TEACHING THEOREM 13
USING A CONTEXUALISED PROBLEM

For the lesson on 16 March 2016

At Colaiste Bhride, Enniscorthy,
To a Second Year Junior Cert. Higher Level class.
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Project Maths Tionscadal Mata Development Team

1. **Title of the Lesson:** Teaching Theorem 13 using a contexualised problem.

2. **Brief description of the lesson:**

   The lesson builds on students' prior learning in relation to geometric similarity. Using a contextualised problem students' learn that the sides of similar triangles are proportional. This new knowledge can be used to solve for missing lengths, provided the triangles are similar.

3. **Aims of the Lesson:**
   - I'd like my students to appreciate that mathematics can be used to solve real world problems.
   - I'd like my students to appreciate that mathematics can be used to communicate thinking effectively.
   - I'd like to foster independent learning in my students.
   - I'd like my students to become more creative when devising approaches and methods to solve problems.
   - I'd like to emphasise to students that a problem can have several equally valid solutions.
   - I'd like to build my students' enthusiasm for the subject by engaging them with stimulating activities.

   - I'd like my students to understand that corresponding sides in similar shapes (specifically triangles) are in proportion.

4. **Learning Outcomes:**

   As a result of studying this topic students will be able to:
   - Identify similarity between shapes, specifically triangles.
   - Use the relationship between corresponding sides in similar triangles to find missing lengths.

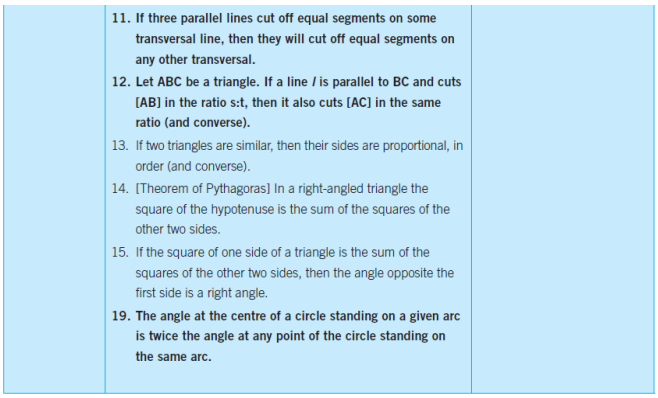
5. **Background and Rationale**

   - The Junior Certificate Syllabus outlines material that is required to be studied during the three years of junior cycle education. The syllabus outlines the material initially in strands, of which there are five, listed below:

     - Statistics and Probability
     - Geometry and Trigonometry
     - Number

- Algebra
- Functions

Each strand in sub-divided into topics where a description of the topic is given (what the student learns about) and learning outcomes are detailed (what the student should be able to do). Strand 2, subsection 2.1 (synthetic geometry) notes that students should be able to use the results of Theorem 13 to solve problems.



11. If three parallel lines cut off equal segments on some transversal line, then they will cut off equal segments on any other transversal.

12. Let ABC be a triangle. If a line *l* is parallel to BC and cuts [AB] in the ratio s:t, then it also cuts [AC] in the same ratio (and converse).

13. If two triangles are similar, then their sides are proportional, in order (and converse).

14. [Theorem of Pythagoras] In a right-angled triangle the square of the hypotenuse is the sum of the squares of the other two sides.

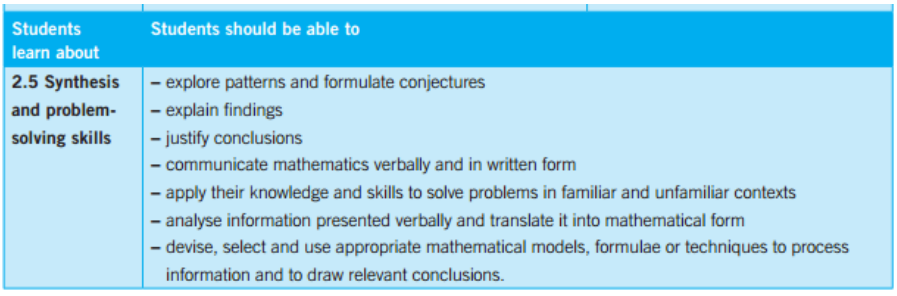
15. If the square of one side of a triangle is the sum of the squares of the other two sides, then the angle opposite the first side is a right angle.

19. The angle at the centre of a circle standing on a given arc is twice the angle at any point of the circle standing on the same arc.

The syllabus notes that teaching and learning methods need to be differentiated in order to meet the needs of all learners (page 11). It is noted that every opportunity to make connections across the strands should be taken (page 11), and that problem solving is 'integral to mathematics learning' (page 10). The syllabus states:

> ."…encouraging learners to share, explain and justify their solution strategies…learners [can] develop robust and deep mathematical understanding as well as confidence in their mathematical ability."
>
> Junior Certificate Mathematics Syllabus Page 11

Page 20 of the Junior Certificate Syllabus also outlines student learning outcomes as they relate to geometry.



