Topic: **Coordinate Geometry and Constructions**
Year group: **3rd Year Higher Level / 6th Year Ordinary Level**

Lesson taught:	12 December 2016
	Moville Community
College	
Class:	3rd Year higher level
Teacher:	Maurice Harkin
Lesson proposal developed by:	Rachel McGuinness,
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	Maurice Harkin,
	and Ryan Monagle

Title of the Lesson:

'Find the centroid of a given rectangle using as many different methods as possible.'

Brief description of the lesson:

By using constructions, algebra, coordinate geometry and geometry students will notice that there are many different methods of finding the centroid of a rectangle when given the four vertices. The students will be challenged to use their knowledge from different strands of the maths curriculum. In some of the more sophisticated methods students will have to really stretch their mathematical knowledge and also their creativity. In presenting their answers to their peers the students will engage their communication skills and hopefully grow in confidence.

Aims of the Lesson:

a. Short term aims:

We would like our students to use a variety of different methods from different parts of the maths curriculum to find the centroid of a rectangle in as many different and creative ways as possible.

b. Long-term goals:

i. We would like our students to appreciate that mathematics can be used to solve real world problems.
ii. We would like our students to appreciate that mathematics can be used to communicate thinking effectively; and that it can be used in a creative manner.
iii. We want students to appreciate that algebra is a tool for making sense of certain situations.
iv. We would like to foster our students to become independent learners.
v. We would like to emphasise to students that a problem can have several equally valid solutions.
vi. We want to build our students' enthusiasm for the subject by engaging them with stimulating activities
vii. We would like students to connect and review the concepts that we have studied already.
viii. We would like to encourage students to develop the confidence to systematically approach a question.

c. Thematic goals

i. We would like students work collaboratively with other students.
ii. We want to incorporate some of the key skills for Junior Cycle into this lesson, including but not limited to: Being Literate, Communicating, Working With Others, Being Creative and Being Numerate.[1]
iii. We would like to support our students in developing their literacy and numeracy skills through discussing ideas.[2]

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[1] The Junior Cycle Key Skills, developed by the NCCA in 2014 can be found here http://www.juniorcycle.ie/NCCA_JuniorCycle/media/NCCA/Documents/Key/Key_Skills_2014.pdf

[2] This Lesson Proposal illustrates a number of strategies to support the implementation of Literacy and Numeracy for Learning and Life: the National Strategy to Improve Literacy and Numeracy among Children and Young People 2011-2020 (Department of Education & Skills 2011).

Learning Outcomes:

As a result of studying this topic students will be able to:

- Plot points on a coordinate plane
- Use relevant coordinate geometry formulae
- Form simultaneous equations and solve them algebraically
- Use constructions
- To effectively use translations
- Extend the shape to solve the problem
- Present logical solutions to their peers, including the rationale (Key Skill: Communicating)

Background and Rationale

a. From Syllabus:

Topic 2.1 Synthetic geometry

- apply the results of all theorems, converses and corollaries to solve problems – prove the specified theorems
- complete the constructions specified[3]

Topic 2.2 Coordinate geometry

- explore the properties of points, lines and line segments including the equation of a line – find the point of intersection of two lines
- find the slopes of parallel and perpendicular lines[4]

Topic 2.4 Transformation geometry

- locate axes of symmetry in simple shapes
- recognise images of points and objects under translation, central symmetry, axial symmetry and rotations[5]

Topic 2.5 Synthesis and problem solving skills

- explore patterns and formulate conjectures
- explain findings
- justify conclusions
- communicate mathematics verbally and in written form

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[3] http://www.ncca.ie/en/Curriculum_and_Assessment/Post-Primary_Education/Project_Maths/Syllabuses_and_Assessment/JC_Maths_English_2013.pdf *Junior Certificate Maths Syllabus (p.18-19)*

[4] Junior Certificate Maths Syllabus (p.20)

[5] Junior Certificate Maths Syllabus (p.20)

- apply their knowledge and skills to solve problems in familiar and unfamiliar contexts – analyse information presented verbally and translate it into mathematical form – devise, select and use appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions.[6]

b. Difficulties students have had in the past with this subject matter?

