Anyone for Pizza?

For the lesson on 18th January 2016
At: Pobalscoil Inbhear Scéine, Kenmare
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Title of the Lesson

Anyone for Pizza? Solving Simultaneous Equations

Brief description of the lesson

Students are posed with a real life problem in a written, verbal and visual format. Through the process of discovery learning and active methodologies they are encouraged to develop possible ways of solving this problem. Students should be able to use simultaneous equations to represent and solve the problem through the process of elimination.

Aims of the Lesson:

Short term goals: For students to be able:

- to convert words into mathematical expressions understanding that letters can be used instead of words or phrases (Key: Being Numerate: Expressing ideas mathematically),
- understand the relevance of key terms such as 'trial and error', 'the difference is', 'substitution', 'subtraction', 'elimination' (Key Skill: Being Literate: Developing my understanding and employment of words and language)
- to solve a system of equations using the substitution method
- to solve a system of equations when no multiplication is necessary to eliminate a variable
- to solve a system of equations when multiplication is necessary to eliminate a variable

Long term goals: We would like our students to be able to:

- apply their knowledge and skills to solve problems in unfamiliar contexts. (Key Skill: Managing Information and Thinking)
- analyse information presented as a story and translate it into mathematical form.
- appreciate that algebra is a tool for making sense of certain situations.
- become independent learners.
- become more creative when devising approaches and methods to solve problems. (Key Skill: Being Creative: Exploring options and alternatives)
- devise, select and use appropriate mathematical techniques to process information and to draw relevant conclusions. (Key Skill: Managing Information and Thinking)

Learning Outcomes:

As a result of studying these topic students will be able to use mathematical reasoning to transform a story into a numerical expression to discover values for variables using the process of elimination.

Background and Rationale

(a) What the students need to learn according to the syllabus;

4.4 Understanding Algebraic Relationships: Features of a relationship and how these features appear in the different representations. Constant rate of change: linear relationships. Non-constant rate of change: quadratic relationships. Proportional relationships.

4.6 Expressions: Using letters to represent quantities that are variable. Arithmetic operations on expressions; applications to real life contexts. Transformational activities: collecting like terms, simplifying expressions, substituting, expanding and factoring.

4.7 Equations and inequalities: Selecting and using suitable strategies (graphic, numeric, algebraic, mental) for finding solutions to equations and inequalities. They identify the necessary information, represent problems mathematically, making correct use of symbols, words, diagrams, tables and graphs.

(b) Difficulties students have had in the past with the subject matter

Teachers Experience*:*

- Students lacks comprehension in relation to the elimination process. They see the process as a series of steps which are to be followed. Thus, without true understanding confusion later arises regarding which variables need eliminating and how to go about eliminating those variables.
- Students are unable to realise that, in some equation, multiplication is unnecessary or on the contrary all the terms in the equation need to be multiplied and not just those to be eliminated.
- Students fail to remember to find the second variable or else do not know how to find the value of the second variable having found the first.
- Students are thrown off by equations not in standard format which require adjusting.
- Students are unable to tackle equations involving fractions.

Chief Examiners Report 2015:

Most candidates also had difficulty solving equations (Question 11(a) and (c)). When required to solve a linear equation in part (a), many simply moved terms, unchanged, from one side of the equation to the other. Candidates also had difficulty solving the simultaneous equations in part (c). Many candidates used trial and improvement here rather than algebraic manipulation, but generally only substituted the correct values into one of the equations rather than into both of them. Those who did use algebraic manipulation often displayed a general idea of what to do, but were unable to carry out the procedures accurately and usually stopped when they had found a value for x only.

(c) The thematic focus of this lesson study, i.e. larger (see above in number 3 for ideas) goals the team will try to address, and why.

- apply their knowledge and skills to solve problems in unfamiliar contexts.
- analyse information presented as a story and translate it into mathematical form.
- appreciate that algebra is a tool for making sense of certain situations.
- become independent learners.
- become more creative when devising approaches and methods to solve problems.
- devise, select and use appropriate mathematical techniques to process information and to draw relevant conclusions.

