Topic: Coordinate Geometry: Introduction to Slope
Year Group: 1st Year

For the lesson on
28/11/2016
At Naas Community College, Naas, Co. Kildare
1st Year class
Teacher: Eadaoin O'Grady
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1. **Title of the Lesson: '**It's a tough climb'

2. **Brief description of the lesson:** Students explore the concept of slope throughout an investigative manner. This school overlooks the Wicklow Mountains and these will be used as a reference point for developing the lesson. Students will be provided with three unique slopes as part of a Mountain and asked to problem solve to conclude on the steepest slope.

3. **Aims of the Lesson (From the Teacher's Perspective):**

Short-time aims: I'd like my students to…

- Connect and review the concepts that we have studied already,
- Understand the relationship between rise/run and the slope,
- Experience meaningful mathematics i.e. that they see a need for what they are studying,
- Become more creative when devising approaches and methods to solve problems (Key Skill: Being Creative[1]),
- I'd like my students to present their solutions to their classmates (Key Skill: Communicating[1]).

Long-time aims: I'd like my students to…

- Appreciate that mathematics can be used to solve real world problems (Key Skill: Being Numerate[1]),
- Appreciate that mathematics can be used to communicate effectively (Key Skill: Communicating[1]),
- Foster my students to become independent learners (Key Skill: Managing Myself[1]),
- Emphasise that a problem can have several equally valid solutions,
- Build my students' enthusiasm for the subject by engaging them with stimulating activities.

[1]These Key Skills are taken from the National Council for Curriculum and Assessment (NCCA): Key Skills of Junior Cycle

4. Learning Outcomes (From the Student's Perspective):

As a result of studying this topic students will be able to:

a) Calculate the slope of a given line in an investigative manner.
b) Apply prior knowledge to a question in order to discover a solution to the problem.
c) Compare and contrast different solutions to the problem in order to articulate a correct possible solution (Key Skill: Being Creative).
d) Critique possible solutions to best select one solution (Key Skill: Being Creative).
e) Utilise problem solving skills to tackle the problem.
f) Present their solutions to their peers (Key Skill: Communicating)

5. Background and Rationale

Teaching the concept of slope can be quite difficult. Teachers can too often jump to provide students with a brief overview of this concept before providing them with the formulae for calculating this value. However, throughout our Reflections on Practice sessions, we are aiming to construct a more investigative way to teach students this concept.

Traditionally, from our experience, students tend not to have difficulties in utilising the slope formulae such as 'the rise over the run' or the coordinate geometry version, however as teachers we must asked if students have a good understanding of what this concept means? Can they see the connection to the change in y and the change in x; do they have a grasp of the mathematics behind these formulae? The Junior Certificate Syllabus states that students should have a good knowledge of 'coordinating the plane and to be familiar with the properties of lines and line segments including slope'.

The thematic goals of this lesson are to establish a strong investigative ability within the students. We want to start a discussion on different areas of the mathematics curriculum and on the difference concepts, rather than just telling students to utilise formulae as means of solving.

6. Research

The research undertaken incorporated a range of Department of Education and Skill guidelines and other curriculum documents published by the Maths Development Team and the National Council for Curriculum and Assessment.

The specific documents that were reviewed in the constructing of this Lesson Proposal include:

- Key Skills for Junior Cycle, National Council for Curriculum and Assessment (2012) Retrieved from: www.**juniorcycle**.ie/NCCA_**JuniorCycle**/.../**key_skills**_oct_2012_WEB_FINAL.pdf
- Teacher Handbook for First Year Students published by the Maths Development Team Retrieved from: www.**projectmaths**.ie/documents/**handbooks/firstyearhandbook**2015.pdf

- Teacher Handbook for Second Year Students published by the Maths Development Team Retrieved from: www.**projectmaths**.ie/documents/**handbooks**/**secondyearhandbook**2015.pdf
- Mathematics Syllabus for Foundation, Ordinary and Higher Level, Department of Education and Skills (2016) Retrieved from: https://**curriculum**online.ie/getmedia/.../JCSEC18_**Maths**_Examination-in-2016.pdf

7. About the Unit and the Lesson

This unit is classed as Coordinate Geometry of the line. The Common Introductory Course for first year, Lesson Idea 1.21, it elaborates that student content should include 'how to coordinate the plane' and also 'how to locate points on the plane using coordinates'. Lesson Idea 1.23 (CIC) states that students should be able to understand 'change and rate of change linked to slope'. Although, slope is not formally included a part of the CIC course, it is expected to be utilised in the topic of patterns and algebra when in First Year.