Students learn about	Students should be able to
2.5 Synthesis and problem-solving skills	– explore patterns and formulate conjectures – explain findings – justify conclusions – communicate mathematics verbally and in written form – apply their knowledge and skills to solve problems in familiar and unfamiliar contexts – analyse information presented verbally and translate it into mathematical form – devise, select and use appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions.

- The Second Year Teacher Handbook section 12 (page 40) outlines possible lesson ideas for the teaching of Ratio and Proportion:
  - Distinguish between proportional and non-proportional situations.

- Solve problems involving proportional reasoning in different contexts.
- Interpret scaled diagrams.

- Students' are usually successful with the rote learning of Theorem 13 and can apply the theorem to questions that are similar to those given as exemplars in traditional maths classes. However, students' often:
  - Apply the theorem without understanding that it works only when triangles are similar. Essentially they see a shape/diagram that they recognise and apply a technique that they have been told will yield the correct answer.
  - Miss the significance of the relationship between the angles in similar triangles and corresponding sides, leading to a poorer understanding of trigonometric ratios.

Thematic Focus:

- To understand that corresponding sides in similar shapes (specifically triangles) are in proportion.

6. Research

- *Junior Certificate Guidelines for Teachers* (DES 2002, Government Publications Sales Office).
- First Year Handbook (PMDT).
- Second Year Handbook (PMDT).
- Third Year Handbook (PMDT).
- *Junior Certificate Mathematics Syllabus* (DES 2016, Government Publications Sales Office).
- www.projectmaths.ie
- www.nrich.maths.org
- Chief Examiners Report on Junior Certificate Mathematics 2006 (SEC 2006).
- *Literacy and Numeracy for Learning and Life* (DES 2011).

7. About the Unit and the Lesson

Unit 2.1 (synthetic geometry) details the theorems that are required learning at Junior Certificate Ordinary Level. This lesson looks to combine the learning outcomes for unit 2.1 with the problem solving skills detailed in unit 2.5 (see section 5 above for further details). Students have completed the Common Introductory Course in First Year and have a working knowledge of the basic concepts of geometry, angle measure, axioms, Theorems 1-6, triangles and the terms parallel and perpendicular. Lesson 1 provides an opportunity for students to develop the skill of disassembling geometric figures. Lesson 2 provides an

opportunity for students to discover the meaning of mathematical similarity. The learning in lessons 1 and 2 form the prior knowledge for the research lesson.

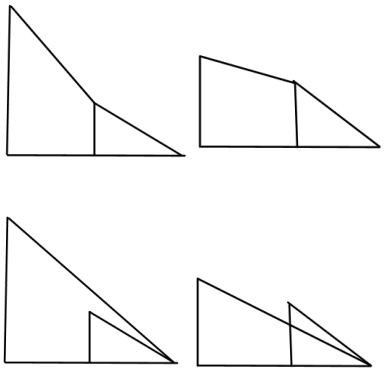
The research lesson uses a contextualised problem that requires students to find missing lengths in a series of similar triangles. The problem is based on the construction of a roof and is presented in phases allowing students to use the skills and mathematical facts they have learned in phase 1, to help them solve more advanced versions of the problem. The problem is designed so that there are multiple solutions, ranging from least to most sophisticated. The most sophisticated solution requires students to use the relationship between corresponding sides in similar triangles to find missing lengths.

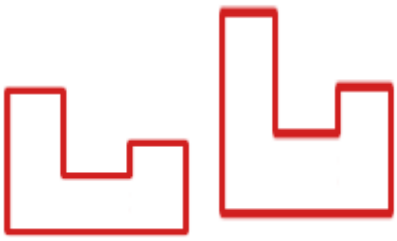
8. Flow of the Unit:

Lesson		# of lesson periods
1	*How many triangles do you see?* See Appendix A for this exercise. This lesson is designed: • To make students aware that their first answer is not always correct. • To encourage flexibility and 'stickability' with respect to geometry problems. • **To encourage students to disassemble geometric figures as a part of the problem solving process.** • To encourage discussion, debate and sharing of reasoning between students.	1-2 Periods
2	*Which shapes are similar?* See Appendix B for this exercise. This lesson is designed to: • **Introduce the concept of mathematical similarity.** • **Make explicit the difference between congruence and similarity.** • **Explore the conditions that must be met in order to label triangles as similar.** • To encourage discussion, debate and sharing of reasoning between students.	1-2 Periods
3	*What is the length of the longest truss?* See Appendix C for this exercise. This lesson is designed to: • **Reinforce the prior learning of disassembling geometric shapes and identification of similarity between triangles.** • **Use the relationship between corresponding sides in similar triangles to find missing lengths.**	Research Lesson 1-2 Periods

