

Introduction to Solving Linear Equations.

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- 1. Title of the Lesson:** Introduction to Solving Linear Equations.
- 2. Brief description of the lesson:** A task will be given to the class, which will require them to explore linear relationships. Using the same number and result they will be asked to equate them in as many ways as possible using a maximum of two operations. Then, when these linear relationships have been discussed at the board, the students will be asked to prove that they are correct. By proving them, they will begin to use inverse operations to solve equations.

3. Aims of the Lesson:

Long Term Goals

- I'd like my students to appreciate that mathematics can be used to communicate thinking effectively
- I'd like my students to appreciate that algebra is a tool for making sense of certain situations
- I'd like to foster my students to become independent learners
- I'd like my students to become more creative when devising approaches and methods to solve problems
- I'd like to emphasise to students that a problem can have several equally valid solutions
- I'd like to build my students' enthusiasm for the subject by engaging them with stimulating activities
- I'd like my students to connect and review the concepts that we have studied already

Short Term Goals

- I'd like my students to understand inverse operations
- I'd like my students to be able to use inverse operations to balance a linear relationship

4. Learning Outcomes:

As a result of studying this topic students will be able identify inverse operations.

They will be able to use inverse operations to solve a linear equation.

They will develop a meaningful understanding of how to balance an equation using algebraic tools.

5. Background and Rationale

Solving linear equations requires students to have an excellent understanding of the principle of balancing the equation using inverse operations. This topic, learned initially in 1st year, is prevalent in the course at every level up to and including senior cycle. From our own experience and from analysis of student responses in exams, it is clear that there are many misconceptions in how to solve equations. The most common way these misconceptions present themselves is through transposition errors, and a failure to use the same operation on both sides. Our belief is that by giving the students an opportunity to come up with the method of using inverse operations to balance equations, we will minimize these errors.

6. Research

We referenced the syllabus as follows:

Junior Certificate Syllabus 4.4: students should be able to

- show that relations have features that can be represented in a variety of ways.

Junior Certificate Syllabus 4.6: students should be able to

- consolidate their understanding of the concept of equality
- solve first degree equations in one or two variables with coefficients elements of \mathbf{Z} and solutions also elements of \mathbf{Z} .

We used a variety of textbooks to look at different ways this topic was being presented in them.

We finally chose as our task a problem from Text & Tests 1 that was given at the end of the chapter.

7. About the Unit and the Lesson

The lesson starts with a task.

“Using any one of the four operations addition, subtraction, multiplication and division, convert the number 6 into the number 18 as many ways as you can.”

Students will then work individually on this task. Their results will be discussed and put on the board. These will then provide the task for the second half of the lesson, where they will be asked to prove that their equation is correct using inverse operations. This forms the foundation of solving equations.

8. Flow of the Unit: Algebra

Lesson		# of lesson periods
1	<ul style="list-style-type: none">• Algebraic expressions, variables, constants etc...	1
2	<ul style="list-style-type: none">• Evaluating expressions	2
3	<ul style="list-style-type: none">• Adding & subtracting algebraic expressions	2
4	<ul style="list-style-type: none">• Multiply algebraic terms	2
5	<ul style="list-style-type: none">• Multiplying algebraic expressions	6
6	<ul style="list-style-type: none">• Research lesson	1
7	<ul style="list-style-type: none">• Solving linear equations without brackets	3
8	<ul style="list-style-type: none">• Solving linear equations with brackets	3