In order to best prepare students for their further study of patterns and algebra, it was decided to include the concept of slope under the heading of Coordinate Geometry. In addressing the learning outcomes of both the syllabus and also of those include in the Common Introductory Course, a problem solving approach, including multiple solutions was constructed.

8. Flow of the Unit:

Lesson		# of lesson periods
1	• Introduction to the plane and plotting coordinates ▪ iPad Activity "Stack the Shelves" • The plane, plotting coordinates, connecting coordinates to make line segments, connecting line segments to make shapes/pictures ▪ Worksheet Side 1 - Maple Leaf Coordinate Picture (Included in Appendix Item 1) ▪ Worksheet Side 2 - Dog Coordinate Picture (Included in Appendix Item 1)	2 x 40 min.
2	• Plotting and labelling points, writing (x, y) coordinates of points, connecting points to make shapes/pictures.	1 x 40 min.
3	• Finding the Slope of a Line ▪ What section of the mountain is toughest to climb?	1 x 40 min. **(research lesson)**
4	• Connecting Slope and Speed ▪ Units of Measurement Review ▪ Distance-Time Graph • Relationship between: Average Speed, Change in Distance	3 x 40 min.

	▪ Distance-Time Graph ▪ Formula	
5	● Review of the unit ● A glance ahead to the next unit: Patterns	1 x 40 min.

9. Flow of the Lesson

Teaching Activity	Points of Consideration
1. Introduction Teacher will welcome the students to class. Acknowledge the presence of the observers in the classroom and comment on their presence. Next the teacher will introduce the lesson and the introduction will consist of a range of activities from the previous learning activities. Students will be tasked to engage in a series of small recap activities based on their prior knowledge of Coordinate Geometry (Line). Teacher will also take the class roll call.	Introduction (5 minutes): Teacher recaps on the prior learning including: ● The plane and plotting coordinates ○ Stock the Shelves iPad Activity ● Plotting coordinates, connecting coordinates to make line segments, connecting line segments to make coordinate shapes/pictures ○ Worksheet Side 1 - Maple Leaf Coordinate Picture ○ Worksheet Side 2 - Dog Coordinate Picture
2. Posing the Task The following question was posed to the students (Appendix Item 2). **The photo shows a mountain with three different sections.** **Which section of the mountain is the toughest to climb?** ● Tell students that they are going to be given 10 minutes to solve a problem. They will be given one minute to ask the teacher questions (after they are shown the problem and before they	Students will not be provided with hints or instructions but will be promote to utilise their knowledge of problem solving skills including drawing a diagram, trial and error, acting it out,

officially start solving it) ● Explanation of Group Work Method: Think-Pair-Share ○ Each student will be provided with a printout of the problem, a placemat, and a magnet/blue tack. ○ Each pair of students will be provided with a larger placemat ○ Students will be given three minutes to **think** individually about solving the problem ○ Students will be given three minutes to **pair and share** their solutions with their team-mate, and decide on their favourite one. ○ Students will be given four minutes to prepare how they will share their solution with the rest of their friends ● Brief talk about the Wicklow Mountains that can be seen out the window. Mention that some parts are tougher to climb than others and therefore more challenging to trek, and that some parts are not as challenging. ● Display Question ● Instruct students that they have one minute to look at the question and to prepare any questions that they might like to ask about it before officially starting to solve it.	eliminate possibilities, draw a table, looking for a pattern etc. Students will receive praise and encouragement throughout the process. Questions including: How can you write that is formal language? What is the pattern that you have here? What do you notice about that table/diagram/shape? What other way can you solve the problem in?
3. Anticipated Student Responses (R) *R1 - Descriptive Passage* Students writing a descriptive passage in English comparing the three line segments and their steepness. *R2 - Graphs and Descriptions* Students identify each graph and discuss their unique steepness in a written manner.	In anticipation of the student responses, a series of A3 posters have been developed to replicate the possible answers. These posters contain limited information as to ensure that students can present to their class mates on their method of solution. Throughout the activity: ● *Teacher to note students responses in order of difficulty from R1 to R9* ● *Group work should prevent students*

R3 - Using Tables Students identifying the arithmetic pattern and the fact that the pattern increases by 2/3 at each unit interval. *R4 - Using Triangles and Comparing* Students will divide the slopes into two unique slopes and then start to discover the connection between slope and triangles. Starting to compare height and width. *R5 - Measuring Angles* Students use a protractor to compare the angles of the three lines and their unique angles of elevation. *R6 - Area of a Triangle* Students calculate the area of each of the triangles and conclude on the steepness based on area. *R7 - Difference in X and Y* Students will develop a sense of a change of x and a change in y. Developing a sense of delta y and delta x. *R8 - Formula* Students conclude on one of the following: $y=mx+c$ $y=riserun$ $m= y2-y1x2-x1$ *R9 – Cut and Compare* Students cut out the lines using a scissor and overlap them to compare steepness.	*getting stuck and/or help them if their solutions are incorrect* ● *Extension problem* for students who finish early