9. Flow of the Lesson

Teaching Activity	Points of Consideration
1. Introduction • Welcome students. • Prior knowledge. o How to disassemble geometric shapes. o Mathematical similarity in triangles.	• Teacher sticks laminated poster on the board from previous lesson (How many triangles do you see?). Teacher sticks disassembled triangles on the board beside the poster. • Teacher sticks two similar shapes on the board from the previous lesson. Teacher asks students to recall why the shapes are similar. • Teacher sticks a third shape (not similar) on the board and asks students for reasons why the third shape is not similar to the first.
2. Posing the Task • Teacher sticks new problem (Phase 1) on the board and hands out hard copies to the students. • The teacher guides the students through the contextualised problem on roof trusses. o Teacher explains that a homeowner wishes to extend his porch and is costing the timber that will be required for the job. o There are some timbers that he can measure and they are shown on the diagram. However, as he has forgotten his ladder and there are some timbers that he cannot measure. Students are asked to figure out the length of the longest truss for the builder. • The teacher asks students to work individually for 10 minutes on the task with the goal of developing as many possible ways to do the problem as possible. Teacher moves around the class examining work and engaging with students when appropriate. • Teacher asks students to the board to illustrate their solutions. Solutions should develop from the least sophisticated to most sophisticated. • Teacher sticks the new problem on the board (phase 2).	• The teacher asks students if they understand the problem. • The teacher asks students to read through the problem on their own (1 minute). • The teacher asks if there are any clarifications needed before individual work begins on the task.

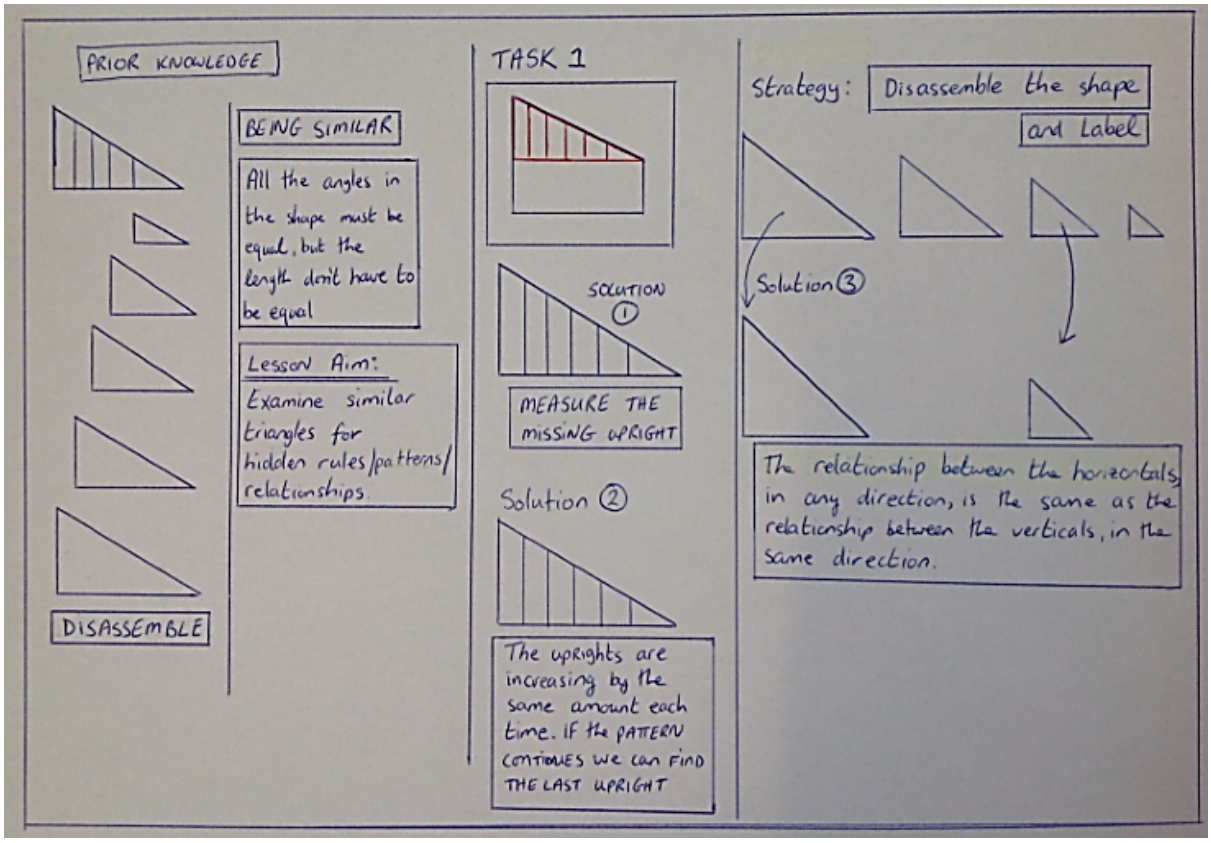
• Teacher may ask students to develop a problem where their new strategy will not work.	• Possible solutions 
3. Anticipated Student Responses Phase 1 • Students may measure the missing lengths. • Students may spot that the lengths of the uprights form an arithmetic sequence. • Students may disassemble the shape, and label the sides of the triangles. o Students may spot that there is a proportional relationship between the sides, particularly between triangles that are not is sequence. Phase 2 • Students may disassemble the diagram. • Students may label the sides of the triangles. • Students may select two appropriate triangles to determine the missing length. • Students may try to use varying pairs of triangles to determine the missing length. Phase 3 • Students may struggle to start this task.	• Students are encouraged to find another method. • Students may be asked if there is a relationship between the horizontals. Students may suggest 1, 1, 1,… teacher may suggest that there is another pattern/relationship. If the student does not disassemble the shape and spot the pattern the teacher may suggest that the prior knowledge could help. • Teacher uses the disassembled triangles to explore the relationship between corresponding sides. This will be referred to as the ‘new strategy’. • Teacher may suggest that the new strategy might help get the problem started. • The teacher should refer to the fact that the triangles must be similar for the new strategy to work. • Teacher suggests to students that they try to alter the original question.