Research (Bibliography)

- Several Junior Cycle Maths textbooks.
- NCCA (2013). Junior Certificate Mathematics Syllabus: Foundation, ordinary and higher level, for examination from 2016. Dublin: DES
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- Maths Development Team Website, www.projectmaths.ie (2015). Maths Counts 2015. [ONLINE] Available at: http://www.projectmaths.ie/forteachers/conferences/maths-counts-2015/. [Accessed 20th October 16]
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About the Unit and the Lesson

The content of this lesson relates to Topics 4.4 Examining Algebraic Relationships particularly the learning outcome which states "decide if two linear relationships have a common value" (as highlighted below), 4.6 Expressions and 4.7 Inequalities. These topics are included in Strand 4: Algebra in the current Junior Certificate Syllabus which broadly aims to:

- make use of letter symbols for numeric quantities
- emphasise relationship-based algebra
- connect graphical and symbolic representations of algebraic concepts
- use real life problems as vehicles to motivate the use of algebra and algebraic thinking
- use appropriate graphing technologies (calculators, computer software) throughout the strand activities.

4.4 Examining algebraic relationships	Features of a relationship and how these features appear in the different representations. Constant rate of change: linear relationships. Non-constant rate of change: quadratic relationships. Proportional relationships.	– show that relations have features that can be represented in a variety of ways – distinguish those features that are especially useful to identify and point out how those features appear in different representations: in tables, graphs, physical models, and formulas expressed in words, and algebraically – use the representations to reason about the situation from which the relationship is derived and communicate their thinking to others – recognise that a distinguishing feature of quadratic relations is the way change varies – discuss rate of change and the y-intercept; consider how these relate to the context from which the relationship is derived, and identify how they can appear in a table, in a graph and in a formula – decide if two linear relations have a common value – investigate relations of the form $y=mx$ and $y=mx+c$ – recognise problems involving direct proportion and identify the necessary information to solve them

4.6 Expressions	Using letters to represent quantities that are variable. Arithmetic operations on expressions; applications to real life contexts. Transformational activities: collecting like terms, simplifying expressions, substituting, expanding and factoring.	– evaluate expressions of the form • $ax + by$ • $a(x + y)$ • $x^2 + bx + c$ • $\frac{ax + by}{cx + dy}$ • axy where $a, b, c, d, x, y \in \mathbf{Z}$ • $\mathbf{ax^2 + bx + c}$ • $\mathbf{x^3 + bx^2 + cx + d}$ **where $a, b, c, d, x, y \in \mathbf{Q}$** – add and subtract simple algebraic expressions of forms such as: • $(ax + by + c) \pm (dx + ey + f)$ • $(ax^2 + bx + c) \pm (dx^2 + ex + f)$ • $\frac{ax + b}{c} \pm \frac{dx + e}{f}$ where $a, b, c, d, e, f \in \mathbf{Z}$ • $\mathbf{\frac{ax + b}{c} \pm \ldots \pm \frac{dx + e}{f}}$ • $\mathbf{(ax + by + c) \pm \ldots \pm (dx + ey + f)}$ • $\mathbf{(ax^2 + bx + c) \pm \ldots \pm (dx^2 + ex + f)}$ **where $a, b, c, d, e, f \in Z$** • $\mathbf{\frac{a}{bx + c} \pm \frac{p}{qx + r}}$ **where $a, b, c, p, q, r \in Z$.** – use the associative and distributive property to simplify such expressions as: • $a(bx + cy + d) + e(fx + gy + h)$ • $\mathbf{a(bx + cy + d) + \ldots + e(fx + gy + h)}$ • $a(bx^2 + cx + d)$ • $ax(bx^2 + c)$ where $a, b, c, d, e, f, g, h \in Z$
4.7 Equations and inequalities	Selecting and using suitable strategies (graphic, numeric, algebraic, mental) for finding solutions to equations and inequalities. They identify the necessary information, represent problems mathematically, making correct use of symbols, words, diagrams, tables and graphs.	– consolidate their understanding of the concept of equality – solve first degree equations in one or two variables, with coefficients elements of **Z** and solutions also elements of **Z** **– solve first degree equations in one or two variables with coefficients elements of Q and solutions also in Q** – solve quadratic equations of the form $x^2 + bx + c = 0$ where $b, c \in \mathbf{Z}$ and $x^2 + bx + c$ is factorisable $\mathbf{ax^2 + bx + c = 0}$ **where $a, b, c \in \mathbf{Q}$ $x \in \mathbf{R}$** **– form quadratic equations given whole number roots** – solve simple problems leading to quadratic equations – solve equations of the form $\frac{ax + b}{c} \pm \frac{dx + e}{f} = \frac{g}{h}$, where $a, b, c, d, e, f, g, h \in \mathbf{Z}$ – solve linear inequalities in one variable of the form $g(x) \leq k$ where $g(x) = ax+b$, $a \in \mathbf{N}$ and $b, k \in \mathbf{Z}$; $\mathbf{k \leq g(x) \leq h}$ **where $g(x) = ax+b$, and $k, a, b, h, \in Z$ and $x \in R$**

Flow of the Unit

Lesson		# of lesson periods
1	• Writing expressions	1 x 60 min.
2	• Solving problems using linear equations	1 x 60 min.
3	• Solving problems using simultaneous equations	3 x 60 min. **(research lesson)**
4	• Introduction to and solving quadratic equations	8 x 60 min.
5	• Solving problems using quadratic equations	2 x 60 min.

