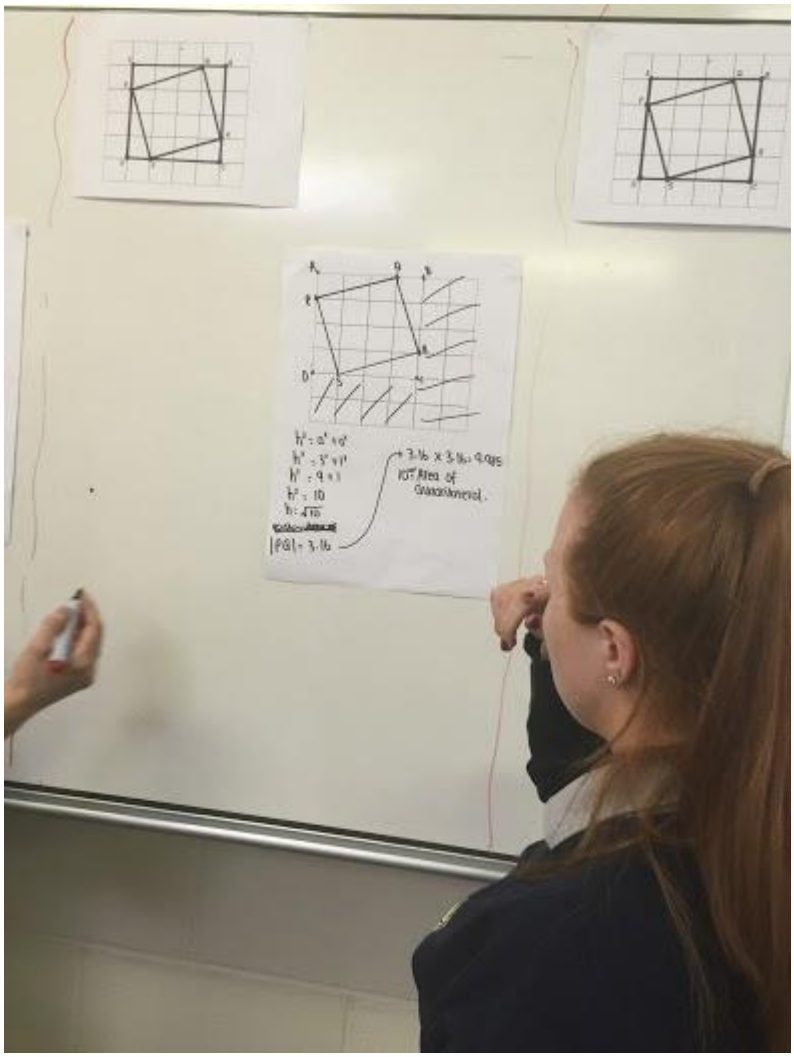


The teacher made a decision to give students ample time and this proved very successful. An interesting process of constructive debate and self-correcting started to take place. Groups started to query whether IPQI could be 3. Doubt started to emerge. Members of groups voiced their doubts to the teacher who asked them to consider some basic issues such as the properties of right angled triangles, type of triangle, the nature of the hypotenuse and the longest side on a triangle.

The teacher asked Reece`s group to present its answer first as it contained the lowest order of complexity from our expected solutions. This group took a simple approach entirely based on area. Their solution was area of square minus area of 4 triangles.