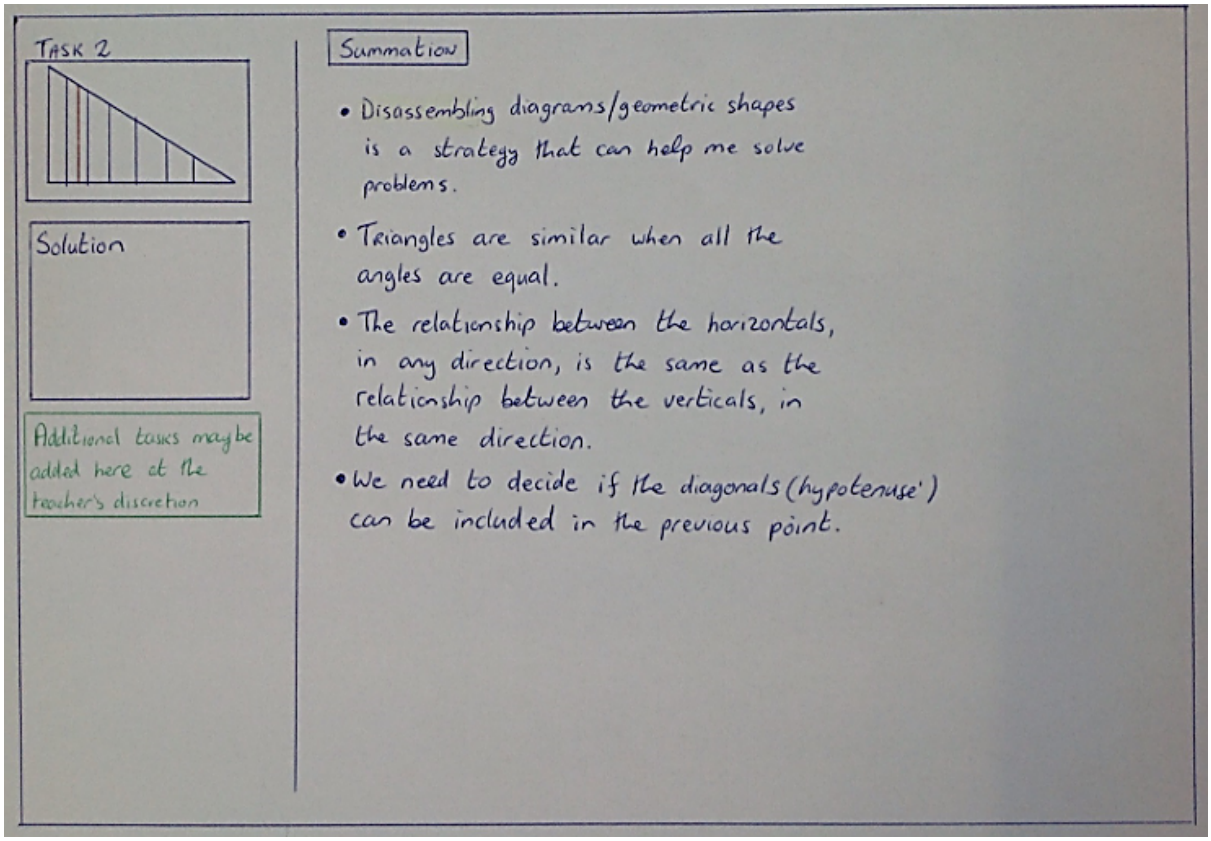
4. Comparing and Discussing Students' solutions should be shared on the board and should develop from least sophisticated to most sophisticated. 1. Measurement 2. Horizontal pattern 3. Upright pattern 4. Corresponding sides	Teacher will focus on the recurring themes of: • Disassembling diagrams • Similarity in triangles
5. Summing up The teacher asks students to write down what they have learned during the lesson. The following should be covered during the lesson summation: 1. That the strategy of disassembling of diagrams/geometric figures is a strategy that can help with problem solving. 2. Triangles are similar when all the angles are equal. 3. When triangles are similar there is a relationship between the sides that can be used to solve for missing lengths. 4. The teacher may ask students to explore whether the new relationship can be applied to the diagonals (hypotenuse').	• Teacher may ask students to express the relationship between corresponding sides in similar triangles in words. • This would likely be a homework task. Alternative homework task if point 4 has been fully explored fully during the lesson: • Further discussion on similarity (although not on the syllabus) may be facilitated by exploring shapes other than triangles. A possible task: Are the following shapes similar? 