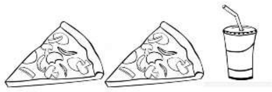
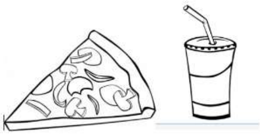
Flow of the Lesson

Prior Knowledge:
Creating and solving algebraic equations

Teaching Activity	Points of Consideration
Introduction A PowerPoint presentation will be used to scaffold the students understanding of Linear Equations in two variables.	
Task 1: Pete's-A-Place (PowerPoint)	
Students will be asked to represent the price of (a) one can of coke (b) one slice of pizza (c) one can of coke and one slice of pizza (d) two cans of coke (e|) two slices of pizza (f) two cans of coke and two slices of pizza, using mathematical expressions.	Teacher encourages students to write their answers on their white boards and hold them up when finished.
Anticipated Student Responses (PowerPoint) a) $c =$ €1.25 b) $p =$ €1.00 c) $c + p =$ €2.25 d) $2c =$ €2.50 e) $2p =$ €2.00 f) $2c + 2p =$ €4.50	Do students realise that doubling of part (c) gives the result to part (f) if they have not seen this. Allow 5-6 minutes.
Task 2: Pizza Shack (PowerPoint)	
Students will be told a story about Tom who goes into Pizza Shack and orders three cans of coke and two pizza slices. These cost him a total of €6.90. Students are given the mathematical expression $3c + 2p =$ €6.90	Students should be given 4- 5 minutes to come up with answers. As students are working on the problem the teacher will turn over the Pizza Shack Menu.

Students are then asked to calculate the price of one can of coke and one slice of pizza?	Teacher also circulates the room encouraging and prompting students who get stuck, or students who finish early. Ask, "are there other answers?"
Anticipated Student Responses $3(1.00) + 2(1.95) = €6.90$ $3(1.50) + 2(1.20) = €6.90$ $3(2.00) + 2(0.45) = €6.90$	Are students coming up with a variety of numbers, using trial & error and substitution which satisfy this equation? Do students understand that the use of trial & error and substitution is not sufficient in this scenario?
4. Comparing and Discussing The teacher asks the first student to come to the **board** to write out and explain his. The teacher asks the class if this is the answer they got and the method they used. The teacher asks a second student to come to the board and to present his/her solution. The teacher asks a third student to come to the board and show the class his/her solution. The teacher asks the class: "Which answers are actually on the Pizza Shack Menu? "What can we learn from these different solutions?" The teacher introduces the terms "trial and error", "substitution". The teacher asks "Do we need more information to answer this question?"	Allow students to discuss their work so that any misconceptions may be dealt with. Do students offer different ways of finding the solution? Are students comfortable explaining their thinking? It is important that students are given time to explain their approach to the problem. Remember to get the students to sign their work at the board. Which methods have most students used? Which methods do they think are more efficient? Do students recognise that trial and error and substitution may not be good methods?
Task 3: Pizza Shack continued (PowerPoint)	
Students will be told about Michael who goes into Pizza Shack and orders three cans of coke and one slice of pizza. These cost him a total of €5.40. Students are given the mathematical expression $3c + p = €5.40$ The following question is now posed: How much for one can of coke and one slice of pizza now?	Can students recognise that the two equations are similar? Do students recognize that the c and the p have the same values in both equations? Can students solve the simultaneous equations?
Anticipated Student Responses R1. Trial and Error and Substitution $3(1.30) + 2(1.50) = €6.90$ R2. The difference is an extra slice of pizza Pizza: $€6.90 - €5.50 = €1.50$ followed by substitution Coke: $\frac{5.40-1.50}{3} = €1.30$	Teacher circulates the room encouraging students to attempt as many methods as they can. It is not expected that every student would use all these approaches but most students should be able to complete the task using at least one method and students should be able to explain their thinking.