- Not knowing how or where to start.
- Constructing Simultaneous Equations
- Plotting points
- Use of the various formula
- Understanding the term centre of a rectangle
- Using construction equipment
- Visualising the problem

c. Thematic Focus of the Lesson

The thematic focus of this lesson is a number of things. Initially we identify that students are finding it difficult to know where to start with a question. We would like to focus on the students gaining confidence in order to attempt the question without initially seeing the pathway to the end solution. The reason for this is we have seen in class that students look towards an example given or previously taught in order to complete the question given. If the question deviates at all from the example given they struggle to initiate any attempt. We hope that from doing the activity that students will gain an insight into the many ways of solving the centre point of the rectangle and hence realise that there are often many ways to go about solving a question.

Research

In preparation for this lesson the following resources were used:

a. Junior Certificate Mathematics Guidelines for Teachers (DES 2002)[7]
b. Second Year Teachers Handbook (Maths Development Team)[8]
c. PISA Mathematics: A Teacher's Guide (2007)[9]

[6] Junior Certificate Maths Syllabus (p.20)
[7] http://www.projectmaths.ie/documents/PDF/JCMathsGuidelines.pdf
[8] http://www.projectmaths.ie/documents/handbooks/secondyearhandbook2015.pdf
[9] http://www.sdpi.ie/inspectorate/insp_pisa_maths_teach_guide.pdf

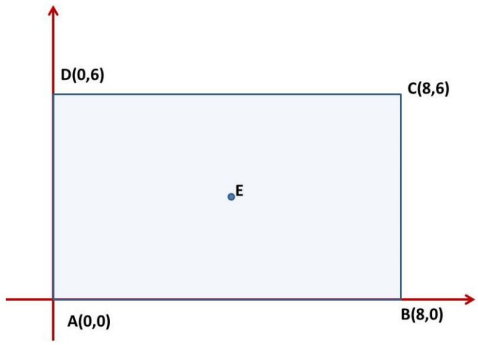
About the Unit and the Lesson

Students will find the centre of a rectangle using Geometry, algebra and construction methods and share their different methods with the rest of the class. They will address some or all of the aims set out in the syllabus and outlined in the 'Background and Rationale' section above.

Flow of the Unit:

Lesson		# of lesson periods
1	• Revision of Coordinate Geometry including midpoint, translations, distance formula, slope, plotting points. • Revision of Constructions, bisection of lines and angles, parallel and perpendicular.	3 x 35 min.
2	• Simultaneous Linear Equations. Forming the equations of lines given the points, finding the point of intersection using two or more line equations	2 x 35 min.
3	• Finding the centroid of a rectangle using as many methods as possible. • Sharing the various methods with the class	2 x 35 min. (double Class) **(research lesson)**
4	• Recap of the research lesson and covering methods that the students did not identify.	1 x 35 min.
5	• Extension of the question but as a square and a triangle.	2 x 35 min. (double class)

Flow of the Lesson

Teaching Activity	Points of Consideration
1. Introduction *(5 mins)* The students have recently completed coordinate geometry and trigonometry. They have completed constructions in the previous year Introduce the problem.	*Students are comfortable with plotting points, the coordinate line formulae, the constructions covered up to this point as well as the properties of shapes.*
2. Posing the Task *(5 mins)* Problem displayed on board. *'Find the centroid of the rectangle in as many ways as possible'*  	The problem is shown on the whiteboard and students are asked to come up with as many different ways of answering the problem as possible. *Observation of students work while they undertake the task will show their understanding of the objective.* *Students are also equipped with a geometry set.*
Students individual work *(10 mins)* Students are encouraged to come up with as many ways of solving the problem as possible while working individually for ten minutes. They are then asked to discuss with the person beside them and come up with another method, this will last approximately five minutes.	Teacher circulates the room assessing students' work to plan how to orchestrate the presentation of students' work on the board and class discussion. Students who get stuck will be encouraged to attempt the problem in a new way. Students who finish early will