4. Comparing and Discussing (15 minutes) R1 - Descriptive Passage R2 - Graphs and Descriptions R3 - Using Tables R4 - Using Triangles and Comparing R5 - Measuring Angles R6 - Area of a Triangle R7 - Difference in X and Y R8 - Formula R9 – Cut and Compare R10 - Other	The discussion will be carefully planned based on the responses generated by the students. Due to time constraints, the teacher will select different student responses that demonstrate different ways of solving this problem. Student's responses will be ranked based on their complexity and any common misconceptions will be examined and discussed. This will be repeated for each possible solution.
5. Summing up In summing up, the teacher will recap on the solutions as constructed by the students. In will be vital that all the solutions are discussed before formulating on the most common solution. The teacher will ask questions to the class on these solutions. It is envisaged that the 'bones' of a formula will emerge and as a result, this will be reinforced.	The teacher will ensure that any common misconceptions will be carefully addressed to dismiss these from the student's knowledge. The teacher will also highlight the use of a variety of solutions to the one question and congratulate students on their success in this lesson.

10. Evaluation

- <u>What is your plan for observing students?</u>
  Students will be observed by three teachers. One of these teachers also teaches in this host school and as a result, the students will be familiar with her. The observing teachers will observe using a multiple of methods namely; Lesson Study App and by hand with pen and paper. The observing teachers will not inferred with the students but will visit different student clusters. Teachers will note any interesting information or solutions. Teachers will also note any misconceptions.

- <u>Discuss logistical issues such as who will observe, what will be observed, how to record</u>

data, etc.

The three teachers will all be responsible for observation throughout the lesson. In particular, the observing teachers will be observing interesting solutions or approaches to the solution of this question. The observers will also listen to the student's use of mathematical literacy and in their ability to orally describe mathematics.

A seating plan of all students has been formulated and as a result, the observers will record data based on the individual name of the students. One observer has decided to utilise a digital approach to the recording of lesson study, where the other observers have decided to utilise a paper record.

Data will be recorded in a swift manner, by using abbreviations and short hand to ensure the capturing of accurate information from each student cluster.

- What observational strategies will you use (e.g., notes related to lesson plan, questions they ask,)?

  The observers will record notes and important issue that relate to the methods for solving this question. Observers will especially note and record questions that students ask, the appropriate use of mathematical literature, any misconceptions and their root causes (where applicable), any potential causes of concerns or evidence of success will also be recorded.

- What types of student thinking and behaviour will observers focus on?

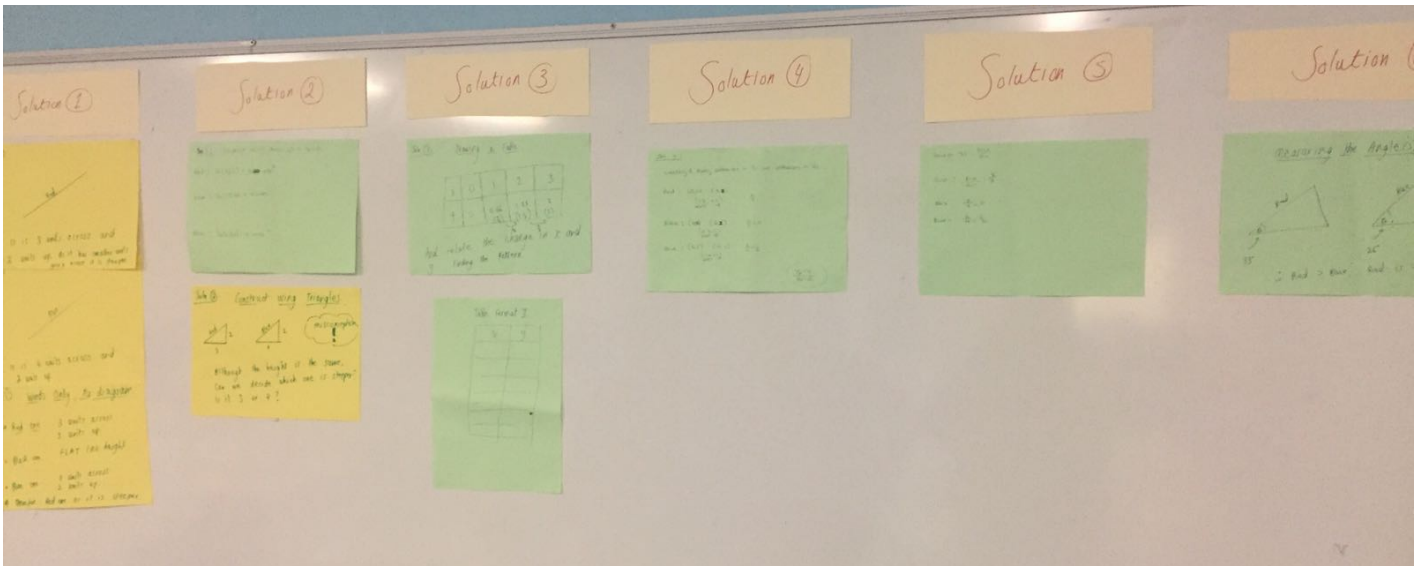
  Observers will focus on the students' abilities of problem-solving and in their utilisation of their problem solving knowledge. Student thinking that involves different approaches and techniques will also be commented on. In particular, students' use of different aspects of Maths will be noted. For example, a trigonometry aspect, a synthetic geometry approach, a logical approach and any other applicable approaches will be focused on.