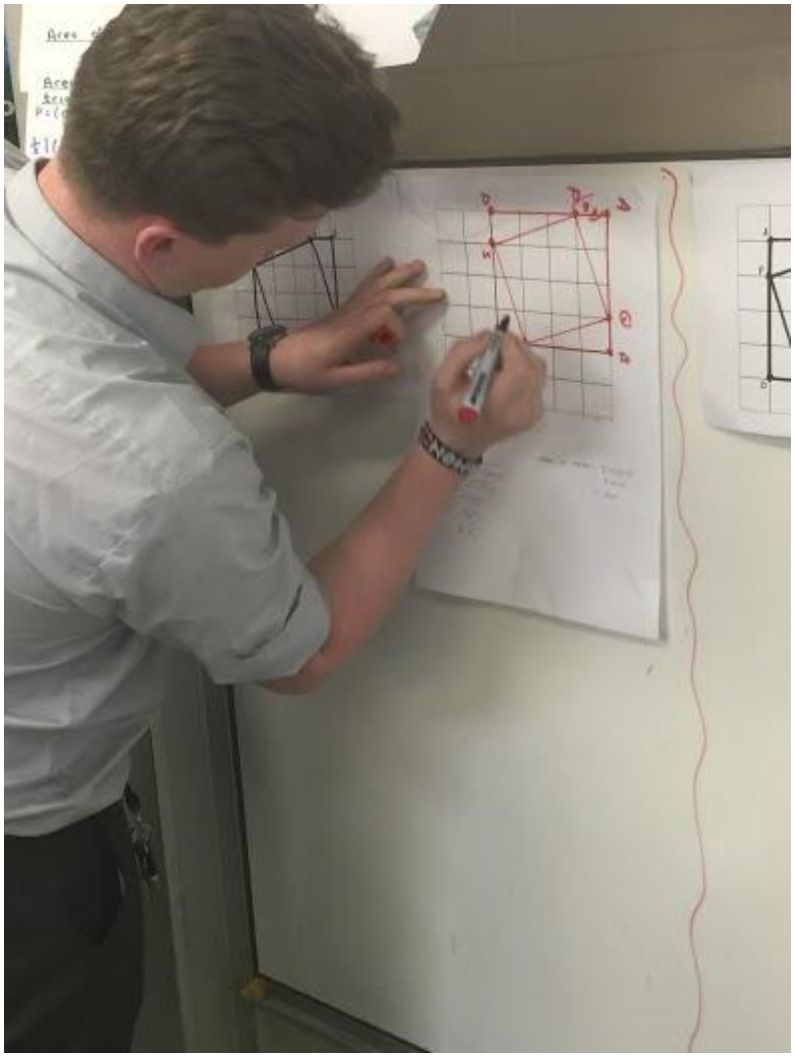


Kayleigh`s group produced a largely correct answer using Pythagoras theorem and the area of the square formula.

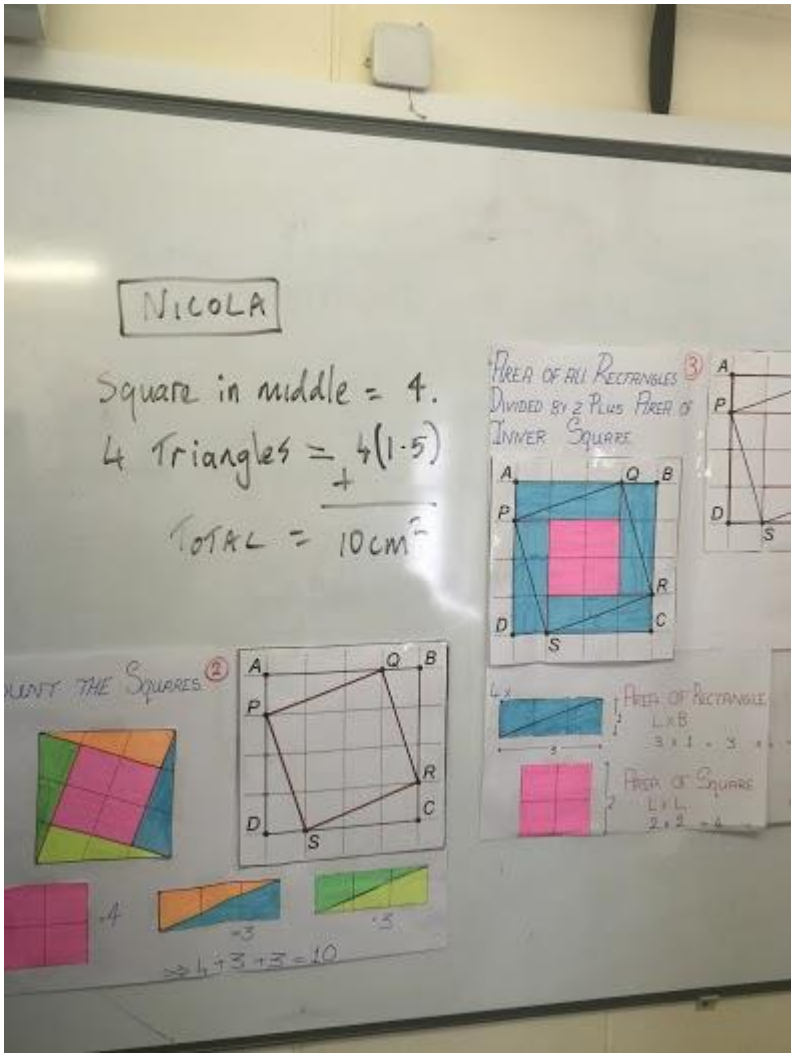


An interesting feature of this group was rounding off. This is probably to be expected at leaving cert ordinary level. Rounding off twice actually gave the correct solution. While this group successfully produced a solution, a particular difficulty arose with its operation. One student who lacked confidence made very little contribution to the debate. She was interested and engaged, but was clearly reluctant to offer any ideas. This is an issue that the teacher and the observers agreed should be addressed in any future similar undertaking.