9. Flow of the Lesson

Teaching Activity	Points of Consideration
<p>1. Introduction. Teacher introduces the lesson by talking about prior knowledge, i.e. the definition of an integer. The teacher also does a warm up activity, a ten question mental maths quiz on addition, subtraction, multiplication and division of integers. When this is finished the teacher will then put the learning outcome for the current lesson ‘solving equations’.</p>	
<p>2. Posing the Task The teacher sets out the task; <i>“Using any one of the four operations addition, subtraction, multiplication and division, convert the number 6 into the number 18 as many ways as you can.”</i></p>	<p>We hope that the prior knowledge given will prevent the obvious questions that may have arisen such as; “What is an integer?”, “What is an operation?”</p>
<p>3. Anticipated Student Responses The task will be broken into two different discussions. The first will concern the number sentences the students come up with:</p> <ul style="list-style-type: none"> - Addition $6 + 12 = 18$ - Subtraction $24 - 6 = 18$ - Multiplication $3 \times 6 = 18$ - Division $108 \div 6 = 18$. 	<p>We anticipate this will take 5 minutes. Then we will ask different students up to the board to write up each of the four in the order addition, subtraction, multiplication and division. We will have prepared headings ‘addition, subtraction, multiplication and division’ and will put them up over each response.</p>
<p>4. Refining the Task The teacher will now replace one integer in each number sentence with a box. Students will be asked if they recognize the boxes. The boxes will be then replaced with letters, introducing the variable into the equation. In the addition number 6 will be covered with a letter. In subtraction 24 will be covered. In multiplication 6 will be covered and in division 108 will be covered. The teacher will set the question ‘how would you find the unknown number if you didn’t already know what it was?’</p>	<p>The students will have 5 minutes to work on this task.</p>
<p>5. Anticipated students responses We anticipated that the first answer would be trial & error / substitution. Under this heading</p>	<p>We will have trial & error prepared as our first heading. We consider this to be the least sophisticated response in this context.</p>

<p>students will use substitution to show that the initial number sentence was correct. Our second anticipated answer is reverse operations. Students will use this method to work back to the missing number.</p> <p> $18 - 12 = 6$ $18 + 6 = 24$ $18/3 = 6$ $18 \times 6 = 108$ </p>	<p>In addressing student's responses under the heading 'reverse operations' we will begin to introduce the preferred notation for solving linear equations.</p> <p> $x + 6 = 18$ $x = 12$ </p> <p> $y - 6 = 18$ $y = 24$ </p> <p> $p \times 6 = 18$ $p = 3$ </p> <p> $m/6 = 18$ $m = 108$ </p> <p>It may be ambitious to think that we will get through all the four operations in this lesson. One or perhaps two of the proofs will be left for homework.</p>
<p>5. Summing up Five minutes before the end of the lesson, we will start our summary. We will draw their attention to the idea of reverse operators being used to find unknown quantities. Get the students to write down in their own words what they learned today.</p>	