R3. Using Subtraction/ Elimination $3c + 2p = 6.90$ $3c + p = 5.40$ $p = 1.50$ $3c + 1.50 = 5.40$ $3c + 1.50 - 1.50 = 5.40 - 1.50$ $3c = 3.90$ $\frac{3c}{3} = \frac{3.90}{3}$ $c = €1.30$ Or $3c + 2(1.50) = 6.90$ $3c + 3.00 - 3.00 = 6.90 - 3.00$ $3c = 3.90$ $\frac{3c}{3} = \frac{3.90}{3}$ $c = €1.30$	Note all students' methods and prepare selected students to be ready to come to the board. Weaker students might be selected during this easier task. Are students able to explain their thinking? Can students recognise that the two equations are similar? Do students recognize that the c and the p have the same values in both equations? Can students solve the simultaneous equations?
Comparing and Discussing The teacher asks three different students to come to the board to present to the class the 3/4 approaches to the problem. The teacher introduces the terms "difference", "elimination". The class may be questioned about the approaches: Which did they use? Which do they think is easiest? Which is most efficient?	Allow students to discuss their work so that any misconceptions may be dealt with. Remember to get the students to sign their work at the board. Are students comfortable explaining their thinking? It is important that students are given time to explain their approach to the problem. Do students offer different ways of finding the solution? Are students comfortable explaining their thinking? It is important that students are given time to explain their approach to the problem. Which methods have most students used? Which methods do they think are more efficient? Do students recognise that trial and error and substitution may not be good methods? Do students express a preference for a particular method and why? Students should be given 5 minutes here to reflect on what they have learned so far.

Task 4: Students are now given the following problem:	
Picture that it is lunchtime, and we walk into a pizza restaurant that advertises the following specials:    **Special 1**: Three cans of coke and four slices of pizza for nine euro. **Special 2:** One can of coke and two slices of pizza for four euro. How much is (i) one slice pizza and (ii) one can of coke? 	Students should be given 10 minutes to complete the problem
Anticipated Student Responses Let c = price of a can of coke, p=price of a slice of pizza **R1. Trial & Error and Substitution** $3c + 4p = 9.00$ $c + 2p = 4.00$ $3(1.00) + 4(1.50) = 9.00$ $1.00 + 2(1.50) = 4.00$ **Can of coke costs €1.00** **Pizza Slice costs €1.50** **R2. Double the second Equation and Subtract** $3c + 4p = 9.00$ $c + 2p = 4.00$ $3c + 4p = 9.00$ $2c + 4p = 8.00$ Subtract $1c + 0p = 1.00$ $c = 1.00$ **Substitution** $3(1.00) + 4p = 9.00$ $3.00 - 3.00 + 4p = 9.00 - 3.00$ $4p = 6.00$ $\frac{4p}{4} = \frac{6.00}{4}$ $p = €1.50$ Or $c + 2p = 4.00$ $1.00 + 2p = 4.00$ $1.00 - 1.00 + 2p = 4.00 - 1.00$ $2p = 3.00$	Teacher circulates the room encouraging students to attempt as many methods as they can. Are students able to complete the task using at least one method? Are students able to explain their thinking? Can students recognise that the two equations are similar? Do students recognize that the *c* and the *p* have the same values in both equations? Can students solve the simultaneous equations? Do students recognize a difference? Are students able to eliminate either of the unknown terms?

$$\frac{2p}{2} = \frac{3.00}{2}$$ $$p = 1.50$$ **Can of coke costs €1.00** **Pizza Slice costs €1.50** **R3. Subtract twice** $$3c + 4p = 9.00$$ $$c + 2p = 4.00$$ $$2c + 2p = 5.00$$ $$c + 2p = 4.00$$ $$c = 1.00$$ **Substitution** $$3(1.00) + 4p = 9.00$$ $$3.00 - 3.00 + 4p = 9.00 - 3.00$$ $$4p = 6.00$$ $$\frac{4p}{4} = \frac{6.00}{4}$$ $$p = €1.50$$ Or $$c + 2p = 4.00$$ $$1.00 + 2p = 4.00$$ $$1.00 - 1.00 + 2p = 4.00 - 1.00$$ $$2p = 3.00$$ $$\frac{2p}{2} = \frac{3.00}{2}$$ $$p = 1.50$$ **Can of coke costs €1.00** **Pizza Slice costs €1.50**	
Comparing and Discussing The teacher asks three different students to come to the board to present to the class the 3/4 approaches to the problem. The class may be questioned about the approaches: Which did they use? Which do they think is easiest? Which is most efficient?	Allow students to discuss their work so that any misconceptions may be dealt with. Remember to get the students to sign their work at the board. Are students comfortable explaining their thinking? It is important that students are given time to explain their approach to the problem. Do students offer different ways of finding the solution? Are students comfortable explaining their thinking? It is important that students are given time to explain their approach to the problem. Which methods have most students used? Which methods do they think are more efficient?