Eamonn`s group quickly spotted the potential of being given some element of coordination in the graph. They proceeded to find |PQ| using the coordinate geometry formula and completed the problem by using the area of a square formula.

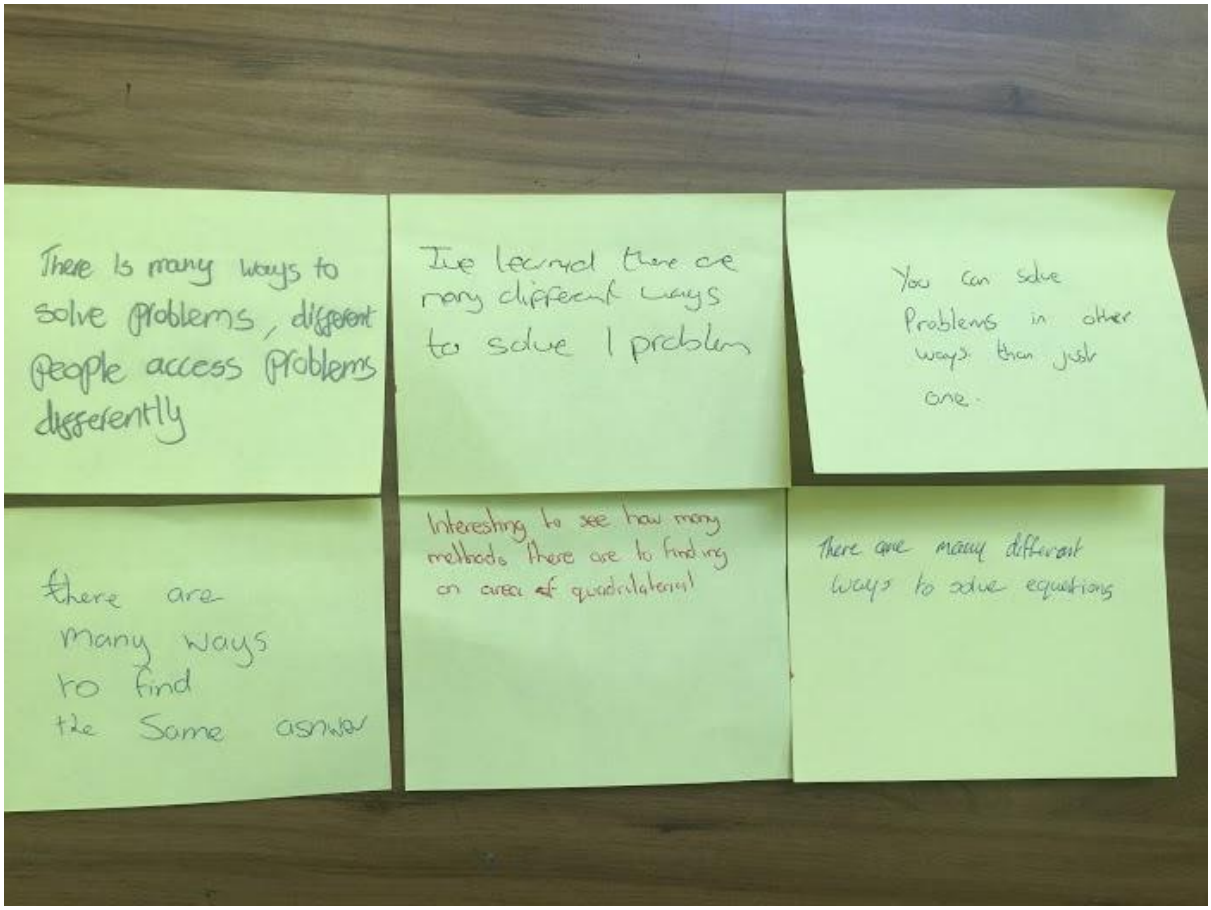


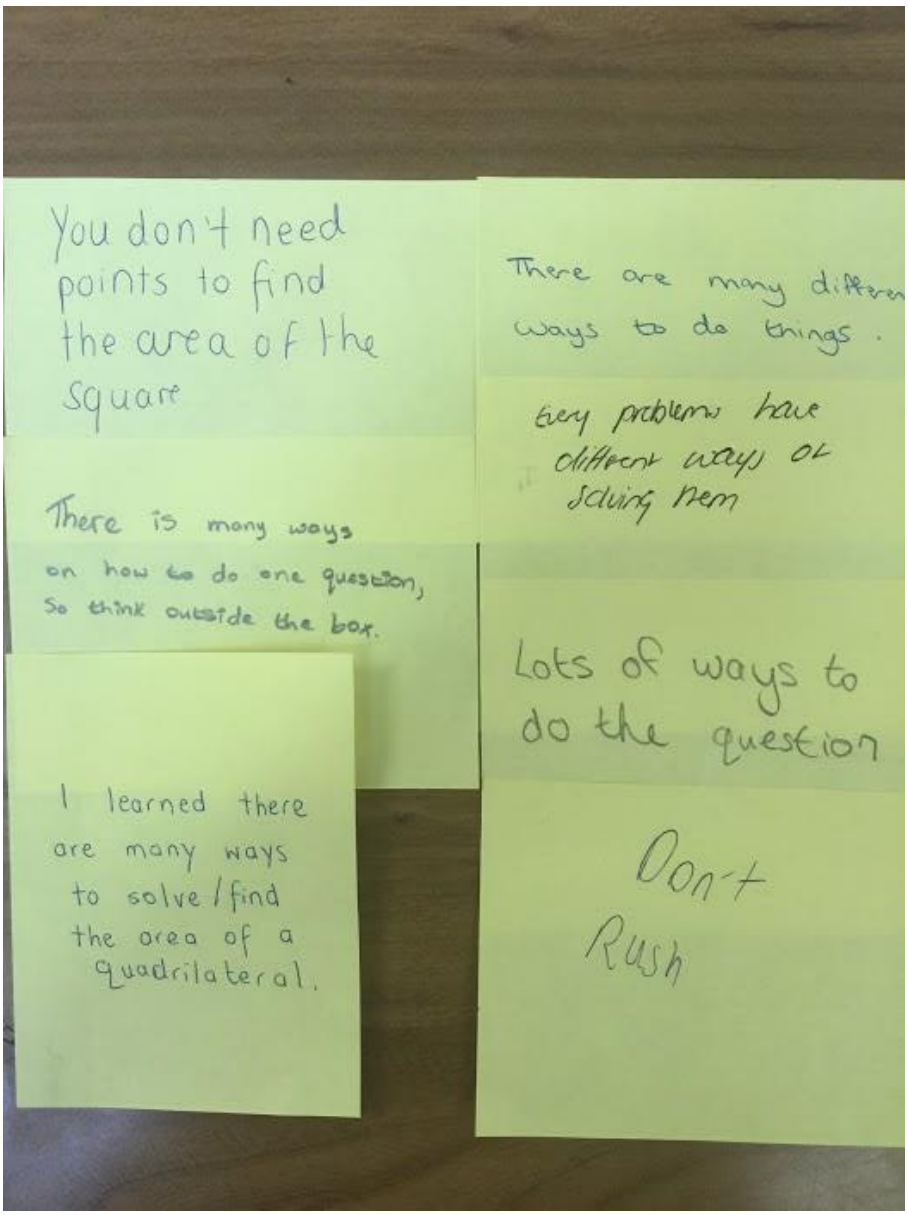
The teacher and the observer of this group noticed how a student, Nicola, who is normally quiet and passive in class was very engaged in the work. This group organised its work and discussion well. Nicola actually presented a second solution offered by this group, requiring further sub-division of the shape.



Unfortunately one group did not get to present their answer as time ran out for presenting their ideas. This was a direct result of the teacher's deliberate decision to give students more time to consider the problem when they ran into difficulty. When considering this after the class, both the teacher and the observers agreed that this decision was worthwhile. The resultant self-correcting and work on the problem by all groups was the most successful aspect of the class.

Toward the end of the class the students were asked to reflect on what they felt they had learned in the lesson. Comments reflected the positive and engaged nature of their work. They highlighted their appreciation of solving problems in a variety of ways. Students also emphasised the importance of being given time to think and to explore potential solutions fully.