10. Evaluation

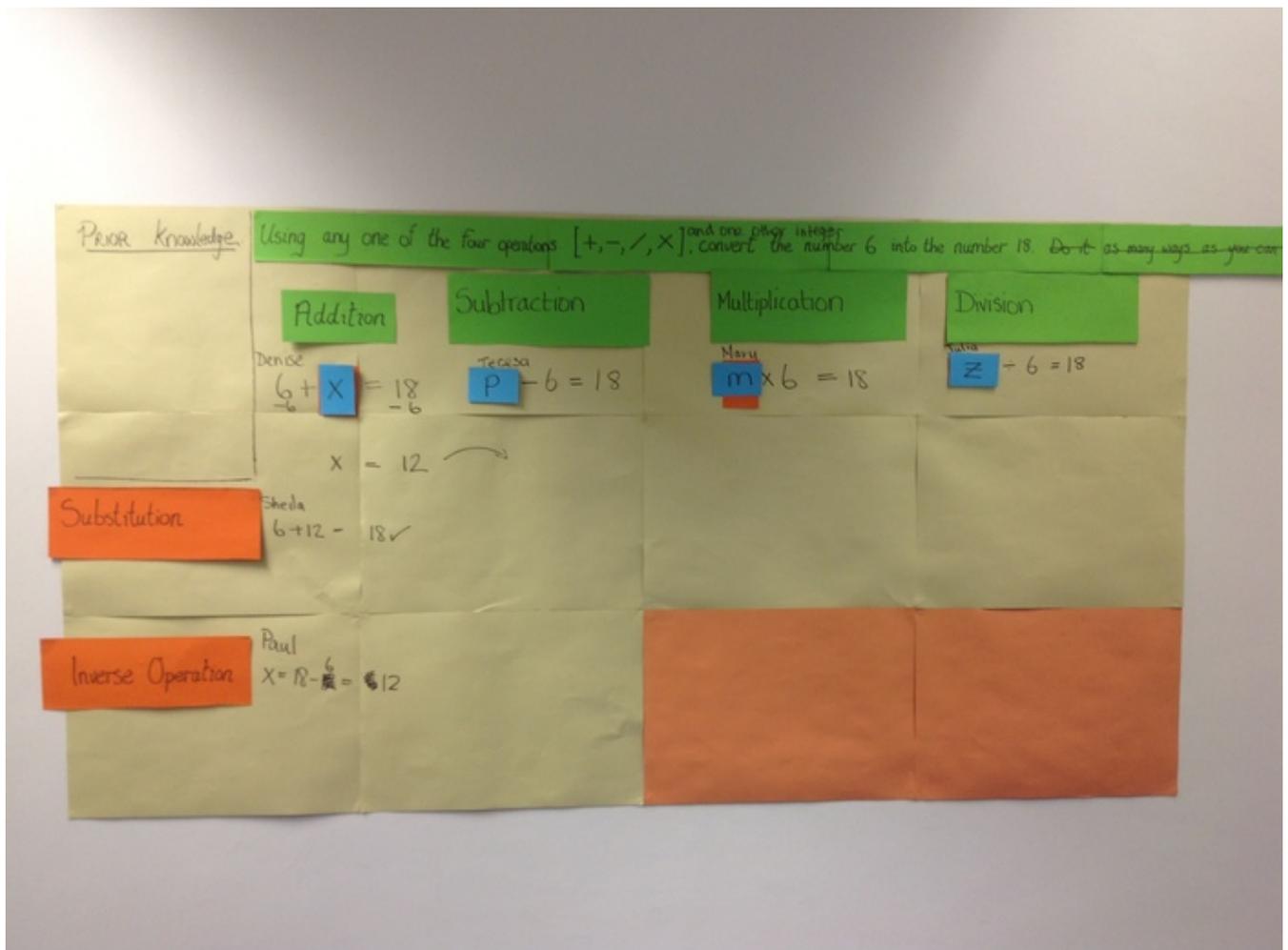
Observation

As there were two observers, we decided to split the class and take half each. We both opted to record our observations using the lesson note app. A plan of the classroom was uploaded onto basecamp in advance of the lesson. We also had a copy of the lesson proposal. We had done a mock-up of the lesson on the fifth night, so had a very clear idea of where we expected the lesson to go. We were looking out for four different number sentences connecting 6 to 18 and using each of the four operations. In the second part, we were looking for examples of trial and error, and reverse operations.

Classroom plan



11. Board Plan



12. Post-lesson reflection

Progress of the lesson

In the first part of the task, all students produced four correct number sentences using the four operations.

In the second part, no student used trial and error and so the teacher did not propose it as a possible method as it was not necessary. All students used the idea of inverse operations to find the value of the variable. However, they did not reference the variable when they were doing so. They simply performed the calculation. For example, for the number sentence $6 + 12 = 18$ they performed the calculation $18 - 12 = 6$. No student produced $x = 6$, or $x = 18 - 12$, $x = 6$. At this point the teacher stepped in and while a student was presenting their solution at the board, guided them towards this method.

Student Learning

The major achievements were that all students identified the unknown quantity as the variable. They all immediately used inverse operations to find the value of that unknown quantity. They were all able to present their work using correct mathematical language at the board. In terms of achieving the goal of the lesson all students identified inverse operations as the method for finding an unknown quantity. However they failed to use them on both sides of the equation. They used the inverse in a separate calculation instead of balancing the equation. We felt that getting students to reflect on what they have learned in the lesson was beneficial to both the teacher, in terms of assessment, and to the student, in terms of reinforcement.