	Do students recognise that trial and error and substitution may not be good methods?
5. Summing up The teacher asks the students to write a reflection on "what did I learn today?" and "what would be a good name for this lesson?" The teacher presents students with homework and explains what they are expected to do.	Do students recognise that different approaches to solving a problem are acceptable and that there is more than one way to solve a problem? What misconceptions are evident?

Evaluation

Peer observation is planned:
There will be four observers in the lesson along with the teacher.
One teacher will move through the room taking photographs of the students' work.
Three other teachers, at the side of the room, will observe and note students' interaction with the class with an emphasis on:

- How are they engaged during each task?
- How are they learning/successfully completing tasks?
- Student questions and responses.
- Quality of independent written work.
- How are they explaining when at the board?
- Evidence that the stated learning outcomes and aims of the lesson are being achieved?

Teachers will discuss their observations on the students learning during the class at a post-lesson valuation meeting, which will take place directly after the lesson is taught.

Board Plan

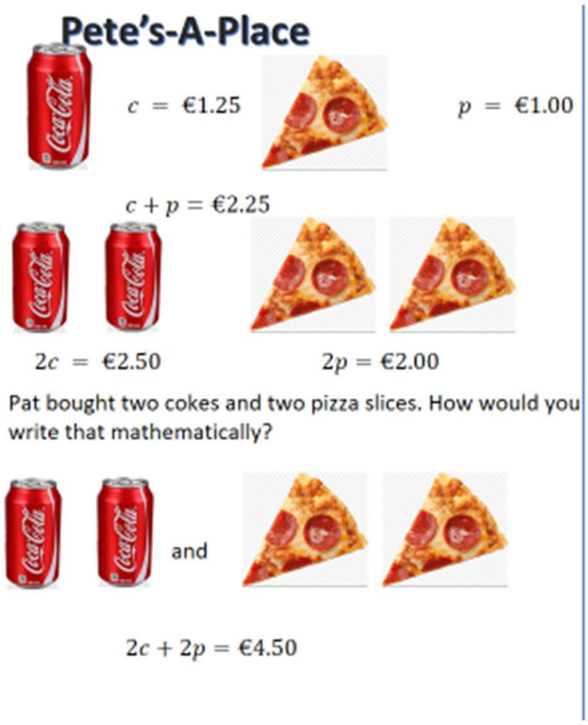


Fig 1: PowerPoint Presentation

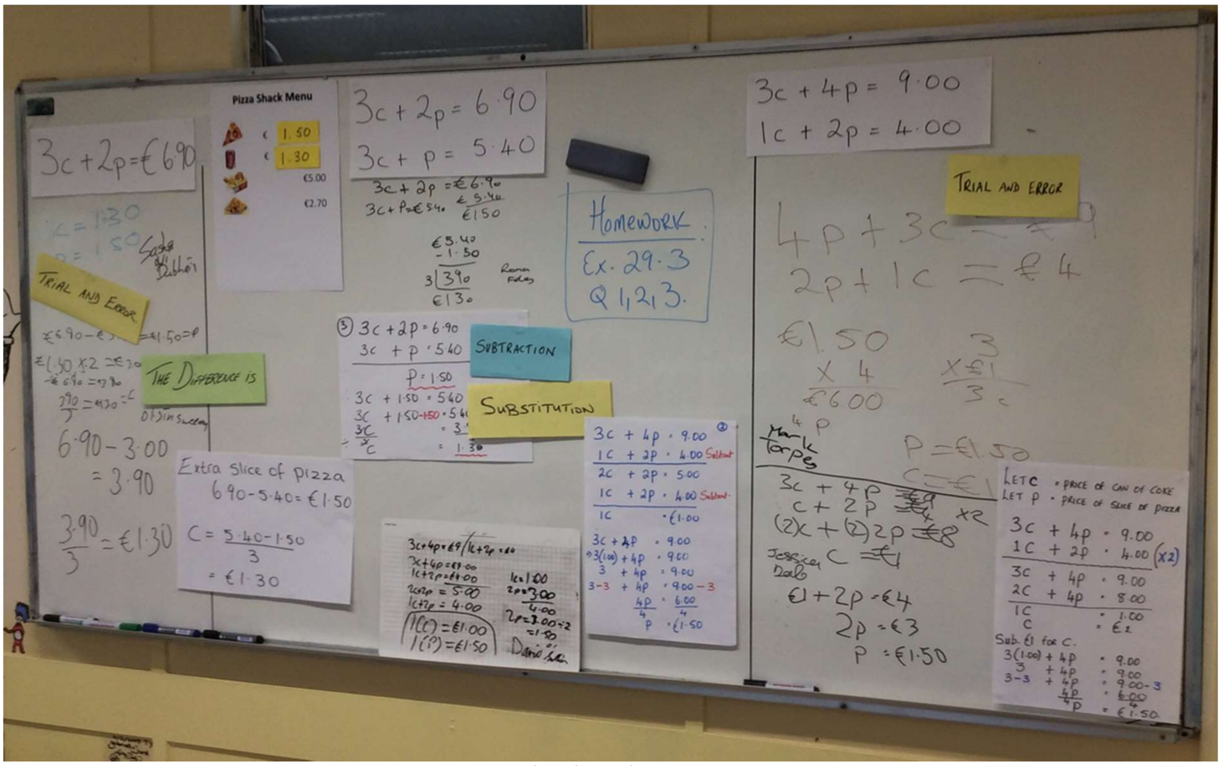
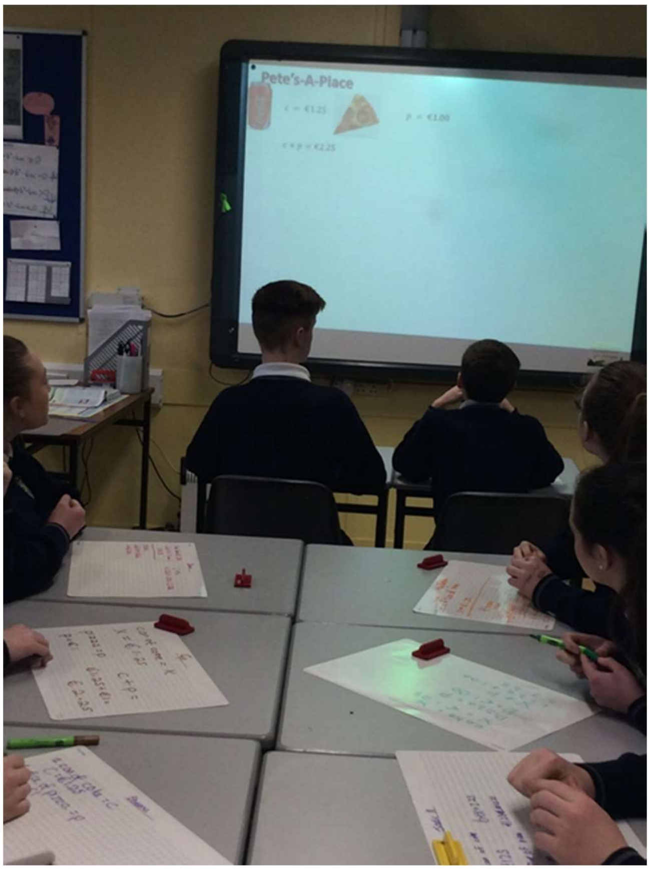


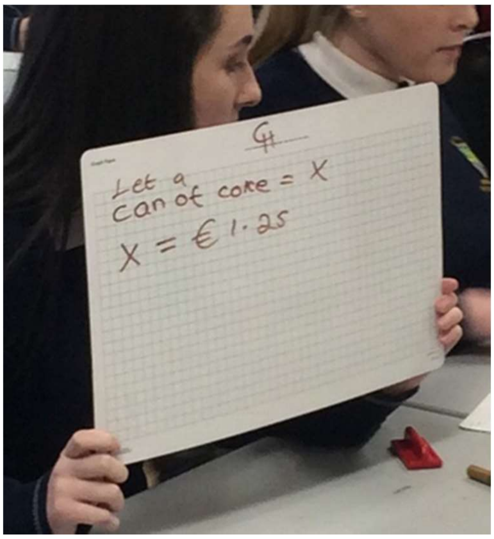
Fig 2: Board with Students' Presentations

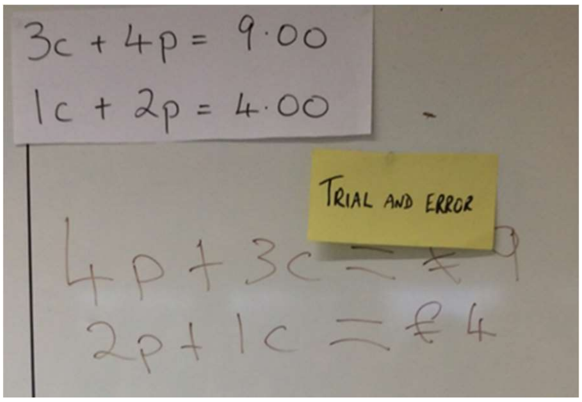
Post-lesson reflection

What are the major patterns and tendencies in the evidence?