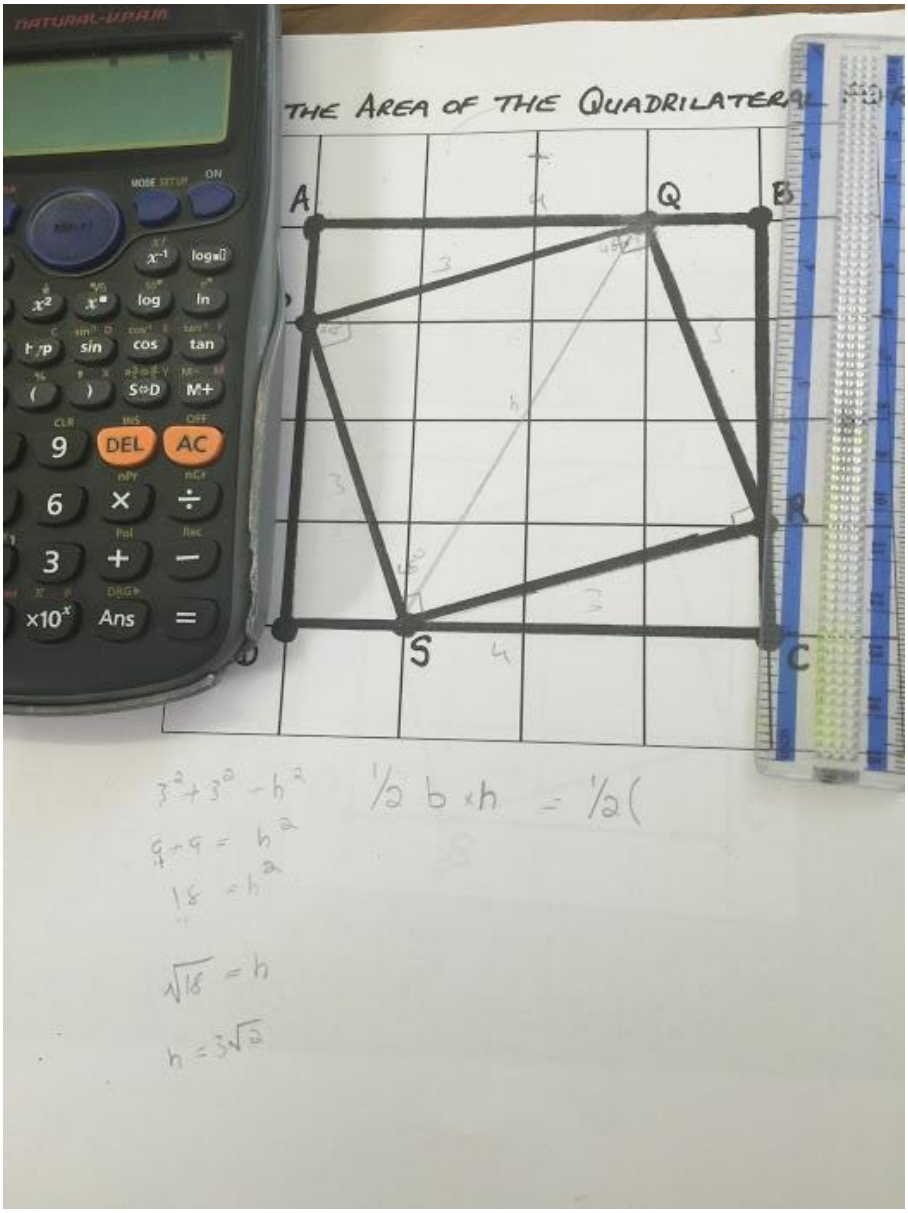

There is many ways to solve problems, different people access problems differently
I've learned there are many different ways to solve 1 problem
You can solve Problems in other ways than just one.
there are many ways to find the same answer
Interesting to see how many methods there are to finding an area of quadrilaterals
there are many different ways to solve equations