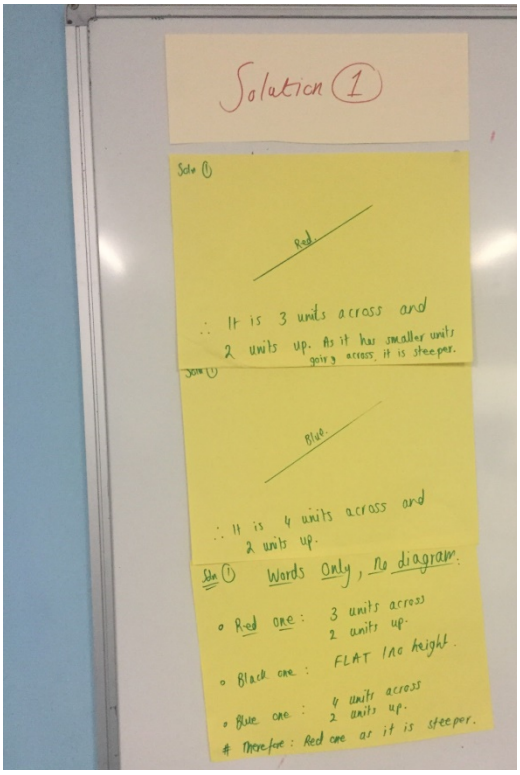
- What additional kinds of evidence will be collected (e.g., student work and performance related to the learning goal)?

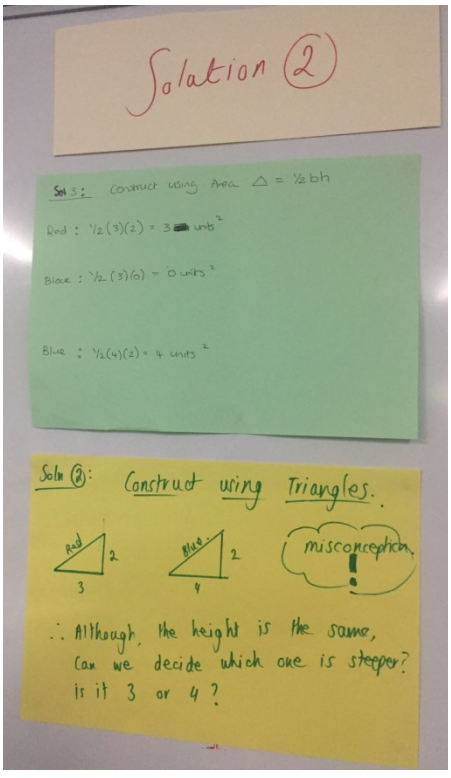
  Students will complete 'show-me' boards on their Think-Pair-Share activity, in addition to the problem sheet and also the larger A3 sized sheets (in which students will answer on the board). All of these will act as evidence of the student's ability to investigate the concept of slope and also their ability to reach the learning goal.

