

## **Many paths from word problems to linear equations**

Main Plan:

|  |                        |                          |
|--|------------------------|--------------------------|
| <b>Date:</b> 22 <sup>nd</sup> January 2016 | <b>Class:</b>          | <b>Room:</b>             |
| <b>Time:</b> 8:00                          | <b>Duration:</b> 70min | <b>Teacher:</b> P. Brady |
| <b>No. of Students:</b>                    | <b>Topic:</b> Algebra  |                          |

### **Lesson plan for first year mixed ability**

**Date of lesson** 22nd Jan. 2016

#### **Aims:**

1. That students see a need for a formal approach, involving variables/unknowns, to solving problems.
2. That students develop confidence in using variables along with numbers in problems requiring solving algebraic equations.
3. They recognise that algebra is being used in everyday life and can simplify a problem into an equation that requires finding an unknown quantity.

#### **Objectives:**

Students will be able to:

1. Recognise and use inverse operations in order to work back to an unknown.
2. Connect and use letters as ‘unknowns’ as a facility for more efficient and effective means to finding numerical solutions to word problems.

#### **Syllabus connections:**

The syllabus states that Strand 3, ‘Number’,

Builds on the ideas about number that learners developed in primary school and facilitates the transition between arithmetic and algebra. (p.21)

In Strand 4, ‘Algebra’, two aspects of algebra are said to be:

A systematic way of expressing generality and abstraction. (p.26)

Two of the objectives are:

Make use of letter symbols for numeric quantities.

Use real life problems as vehicles to motivate the use of algebra and algebraic thinking. (p.26)

### **Prior knowledge:**

1. To have some experience of the meaning of “identity” in the sense of a number that leaves a value unchanged under a certain operation e.g.  $0 + 2 = 2$ ;

$$1 \times 3 = 3.$$

2. Know which operations are the inverse of each other.

Recognise the place holding, rather than fixed, value of variables as demonstrated in previous teaching of algebraic expressions.

3. Use of balance to demonstrate equivalence of 200 millilitres of water and 200 g weight etc.

4. Circle diagrams with numbers back and forth. Back track table.

5. Think of a number problems.

Equipment available:

Balance; mini shapes and arrows

### **Outcomes:**

At the end of this lesson students should be able to:

1. Interpret word problems and translate them into algebraic equations.

2. Solve word equations.

3. Justify/Verify their answers.

4. Be able to explain, represent, their method (thought processes and reasoning).

Teacher will observe any misconceptions and incorporate ways to tackle them in the next lesson(s).

### Possible problems- ‘meaningful task(s)’

When a certain number is multiplied by 5 and the answer is added to 45 the total is 110. What is the number?

When a certain number is multiplied by 6 and then 18 is subtracted from the answer the total is 54. What is the number?

If Mike was 3 times his present age he would be 8 years older than his uncle, who is 37. How old is Mike?

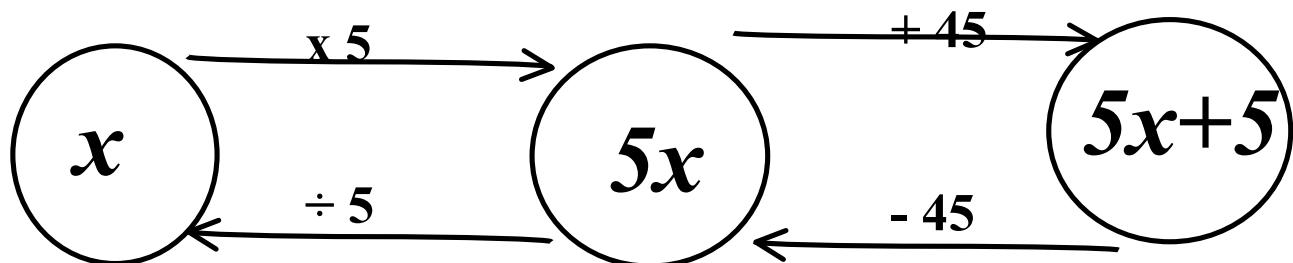
The larger bag of potatoes is 3 times heavier than the smaller bag. The total weight of the two bags is 16 Kg. What weight is the smaller bag?

### Organising and managing discussion

Choice of solutions and the order of their presentation:

1. Intuitive: Allow for a ‘just knowing’ approach where explanation is lacking.
2. Trial and error (pick any number and see if it gives the required answer)
3. Backtracking (opposite/inverse operations) using such methods as a table or  
circle diagrams or numerical representation

| Rules for backtracking (opposite operations) |                |
|--|----------------|
| Original action                              | Inverse action |
| X 5  |                |
| + 45   |                |
| ↓  | ↑              |



### Numerical representation:

$$110 - 45 = 65 \quad 65 \div 5 = 13 \quad \text{check: } 13 \times 5 + 45 = 110$$

4. Formal equation with or without stabilisers, or some use of the balance method-whatever you do to one side you do to the other. (as described in the PM teaching and learning plan).