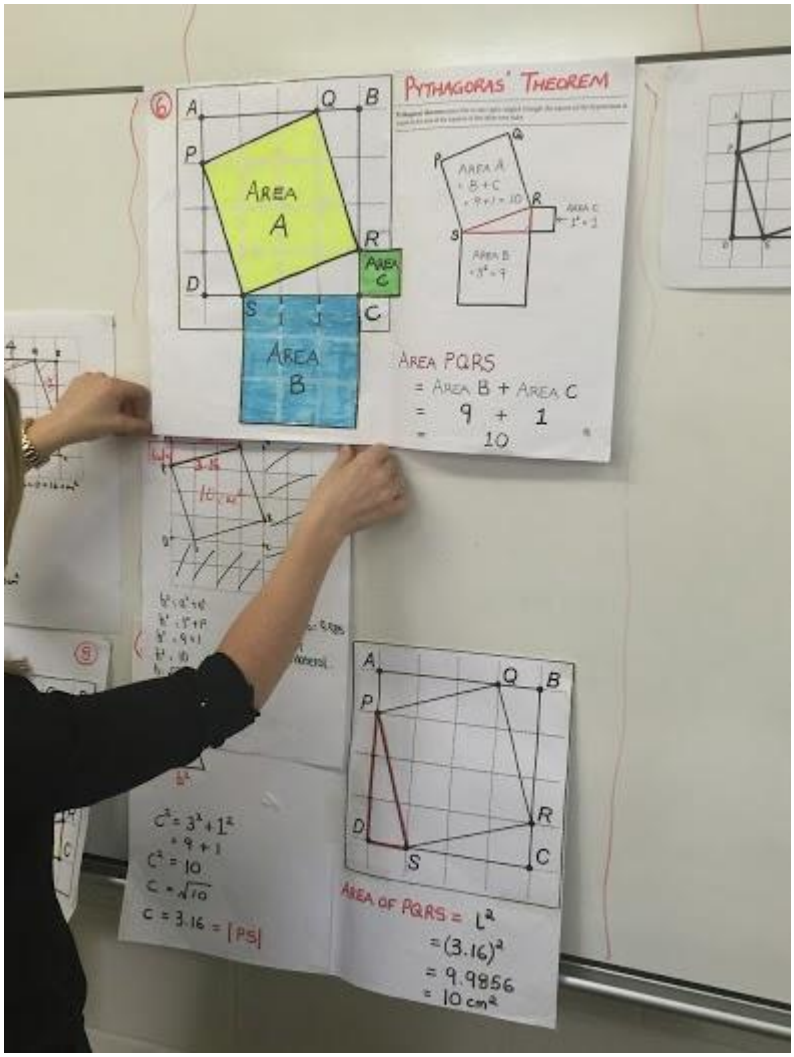


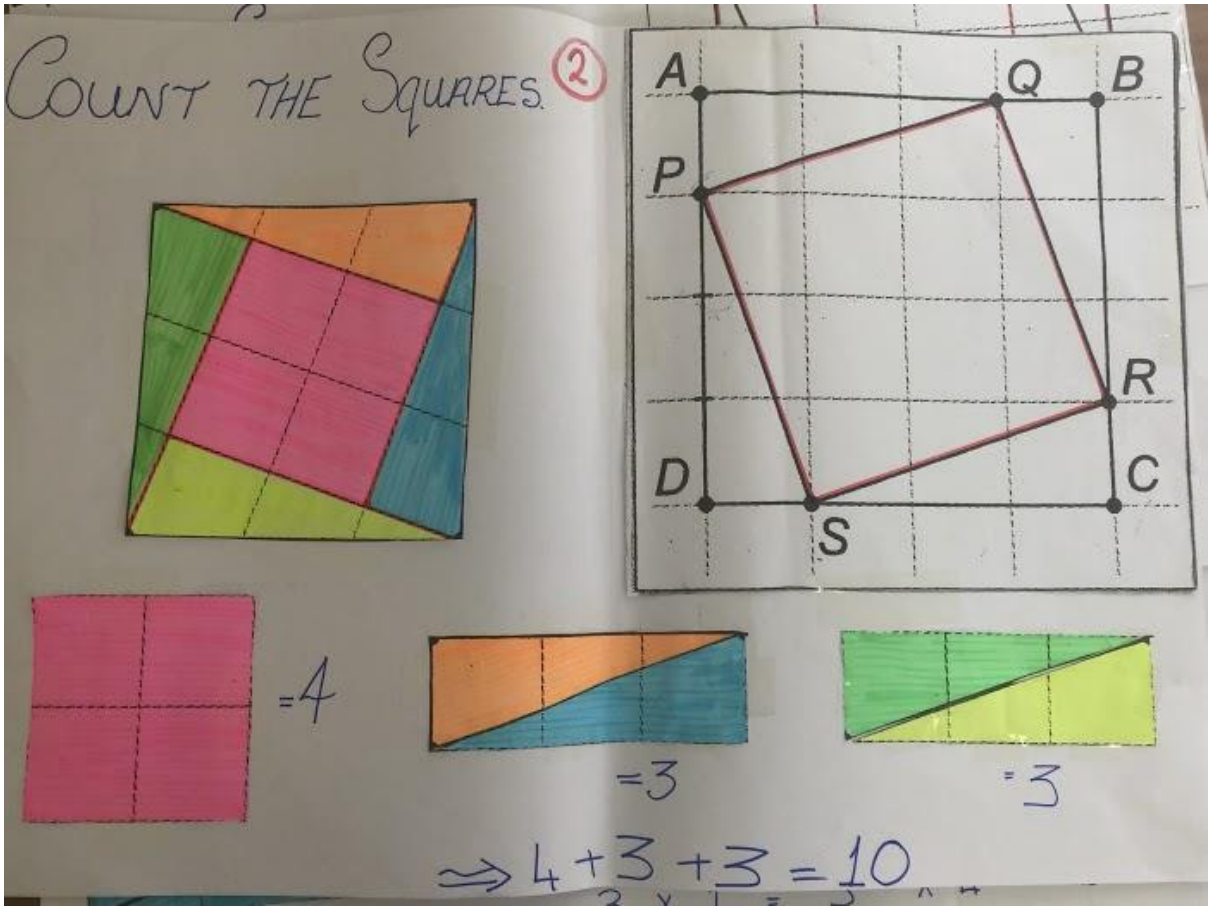
To conclude, a number of expected solutions were not developed by the groups in the allotted time, e.g. subdividing the shape, area of a triangle formula in coordinate geometry. These solutions are included in the white board plan and appendix. They also formed a basis of a follow up class, where the students were asked to consider multiple approaches and to cross check the answers. These are key problem solving skills.