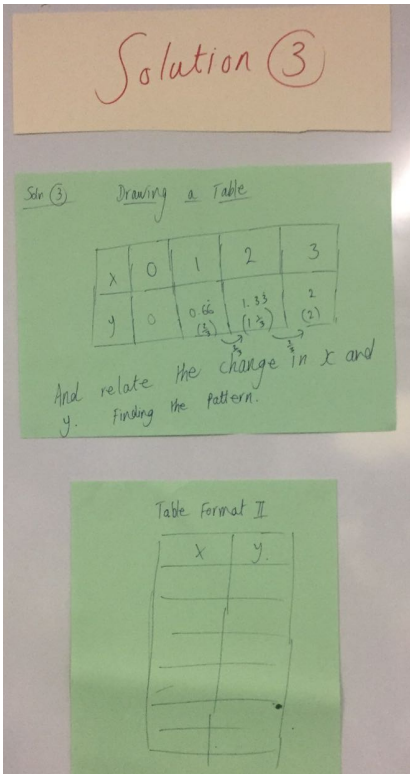
11. Board Plan

The following pictures are the suggested plan for the board work:









Solution (4)

Sol 4 :

Labelling & finding differences in Ys and differences in Xs

Red : (0,0) (3,2)
(+3 across, +2 up) $\frac{2}{3}$

Black : (3,2) (6,2) $\frac{0}{3} = 0$
(+3, 0 across, 0 up)

Blue : (6,2) (10,4) $\frac{2}{4} = \frac{1}{2}$
(+4 across, +2 up)

$\frac{Y_2 - Y_1}{X_2 - X_1}$

Solution (5)

Solution 5: $\frac{\text{Rise}}{\text{Run}}$

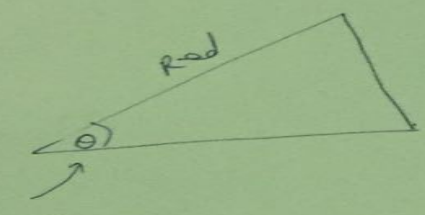
Red : $\frac{\text{Rise}}{\text{Run}} = \frac{2}{3}$

Black : $\frac{0}{3} = 0$

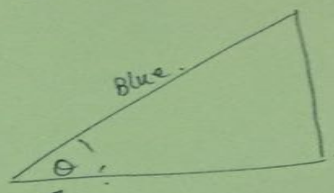
Blue = $\frac{2}{4} = \frac{1}{2}$

Solution (6)

Measuring the Angle(s) :



33°



26°

∴ Red > Blue, Red is steeper.

12. Post-lesson reflection

- What are the major patterns and tendencies in the evidence? Discuss

The major and initially pattern that emerged was the use of words and descriptions. The majority of students used descriptors such as '2 across and 3 up' and there was a strong connection to mathematical literacy words including 'steepness', 'slope', 'altitude', and 'gradient'. Throughout the discussion, one student was trying to connect the 'up' and 'across' values of the line segment together, in order to develop a formula.

As students were provide with geometry sets, another major pattern was students measuring the angles of each of the three lines and comparing that the greater angle will be the tougher one to climb. This solution enables students to develop an understanding of the flat line and the slope value of zero.

- What are the key observations or representative examples of student learning and thinking?

Initially the students connected external variables into their solutions. These included the use of logical, that the last section of the mountain will be the tough due to the impact of tiredness and the lack of oxygen at a higher peak of the mountain. Students also were quick to examine the impact of energy on the hiker. Students were unexpectedly connecting this question to geography and science, which was excellent from a cross-curricular point of view and could allow for further development of lesson.

- What does the evidence suggest about student thinking such as their misconceptions, difficulties, confusion, insights, surprising ideas, etc.?

The evidence suggests that students were aware of the fact that the length of the line does not always refer to its steepness. The team are confident that all students are now aware of this misconception. Additionally, there were some difficulties in measuring angles and measuring the correct angles of the line. This aspect of the course was not formally covered by the class teacher and as a result, the students were relying on their knowledge of geometry from Primary School.

One surprise was in the use of the student's mathematical language, the common occurrence of the word 'gradient' was noted intermediately. Furthermore, there was one solution that the team did not recognise during their preparations. One student, during the discussion, decided to extend both of the lines and as a result, the steeper slope was evident. Another student, during the discussion started to connect this question to a ratio solution, which examined the constant increase in the

pattern. However, as the pattern was increasing by *23* this caused some difficulties in his calculations.

- In what ways did students achieve or not achieve the learning goals?

Students achieved the majority of the goals and at the end of the lesson; a formula for slope was achieved. However, some assistance and input from the teacher was needed during the discussion stage. The closest the students got to discovering this formula was in describing the connection between 'up' and 'across' of the boxes. On conclusion, the main goal was achieved to a certain point and it is recommended that more time (or another class) would allow for the formal concept of *(x1,y1)* and *(x2,y2)*.

Students did utilise different methods to approach this question, which was fantastic. The discussion of different strategies and methods, along with the dispelling of misconceptions, was agreed at the 'share' stage of the Think-Pair-Share activity. Additionally, the team would probably remove the geometry sets from the students' tables and it would be preferred to focus on a co-ordinate geometry approach rather than the use of angles.

- Based on your analysis, how would you change or revise the lesson?