The observing teachers noted that a positive attitude was established at the start of the lesson. By providing the students with a short ‘warm up’ activity (see task 1) in the form of a question, rather than a statement, students immediately became engaged and curiosity was evoked. As a result, this induced a problem solving thrust towards what followed in the lesson and students became comfortable with writing mathematical expressions quite quickly. For example, the use of c=the price of a can of coke and p=the price of slice of pizza. When students came to task 4, they immediately converted to using Mathematical expressions and some used the commutative property of addition $3c + 4p = 4p + 3c$.



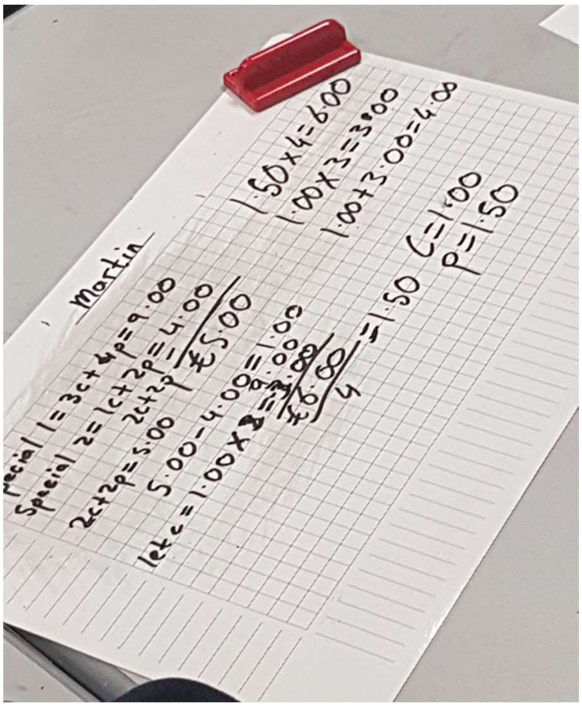


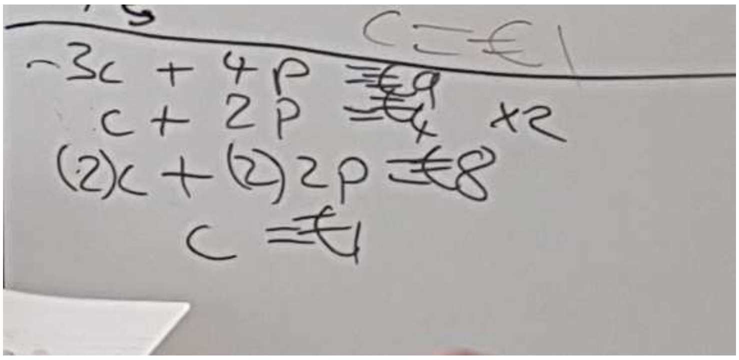


What are the key observations or representative examples of student learning and thinking?

It was clear from the outset that, through the use of a 'whole class interactive approach', genuine problem-solving and understanding was at the core of this lesson. The lesson was done with pace in order to sustain a buoyant feel and students were given a marked amount of autonomy and control over the organisation and delivery of their learning. As a result:

- Students identified variables easily i.e. let c=the price of a can of coke and let p=the price of a slice of pizza.
- Students translated the real life situation into Mathematical language.
- Students quickly learned that mathematical equations can be added/subtracted to/from each other, and that they can be multiplied by a constant.
- Students solved a system of equations when no multiplication was necessary to eliminate a variable.
- Students solved a system of equations when multiplication was necessary to eliminate a variable.





What does the evidence suggest about student thinking such as their misconceptions, difficulties, confusion, insights, surprising ideas, etc.?