10. Evaluation

The lesson is to be observed by two teachers and a Regional Development Officer from the Project Maths Development Team. During the lesson one teacher may navigate the room observing students' work. Samples of work may be photographed. Samples of work may be collected after the lesson. A second teacher may record teacher-student interactions. In particular, evidence that students are assimilating learning from lessons one and two, to aid them during the Research Lesson. Evidence that students are seeing the relevance and usefulness of new strategies should be evident in their efforts to apply the new strategies to extensions of the original problem.

11. Board Plan





12. Post-lesson reflection

Analysis of Lesson Aims

- All of the lesson aims were achieved; however, the degree of success varies depending on the aim being analysed. More specifically;
    - There was a high degree of success with respect to students being more creative when devising approaches to solve problems.
    - There was strong evidence of students taking control of their own learning and becoming independent learners.
    - Students were stimulated and engaged by the task. It was recorded that 65% of class time was student trying to solve the task.

  It was agreed during the post lesson reflection that more frequent use of lessons that promote learning through problem solving are needed to fully achieve lesson aims such as:
    - An appreciation that maths can be used to solve real world problems.
    - An appreciation that maths can be used to communicate thinking effectively.
    - An appreciation that a problem can have several equally likely solutions.

Analysis of Learning Outcomes

1) Identifying similarity between triangles.

- Students had learned about similarity of triangles before the Research Lesson was conducted. The concept of similar triangles was revised as a part of the prior knowledge of the lesson. The majority of students understood the concept, however, not all students saw the presence of this mathematical fact in the task until it was identified for them.
  Similarity in triangles and the disassembling of geometric shapes formed part of the prior knowledge for the task also. Although students had learned these skills prior to the Research Lesson, and were reminded of them during the lesson introduction, some students did not use these skills until prompted to do so by the teacher during the class. This may suggest that students do not see what they learn as a set of skills that can be applied across topics, but rather as individual pieces of information that are relevant only to a particular question/task.

2) Using the relationship between corresponding sides in similar triangles to find missing lengths.

- Between 30%-35% of the class arrived at the correct solution to the original task using the relationship between corresponding sides of similar triangles.
- There was a large uptake of this method by the students in the class when given task two (addition of the extra truss).

Key Observations

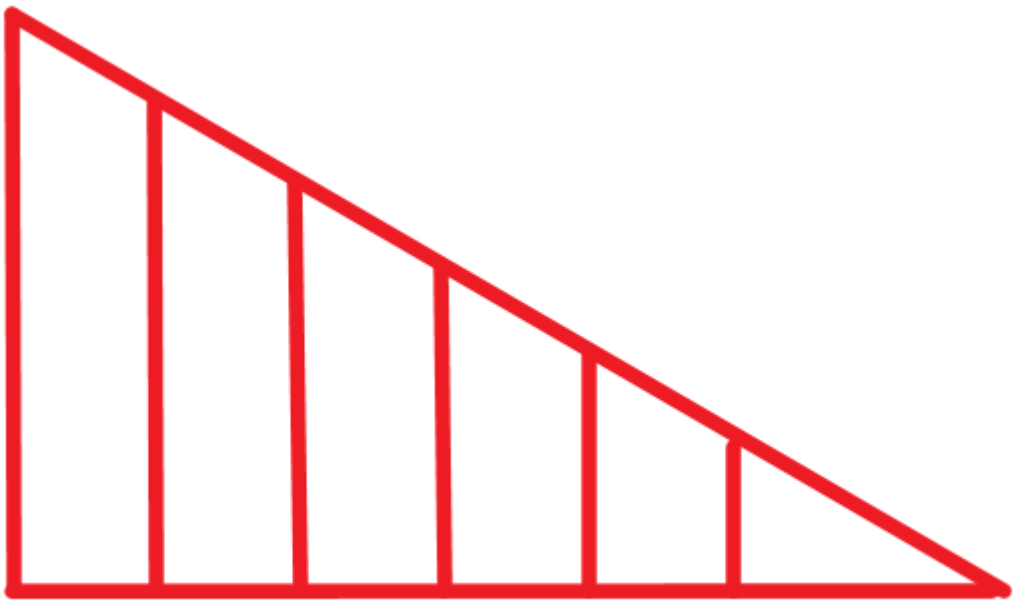
- Students' understanding of scaling is very good. Many students measured the trusses on the diagram in centimeters, identified the scaled relationship between their measurements and the information on the diagram and used it to calculate the length of the missing truss.
- Students' ability to manipulate ratios to calculate missing lengths is very good. A minority of students used the unitary method; the majority used their intuitive understanding of the relationship between the numbers (lengths).
- The accuracy of labeling of diagrams was very good.
- Central to the task is the identification of a multiplication pattern between triangles. There is evidence to suggest that students' have a tendency to be limited to addition when trying to identify patterns.
- Some students applied the formula for the area of a rectangle when dealing with a trapezium.