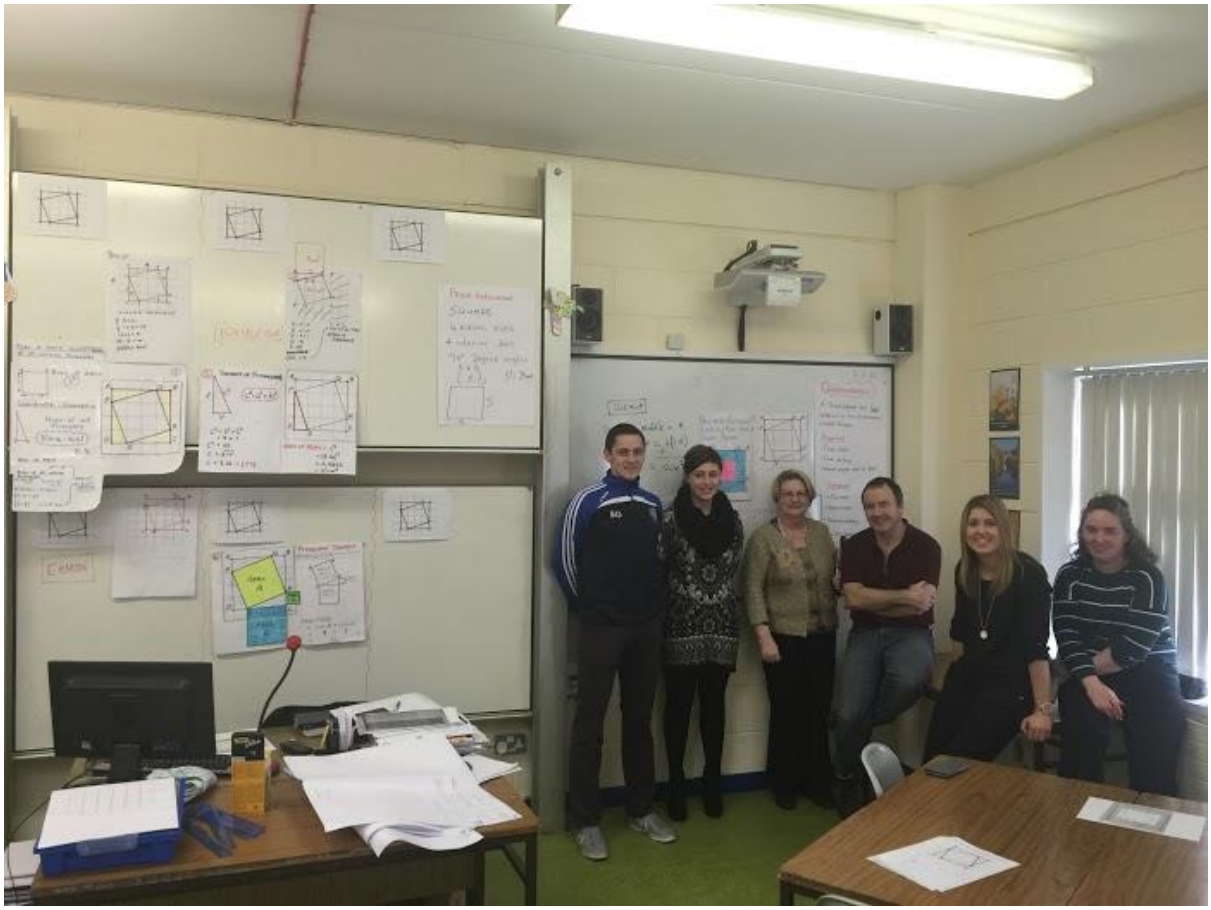
Teachers:
Maureen O'Donnell
Steven Gibson
Caitríona O'Malley
Paddy Flood
Vivienne Doherty
Leah Harkin