Chosen students record a solution on the board and a prepared solution is placed beside each. This and the related discussion will help students to focus on the development of methods to solving the problems and provide visual aids to the increasing abstract nature of solutions.

Students will be encouraged to take notes during the presentations so they can extend/elaborate upon their own approaches.

### Resources:

Teaching and learning plans

<http://www.projectmaths.ie/documents/T&L/IntroductionToEquations.pdf>

<https://www.mathsisfun.com/algebra/index.html> for balance simulation

### RESEARCH:

**Hoffmann, M. and Powell, A. (1992)** 'Circle expressions and equations: multivalent pedagogical tools', in: Pimm, D. and Love, E. eds. **Teaching and Learning School Mathematics**. London: The Open University.

## WORKSHEET

Solve the following problem in as many ways as you can, showing all your work. Verify your answer.

**When a certain number is multiplied by 5 and the answer is added to 45 the total is 110. What is the number?**

**Homework:**

Problem 2:

**If Mike was 3 times his present age he would be 9 years older than his uncle, who is 39. How old is Mike?**

**On the day:**

- Student handouts including problem(s) to be attempted and a short questionnaire to be completed at end of class to obtain student feedback.

Questions:

- What did you learn today?
- Did you enjoy this lesson? Why?
- What could be improved?

- Recorder/video camera
- Students' work – take pictures
- Split class in 2 (Mieke and Eilish): write down & take pictures of what we observe and record misconceptions and students' comments/chat when solutions are presented
- Select students to present on whiteboard, based on showing various methods in order of increasing formality.
- As part of the Japanese 'kyozai-kenkyu' approach, lesson preparation includes studying possible misconceptions/misunderstandings that students might have. These are now analysed through discussion and use of prepared cards and teacher recording on the board, helping to visualise the conversation and topic development.

- This ‘Ceardaiocht’ continues with the weaving of the various approaches for the students into a rich tapestry of strategies, skills and connections, gleaned from each other’s insights.
- This is carefully crafted by the teacher directing attention to similarities and differences among solutions and their elegance/sophistication/transferability.
- The homework allows the students an opportunity to consider a similar problem in the light of what they have gained from the day’s class. It is intended that they are better equipped to consider process more than product, conceptual as well as procedural learning and relational as well as instrumental understanding.

| Time   | Teacher activity   | Student Activity   |
|--------|--|--|
| 8:00am | Homework   | Students take out homework and collect it.   |
| 8:05am | <p>Introduction to the class:</p> <ul style="list-style-type: none"> <li>- Our goal for today is to solve word problems not just to get the answer, but to find as many different ways as possible to solve the problem.</li> <li>- “Give me some words related to algebra, we have been doing?”</li> <li>- As we write down those words, check whether you understand those words.</li> </ul> | Students will come up with different words: Unknown, variable, term constant, integer, rational number, natural number, equation, equal, algebraic equation, linear equation, expressions, balance, formula, equivalent, substitute, identity $4 + 2 = 6$ .  |
| 8:15am | <ul style="list-style-type: none"> <li>- I have worksheet for you with a problem. The problem might be quite easy for you but the aim is to find as many different ways as possible to solve the problem.</li> <li>- Make sure you write your name on the top.</li> <li>- You have 10 minutes to come up with as many different ways as possible.</li> </ul>                                   | <ul style="list-style-type: none"> <li>- Students distribute the worksheets.</li> <li>- Students solve the problem in as many ways as possible. <ul style="list-style-type: none"> <li>○</li> </ul> </li> </ul>  |
| 8:17am | <ul style="list-style-type: none"> <li>- Teacher walks around, observes students work and selects the students who will be asked to present their solution on the board.</li> <li>- “Hands up if you have done as many as you can.”</li> <li>- “Hands up if you need a few more minutes.”</li> </ul>   | <p>Possible solutions:</p> <ul style="list-style-type: none"> <li>- Students pick a number and check the answer. Trial and error.</li> <li>- Backtracking <ul style="list-style-type: none"> <li>○ Numerical presentation</li> <li>○ Circle diagram</li> <li>○ Table format</li> </ul> </li> <li>- Write an equation and solve it. <ul style="list-style-type: none"> <li>○ Balancing method</li> <li>○ Stabilizers</li> </ul> </li> </ul> |
| 8:30am | - Teacher calls particular students to   | Students write down their solution on  |

|        |  |   |
|--------|--|---|
|        | <ul style="list-style-type: none"> <li>the board to present their solution</li> <li>- Discussion/connecting concepts</li> </ul>  | <ul style="list-style-type: none"> <li>the board and explain their answer to the class.</li> <li>Observing what teacher writes/draws on board.</li> </ul> |
| 9:00am | <b>Evaluation</b> <ul style="list-style-type: none"> <li>- What did you learn in this lesson?</li> <li>- Did you enjoy the lesson? Why?</li> </ul>                         | Students write their learning on the evaluation sheet   |
| 9:05am | <ul style="list-style-type: none"> <li>- Hand out sheets with Homework: solve 2 word problems in as many ways as possible.</li> <li>- Write homework in Journal</li> </ul> | Students selected to distribute homework sheets.  |