Adjustments to the Lesson

- Lesson number 2 in the series has been adjusted so that it can be linked more easily to the task in the Research Lesson. Exercise 1 is now exclusively based on right angled triangles.
- It might be useful to ask the students to write out a reflection on what they learned during the class.

Appendix 1 (a).

How many triangles do you see?



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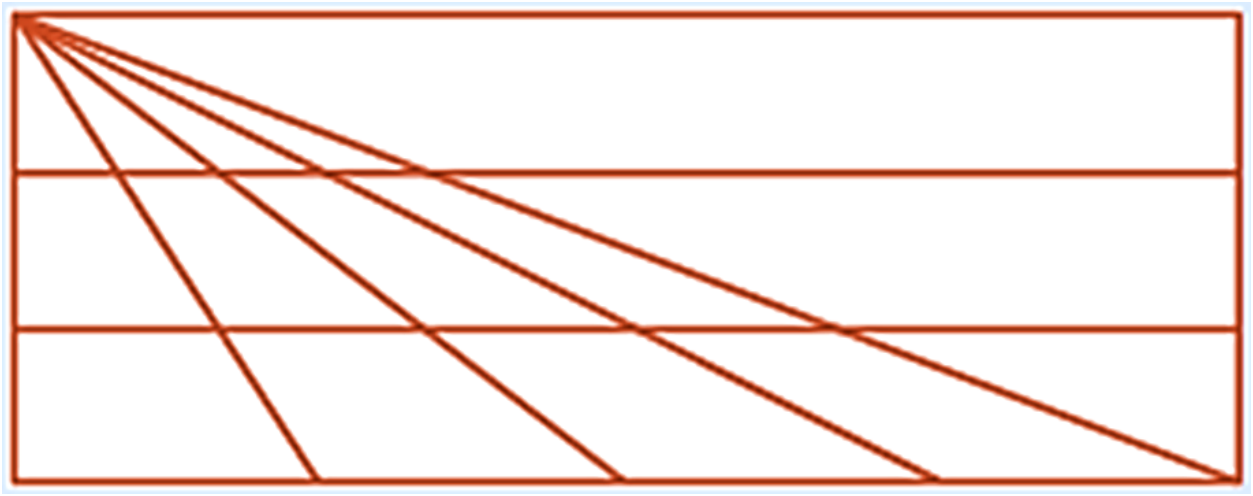
Appendix 1 (b) Extension Activity.

How many triangles do you see?