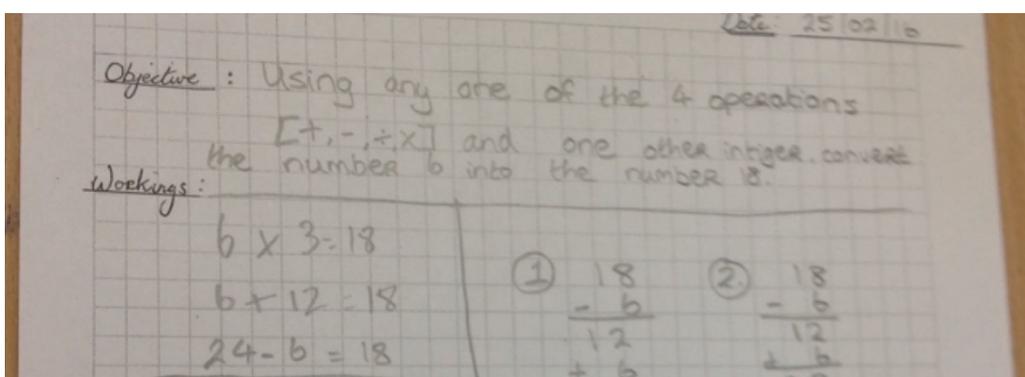
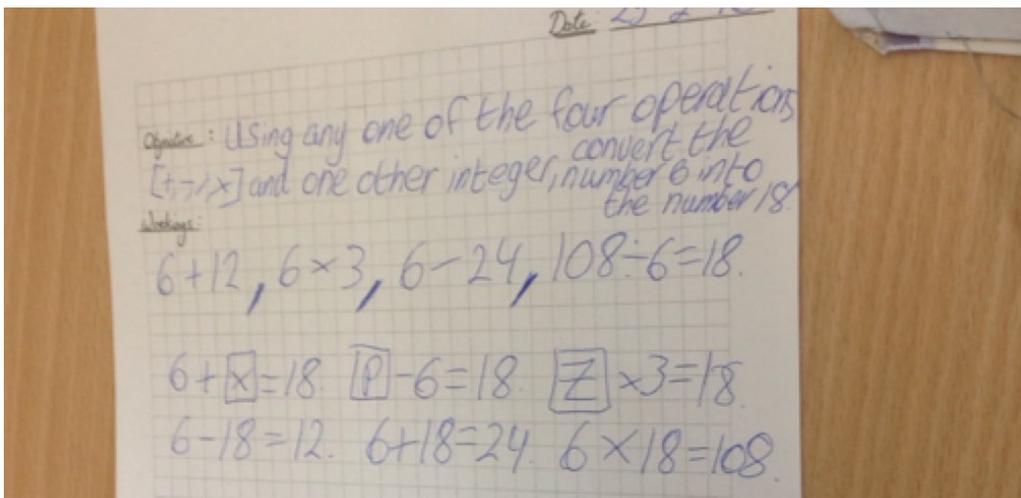
Reflection on lesson proposal

In hindsight we feel we may have been overly ambitious in our expectations of the students. To achieve the learning outcome, in terms of balancing the equation, may have been an unrealistic goal. We do feel however that their success with this skill in subsequent lessons was partly due to our emphasis on inverse operations for this lesson. On reflection we feel that this lesson proposal is more suitable for introducing and reinforcing the idea of using inverse operations rather than as an introduction to the balancing method. In future classes, where this topic was being taught, we would use it for this purpose.

Benefits for teachers from engaging in the 'Reflections on Practice' process.

1. Opportunity to collaborate with colleagues.
2. Focusing on one aspect of one topic in order to plan a lesson gave us the opportunity to see it through the student eyes. When anticipating students responses was where we experienced our greatest challenges and also where we gained our greatest insights.
3. Taking the time to break down the topic in order to plan the lesson was of great benefit. Rather than presenting the students with a method, they were being asked to build on their prior knowledge to discover this method. As teachers we needed to understand what the prior knowledge was for this topic and how they could build on it.
4. Although the process was time consuming, planning lessons this way will probably save teaching time in the long run as students will have strong foundational knowledge of the topic covered.

Examples of student work and student reflections



What I learned today:

How to get a missing number using inverse operation.

What I learned today:

If you do something to it on one side you must do the same thing to the other side.