On reflection, time was the biggest issue faced throughout our lesson study. The host school has forty minute lessons and as a result the team concluded that we would have liked more time to further advance the discussion. The team recognised that a one hour class for Lesson Study would have been ideal. The team suggested removing the individual element from the Think-Pair-Share as a way of increasing the time available for the discussion and to allow more time to advance the rise over the run formula for slope.

Initially the team has thought the inclusion of a 0.5 scale on the XY plane would enable students to gain familiarity with rational numbers; however the majority of students counted the boxes on the plane rather than recognising that every two boxes was one unit. On reflection, as these students are still advancing their fundamental knowledge of mathematics, the team would consider the removal of the 0.5 scale.

As this school is an iPad school, it is suggested to involve some individual/group work on Lesson Study that also was connected to students using their iPad. The team did not conclude on the exact use of this resource. There was a small numerical error on the board during the discussion, which

was unnoticed for a period of time; it is inevitable that there will always be some human error involved. Once noted, this error was swiftly rectify and altered.

Lastly, the team sought some form of individual reflection on the process as a whole, it was suggested that maybe ‘two stars and a wish’ would be utilised in further studies.

- What are the implications for teaching in your field?

On reflection, all of the team members have concluded that they prefer and enjoyed the approach of ‘asking one question’ and waiting for several solutions to be generated by the students on their own knowledge. This approach of Lesson Study enabled the students to draw together different aspects of their mathematical knowledge and to fortify their ability to be problem solvers.

The teachers strongly agreed with the ‘training’ of students in the paradigm that there are (normally) more than one way to find a solution to a mathematical problem. The inclusion of both individual work and group work enabled the students to progress on their ability to seek clarification on mathematical concepts and to utilise their mathematical literacy in the process. It also highlights that the answer is not the most important element of mathematics, but the workings are fundamental.

Lesson Study also has enabled students to progress on the ‘Key Skills’ of the curriculum, such as their ability to present on their work and to work with others. In addition to other skills such as communication, bringing an idea from conception to realisation and being creative.

Overall, all members stated that they will continue to teach throughout the medium of Lesson Study. It was highlighted about the amount of preparation needed to construct such a high standard lesson and as a result the inclusion of Lesson Study once throughout a topic or a term was suggested. The ‘recycle’ of previous Lesson Study plans to suitably meet the needs of other class groups and years was also discussed as a viable option.

13. Appendix Items:

Item 1: Prior Knowledge Worksheet One, this worksheet was generated by www.mathworksheets4kids.com

Plotting Points

A) Write the point that is located at each ordered pair.