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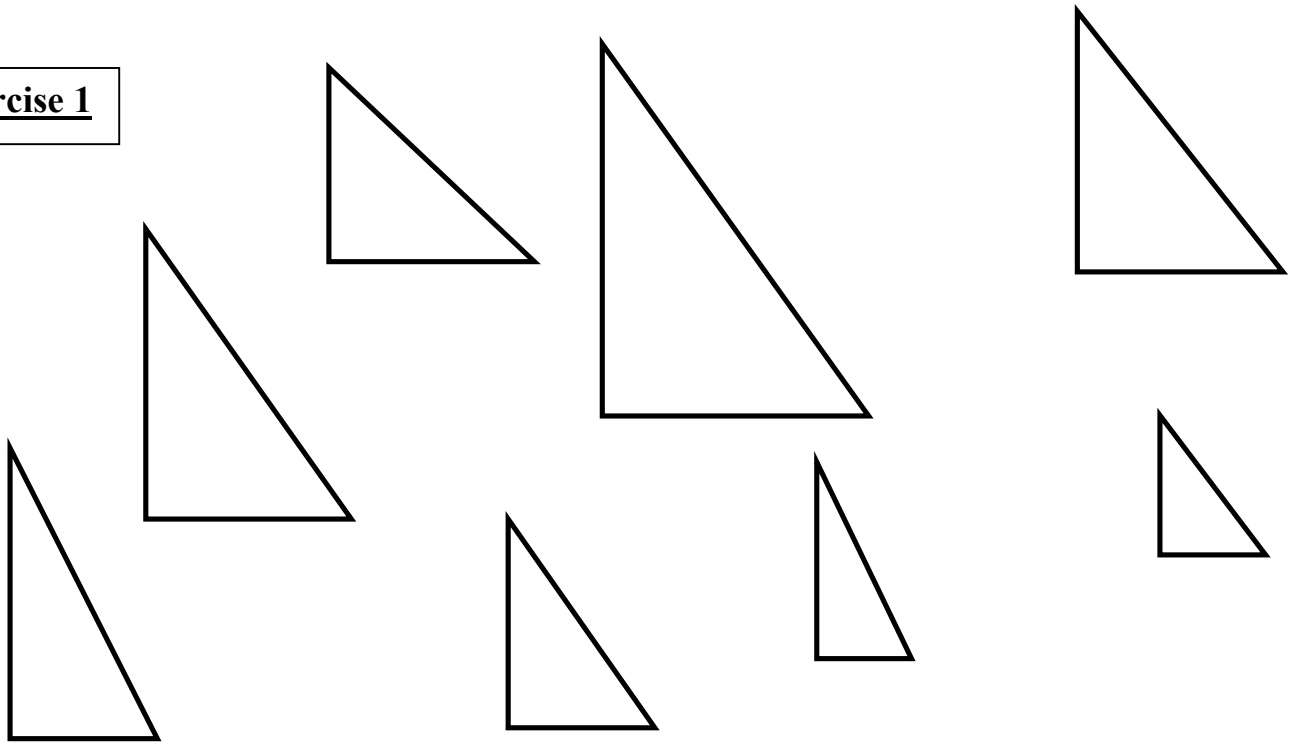
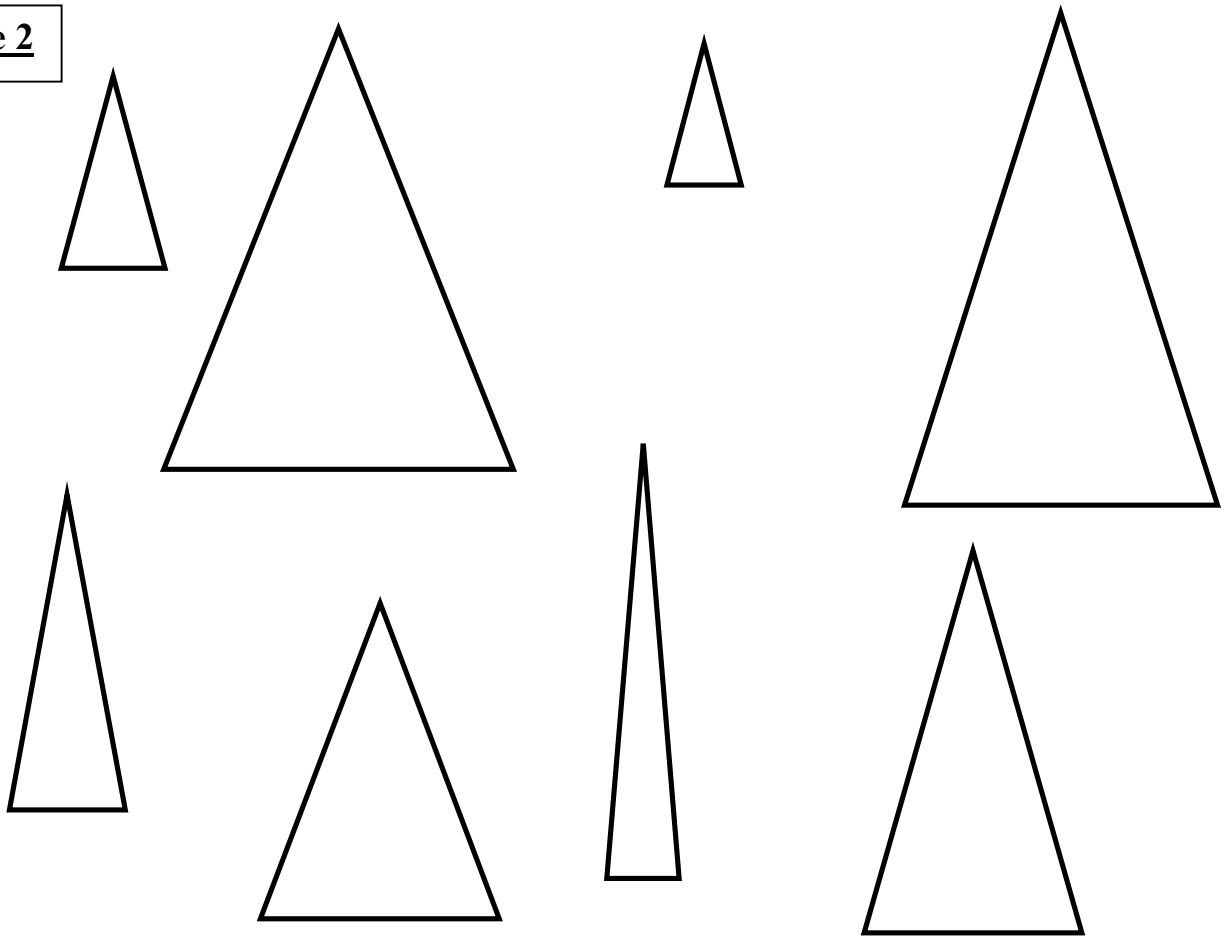
Appendix 2 (a).