It was extremely interesting to notice the various types of misconceptions or preconceived notions that existed within the minds of some students. Research indicates that new concepts cannot be learned if alternative models already exist in the learner's mind. Therefore, the use of the mini-whiteboards was particularly useful in this case as it allowed the teacher to easily identify difficulties and swiftly rectified the problems. Such difficulties included:

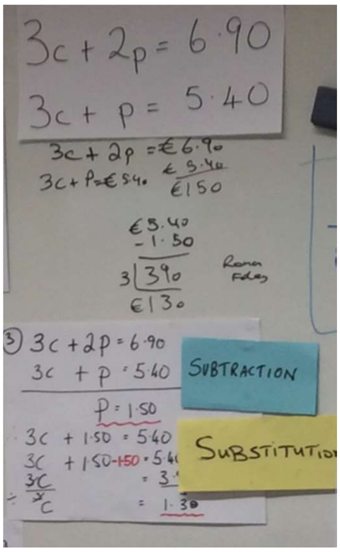
- Some students thought that unknown values added together resulted in a squared unknown. $c^2 = c + c$
- $c + c = 1.25 + 1.25$. Some students did not recognised that multiplication is repeated addition.
- Students did not recognise that there were various solutions to$3c\ + 2p = €6.90$. They felt that more information was needed to find the actual price of one can of coke and the price of one slice of pizza.

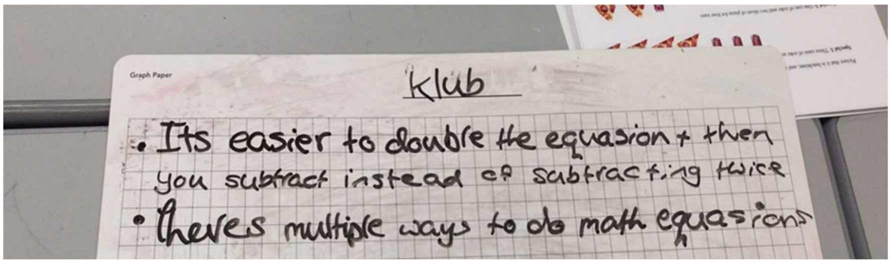
In what ways did students achieve or not achieve the learning goals?

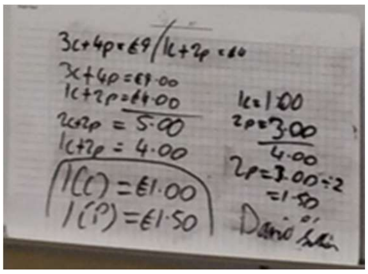
Overall, the lesson was managed very well and smooth transitions took place effectively. Having employed careful planning and much in depth thought prior to this lesson it enabled the teacher to be extremely sensitive to how the lesson was progressing. Consequently, this lead to the achievement of many learning outcomes.

Students achieved the following:

- convert words into mathematical expressions understanding that letters can be used instead of words or phrases
- ability to manipulate mathematical equations, see that two linear equations can be added, subtracted, and multiplied by a constant
- understood the relevance of key terms such as ‘trial and error’, ‘the difference is’, ‘substitution’, ‘subtraction’, and were able to apply these terms to a system of equations
- to solve a system of equations when no multiplication is necessary to eliminate a variable
- to solve a system of equations when multiplication is necessary to eliminate a variable
- to solve a system of equations using the substitution method







Based on your analysis, how would you change or revise the lesson?

One of the most obvious improvement would be to **s**pread the lesson over two periods. It was noted that this lesson was quite ‘heavy’ in relation to content being covered and rarely offered the students any break from intense concentration. Spreading the lesson out over two periods would give the students more time to process the information and reflect on their learning. After all, reflection allows students to organise the content and as a result become producers and not just consumers of knowledge.

What are the implications for teaching in your field?

Having discussed, planned and reflected on this lesson there is no doubt that a teacher's pedagogical content knowledge (PCK) plays a vital role in the quest to achieve optimum student progress. Schulman defined PCK as "that special amalgam of content and pedagogy that is uniquely the province of teachers, their own special form of professional understanding". The two main components that Shulman distinguished in PCK were, on the one hand, the most useful forms of representing the topics in one's subject area and, on the other hand, an understanding of what makes the learning of these topics easy or difficult for students. Based on these ideas it became increasingly obvious that the way in which topics are approached, identification of possible misconceptions and the sequence in which topics are addressed is paramount. Hence, it was decided that the unit should be approached in the following way:

1. Revision of CIC Algebra
2. Introduce Linear Equations (i.e. one equation with one unknown) with real life problems
3. Practise linear equations
4. Introduce Simultaneous Equations (i.e. two equations with two unknowns) with a real life problem. Scaffold the lesson to allow students to see the necessity for elimination to solve a system of equations when no multiplication is necessary to eliminate a variable.
5. Use a real life problem to create & solve a system of equations when multiplication is necessary to eliminate a variable
6. Practise Simultaneous Linear Equations