1) (-1, -4) _____ 2) (-3, 3) _____

3) (2, 5) _____ 4) (5, 1) _____

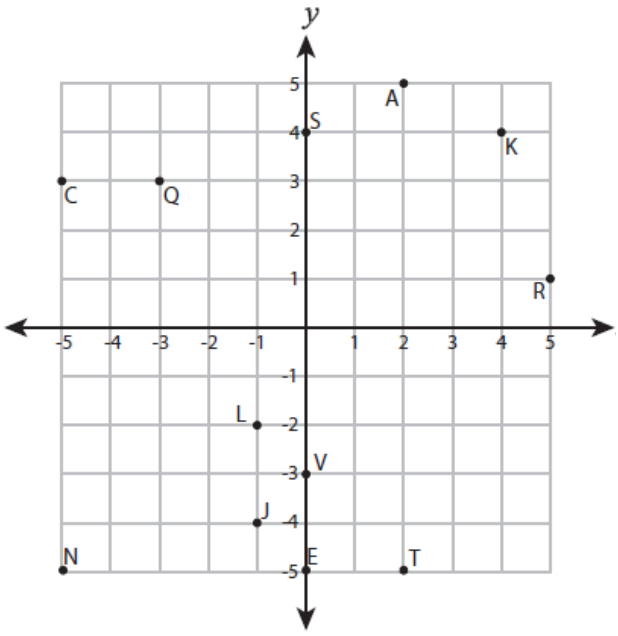
5) (0, -5) _____ 6) (-5, -5) _____

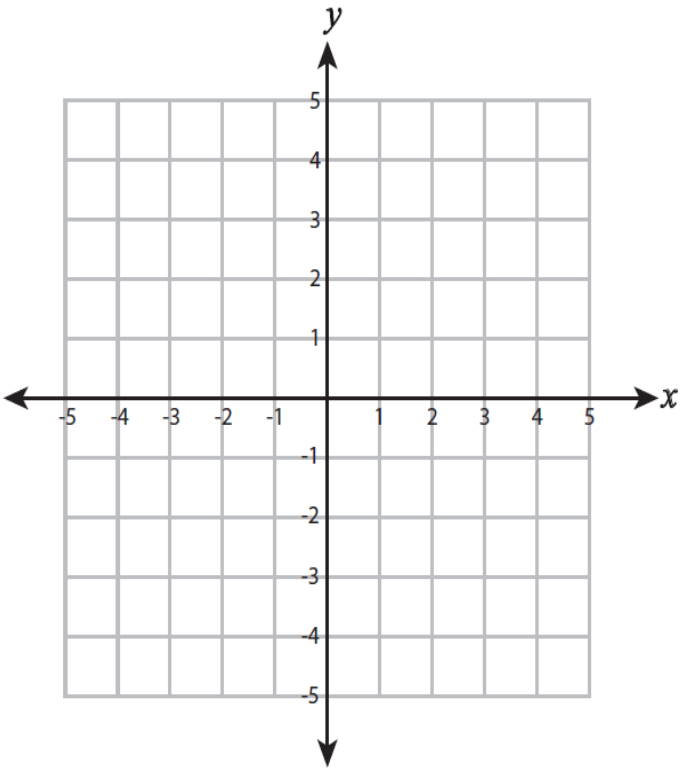
B) Write the ordered pair for each point.

7) L(___ , ___) 8) K(___ , ___)

9) V(___ , ___) 10) T(___ , ___)

11) C(___ , ___) 12) S(___ , ___)





C) Plot each point on the coordinate grid.

13) H(-5, -3)

14) Z(0, 1)

15) D(4, -4)

16) P(-4, 2)

17) Y(3, 5)

18) M(-3, -1)

D) Draw each shape on the coordinate grid.

19) Draw ◯ at (-1, -4)

20) Draw △ at (-2, 5)

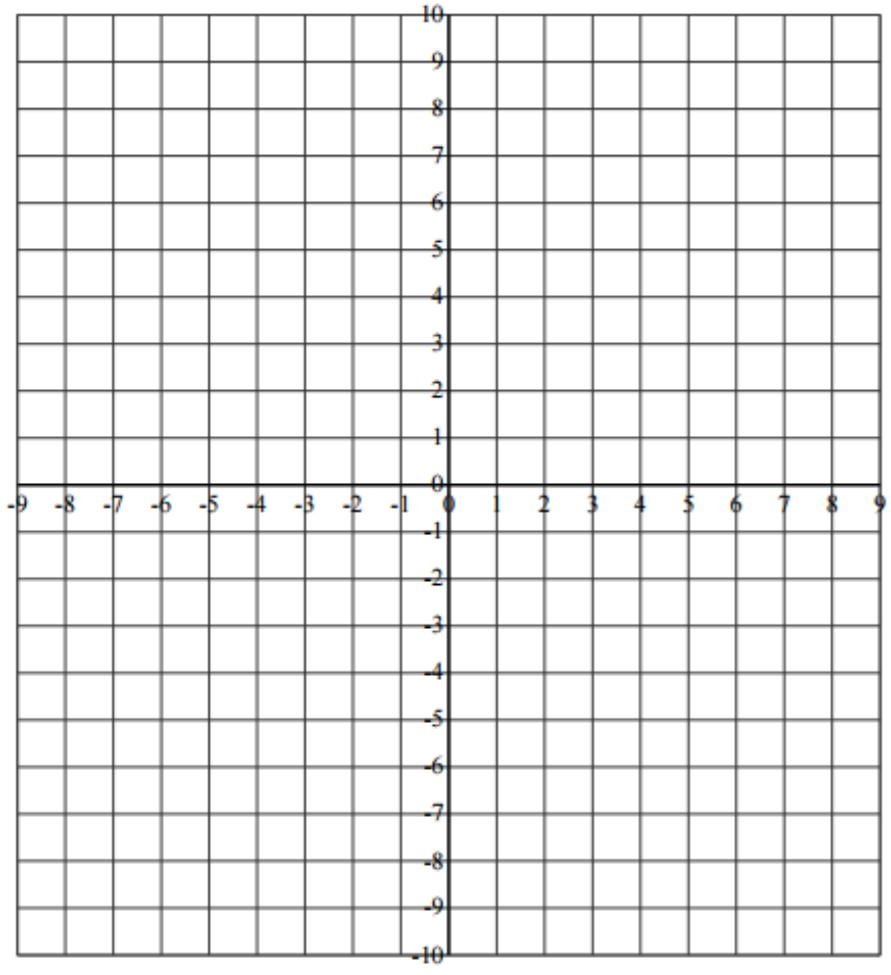
21) Draw ☆ at (5, -5)

Item 1: Prior Knowledge Worksheet Two, this worksheet was generated by www.Math-Drills.com

Coordinate Grid Picture (A)

Plot the points in the order given and join them with line segments.