Which shapes are mathematically similar?

Exercise 1

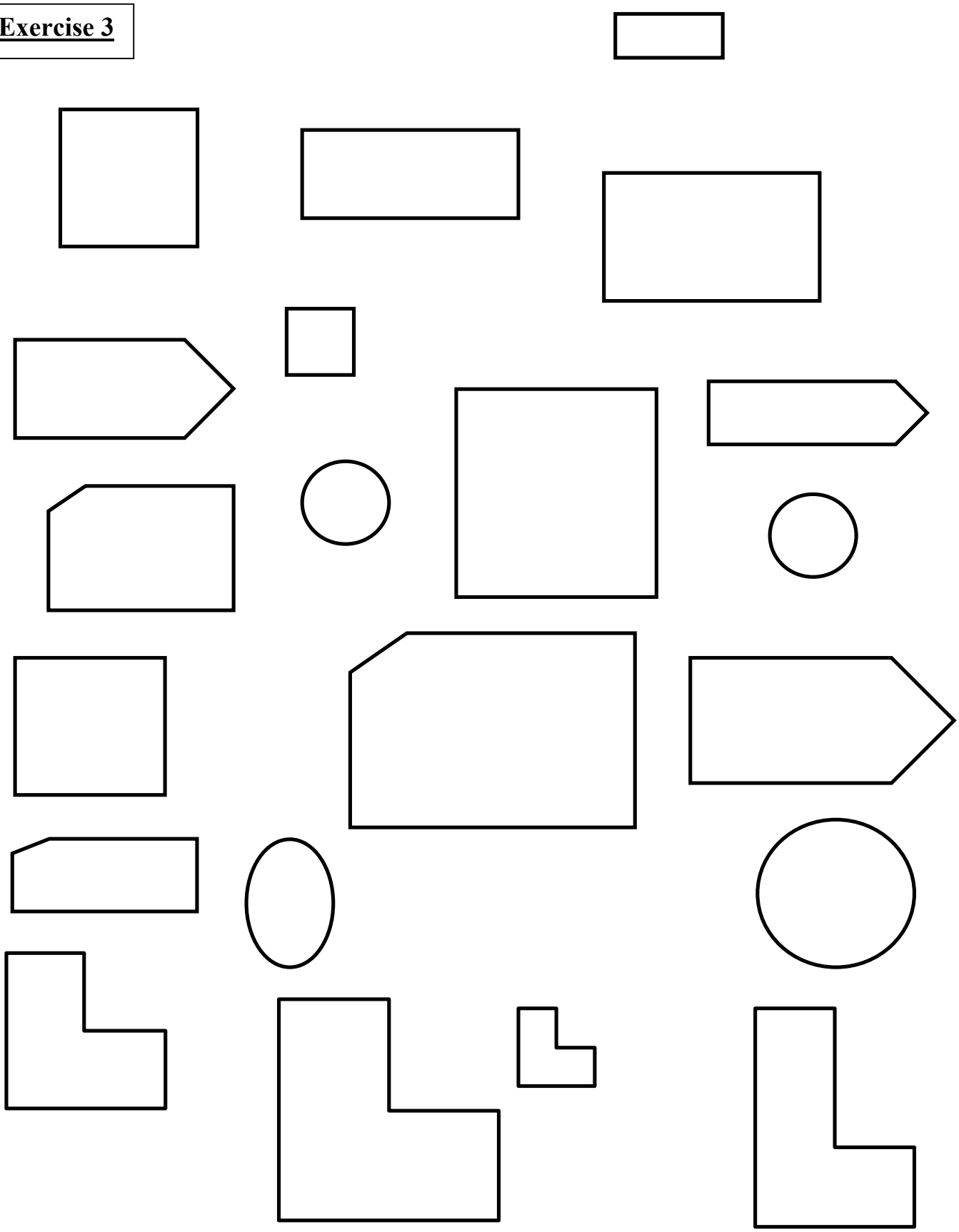


Exercise 2



Which shapes are mathematically similar?

Exercise 3



Appendix 3.

Find the length of the longest truss.

Stephen has recently purchased a new house and has decided to extend the front porch area. He wishes to calculate the amount of timber that he will need to buy so that he can factor this into his costings. He has a measuring tape but no ladder. Using his measuring tape he has managed to take some measurements. They are recorded on the diagram. Can you help him calculate the lengths of the uprights that he cannot reach because he has no ladder?