	be encouraged to find more inventive solutions to the problem.
3. Anticipated Student Responses *(10 mins)* ● We expect students to solve the problem using some of the following methods: ● Draw the diagonals ● Simultaneous equations of the diagonals. ● Bisect 2 sides (construction) ● Translation, half of (0,0) $\longrightarrow$(8,6) ● Midpoint of [AC] or [BD] ● Midpoint of [AB] gives x coordinate and midpoint of [AD] gives y coordinate. ● Extend the rectangle to a square and then bisect two of the angles by construction. ● Draw one diagonal and then construct the perpendicular bisector or one of the sides, intersection point will be E. We expect students to struggle with finding more than three ways to solve the problem.	Teacher will walk around the room identifying correct solutions (and possibly a common incorrect result). Then invite the students to the board to display the various solutions. Similar responses will be grouped together by the teacher. Incorrect solutions will also be grouped by type (if applicable), to clear up any misconceptions.
4. Comparing and Discussing - Céardaíocht (20mins) Students are called up to the whiteboard to share their solution with the class, outlining the rationale. They must show their workings and use the proper instruments where necessary. The discussion will then be around the various ways in which students completed the problem. The teacher will facilitate classroom discussion by encouraging students to identify the differences and similarities between each presentation.	A range of methods and concepts can be employed when solving any mathematical problem. Students who benefit from the lesson will display an increased use of multiple disciplines when approaching problems.
5. Summing up Summary of the lesson by questioning students on their understanding of the inter-relatedness of the topics.	The depth of this closing discussion will be dependant on how much time is remaining in the lesson.

Emphasise that a problem lie this can be approached from a number of different perspectives and can be answered using prior knowledge from many areas of the course even though it may be seen to be attached to a certain area. Follow up activity: Give an additional problem: finding the centre of a triangle.	In the post-lesson discussion we will evaluate the lesson and determine if we consider the lesson to have been a success and see which areas we can improve on.

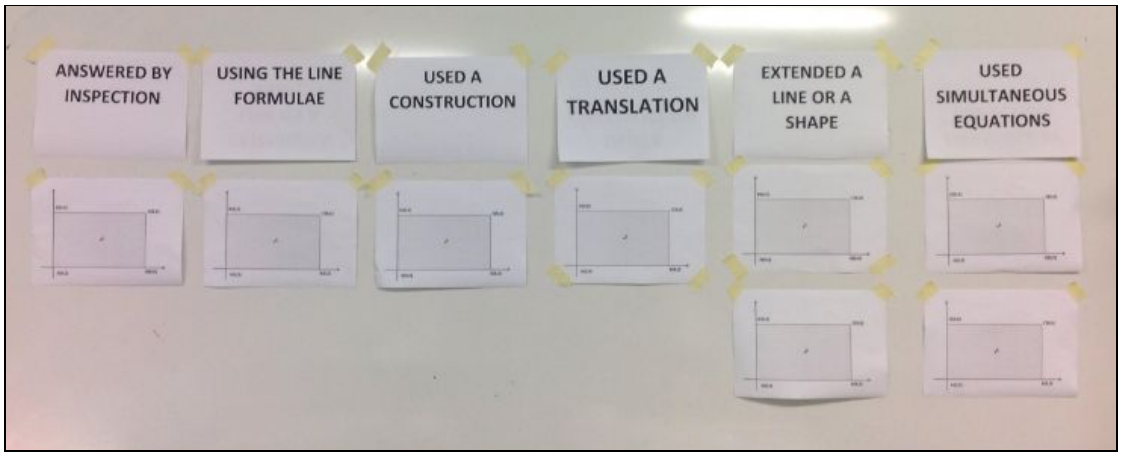
Evaluation: Plan for observing students

- A seating plan is provided by the teacher.
- Three teachers will observe the lesson being taught by the lead teacher. LessonNote will be used by one teacher and the others will observe by taking notes.
- Observing teachers will not interact with students in any way that will steer students in a particular direction in their work.
- Student responses (including both correct responses and erroneous) recorded with particular regard for the range of responses and their frequencies. Written record kept by observing teachers along with Lesson note app.
- Observers will focus on the level of student confidence when approaching and when answering an unfamiliar question.
- Students work will be collected and photographed at the end of the lesson.
- The observers will focus on some of the behaviour outlined in the table:

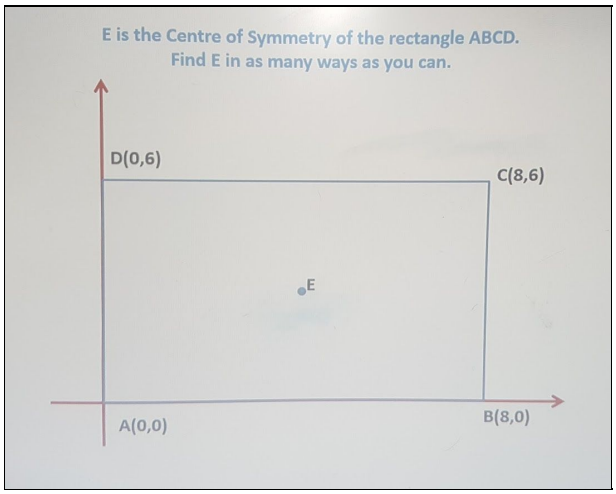
Introduction of the Task	○ Was the wording of the task clear? ○ Did the students ask any questions about the task prior to beginning?
Individual Work	○ Are there any prompts required? ○ Could a student complete a certain problem if they cooperated with a neighbour? ○ Are prompts required? ○ What skills do they employ when answering the question? ○ How long did students spend on the task? ○ Are the students comfortable using the geometry instruments? ○ Do the students persist with the task?
Discussion	○ Are students willing to present their work to their peers? ○ Are students attentive to what is happening at the board

	when their peers are presenting? ○ Do the students ask questions of one another? ○ Are clarifications needed to presenter's board work? ○ Did the discussion promote student learning?

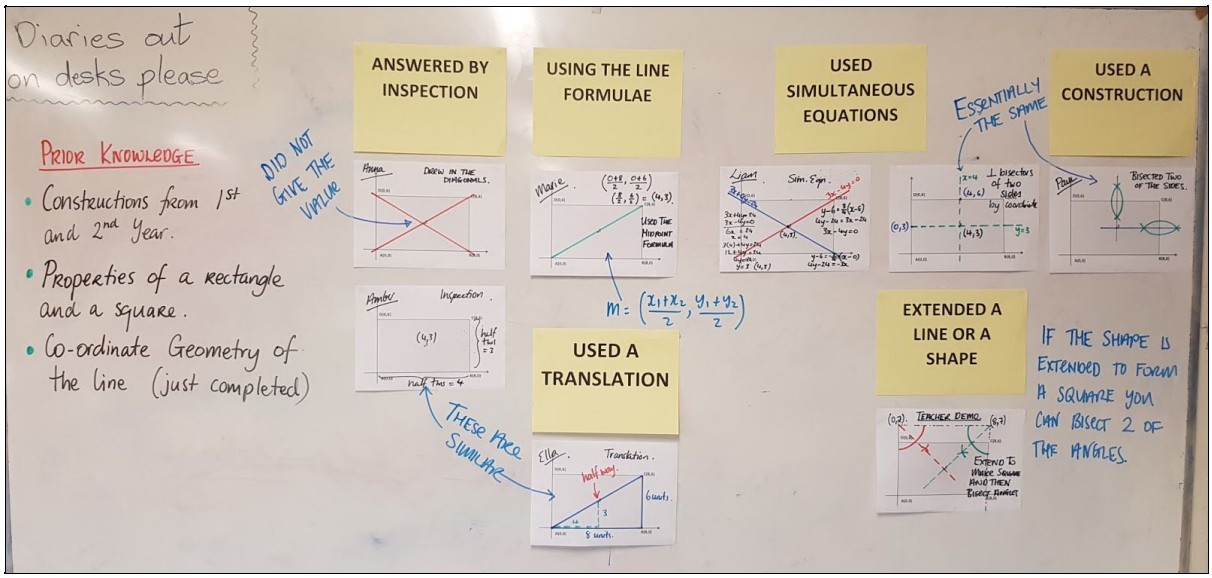
Board Plan



Board plan from Meeting 3



Boardwork: board posing the problem



Boardwork: Student responses grouped (end of the lesson.