(1, -3), (5, -4), (4, -3), (9, 1), (7, 2), (8, 5), (5, 4), (5, 5), (3, 4), (4, 9), (2, 7), (0, 10), (-2, 7), (-4, 8), (-3, 3), (-5, 6), (-5, 4), (-8, 5), (-7, 2), (-9, 1), (-4, -3), (-5, -4), (0, -3), (2, -7), (2, -6), (1, -3)



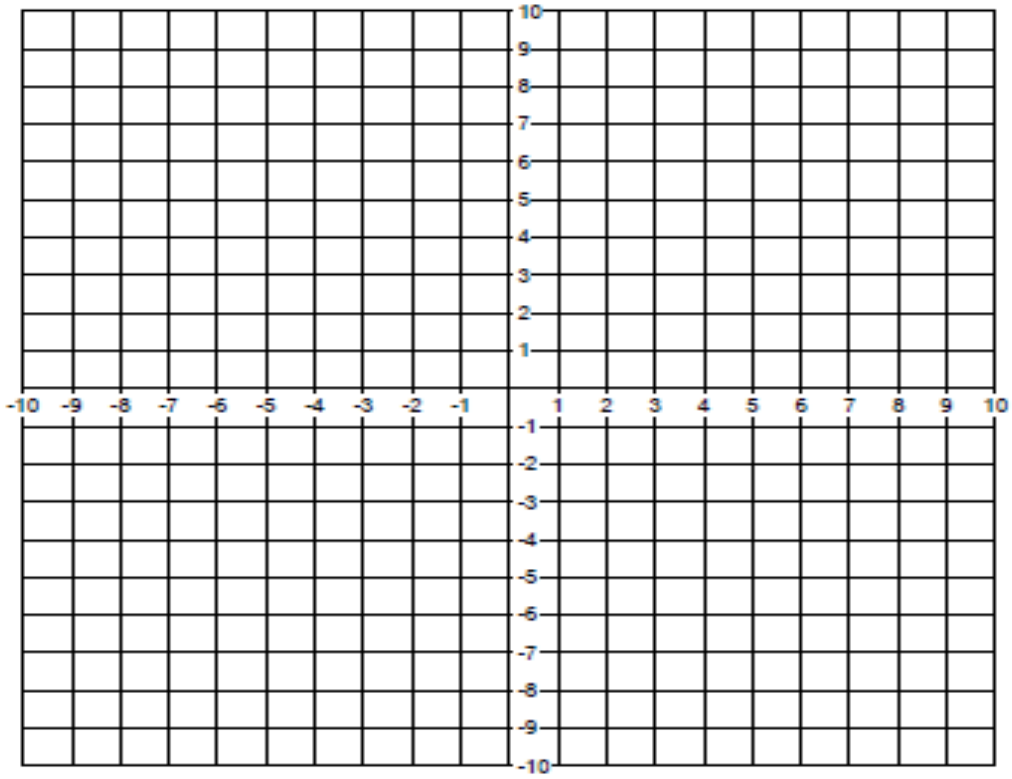
Math-Drills.com

Item 1: Prior Knowledge Worksheet Three, this worksheet was generated by www.worksheetworks.com

Coordinate Picture

Name: _____ **Date:** _____

There is a picture hidden in this grid. Connect the points with lines to reveal it.



Line 1: (6,2), (6,3), (7,2), (6,2) **Line 2**: (-7,2), (-9,-2), (-8,-5), (-8,-7), (-7,-10), (-5,-10), (-6,-7), (-4,-4), (-1,-5) **Line 3**: (2,-5), (3,-9), (5,-9), (4,-6), (5,-4) **Line 4**: (4,-1), (6,1), (4,1), (6,-1) **Line 5**: (-4,-1), (-4,-4), (-5,-6), (-4,-9), (-2,-9), (-3,-6), (-2,-5) **Line 6**: (3,2), (4,3), (4,2), (3,2) **Line 7**: (1,3), (-1,2), (-6,2), (-8,4), (-9,8), (-10,6), (-10,3), (-8,0) **Line 8**: (0,0), (2,6), (0,8), (3,9), (3,6), (7,6), (7,9), (10,8), (8,6), (10,0), (5,-2), (0,0) **Line 9**: (8,-1), (6,-3), (4,-5), (2,-5), (1,-7), (2,-10), (0,-10), (-1,-6), (-1,-3), (0,-2) **Line 10**: (3,0), (5,-2), (7,0)

Item 2: The question for this Lesson Proposal

The photo shows a mountain with three different sections.

Which section of the mountain is the toughest to climb?