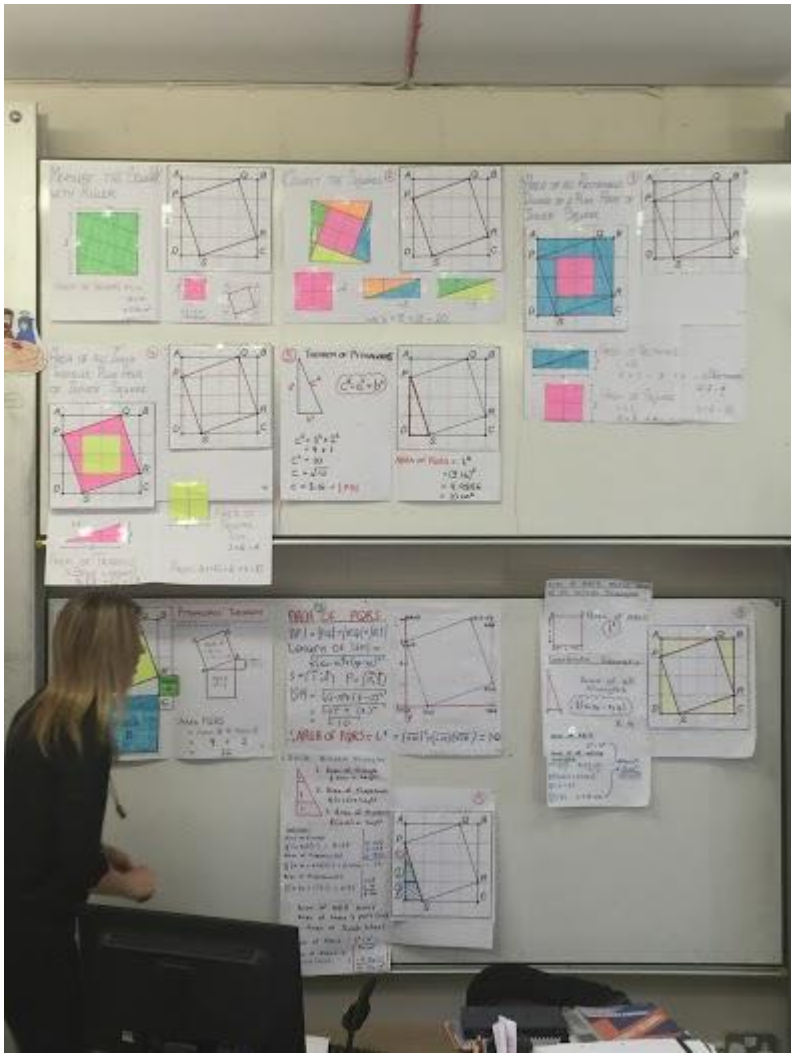
Appendix


THE AREA OF THE QUADRILATERAL
A
Q
B
R
S
C
DEL
AC


PYTHAGORAS' THEOREM
AREA A
AREA B
AREA C
AREA PQRS
= AREA B + AREA C
= 9 + 1
= 10
AREA OF PQRS = L²
= (3.16)²
= 9.9856
= 10 cm²


COUNT THE SQUARES. ②
A
Q
B
P
R
D
C
S
=4
=3
=3
⇒ 4 + 3 + 3 = 10






AREA OF PQRS