Post-lesson reflection

What are the major patterns and tendencies in the evidence?

The group correctly anticipated during the Meitheal Machnaimh process of Lesson Study that the students would focus primarily on the method of solution that pertained to the formulae in coordinate geometry of the line; as these methods are the least sophisticated and students had recently completed this chapter. The group also identified the weakness of using the drawing instruments in the geometry sets. Students in general did not come up with a solution using a construction, unless they also studied Technical Graphics.

What are the key observations or representative examples of student learning and thinking?

A few of the students were very slow to get 'on task' when asked to work individually. It is not that they did not understand what was being asked of them as the teacher further explained to students who were slow to start what was being asked of them; it was more a case of not really knowing where or how to start. There were 4 teachers in the room and students were unwilling to put pen to paper at all in some cases as they did not want to write anything on the page that was not correct.

Every student got at least one correct solution but, as the group had anticipated, few of the students got more than three correct solutions in the given time. When the teacher asked them to share their knowledge with the person beside them this encouraged a few more attempts.

What does the evidence suggest about student thinking such as their misconceptions, difficulties, confusion, insights, surprising ideas, etc.?

No student extended the rectangle to a square and then bisected the angles to find the centre, this wasn't too surprising as relatively few students used a construction of any kind. Perhaps surprising was that only a couple of students formulated the equations of the diagonals and then found the point of intersection using simultaneous equations. We put this down to it being quite a time consuming answer and they were aware that they only had a limited amount of time to work on their own.

In what ways did students achieve or not achieve the learning goals?

The learning goals were achieved in the respect that everyone in the room was able to come up with at least one correct solution. We correctly predicted that the construction method for answering the problem would not be popular, in fact only students also studying Technical Graphics attempted any sort of construction. The students worked effectively when they were paired off with their neighbour, this proved that they could communicate and work well together as well as individually.

The highlight of the lesson was inviting the students up to the whiteboard to share their solutions with the class, this proved to be a motivating factor and the fact that everyone in the class had at least one correct solution added to this novelty; nobody minded being called up as they all had something to contribute to the class.

Based on your analysis, how would you change or revise the lesson?

Students showed that they have a reasonably good understanding of the topic of coordinate geometry and were able to use the formulae with a good degree of confidence; this was mainly due to the fact that this was the section of the course the students had covered most recently, the group felt. If we were to teach this lesson in future we would include constructions as part of the flow of the unit, as we felt this was a weakness in the answering. Time constraints were also a little bit of a difficulty; the problem formulated by the group allowed for such a variety of answers that in the summing up and Céardaíocht, the group felt it was a little rushed.

What are the implications for teaching in your field?

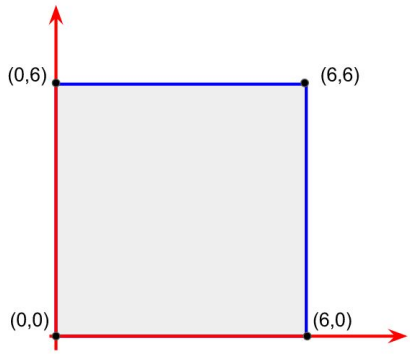
We understand that while carrying out the lesson proposal planning, the question should be clear and simple. Every student should be able to give some sort of correct answer, no matter how simplistic. When we formulated the problem and during the Meitheal Machnaimh process the group were surprised by the amount of possible correct answers we were able to come up with - we thought this might be a good thing in relation to our problem as it would open up the solution to different strands included in the maths curriculum.

Students learn more independently than we give them credit for, and they are more creative than we sometimes allow them to be in class. They really enjoyed sharing the solutions with the rest of the class and were excited to be called up to the whiteboard.

Everyone involved found the process worthwhile and will use this process in future lessons.

Follow Up Activity

Find the centroid of the given square in as many ways as you can



Find the centroid, circumcentre, orthocentre, and incentre (all the same) for the equilateral triangle shown in as many ways as you can.