Topic: Theorem of Pythagoras
Year group: Second years

Lesson plan taught: 27th Jan 2017
At Carrigaline Community School
Niamh O'Flynn and team 2nd years
Teacher: Niamh O'Flynn
Lesson plan developed by: Shaun Holly, Don O Shea, Dee Cahill

Grazing Gazelles

1. **Title of the Lesson: Grazing Gazelles**

2. **Brief description of the lesson:** Investigating and comparing the areas of squares on the sides of a right angles triangle to develop a conceptual understanding of Pythagoras' Theorem

3. Aims of the Lesson:

Long-term goal: We wish for the students:
- to understand the Theorem of Pythagoras so that they can apply the relevant concepts.
- to develop strategies for dealing with problems.
- to develop capacity for logical explanation, justification and communication.
- to become more creative when devising approaches and methods to solve problems.
- to understand that a problem can have several equally valid solutions.
- to feel encouraged and enthused with the subject through engaging in activities.

Short-term goal: We wish for the students:
- to gain an understanding of Pythagoras' Theorem.
- to develop meaning for the application of Pythagoras' Theorem.
- apply prior knowledge of shapes and their area and link this to Pythagoras' Theorem.

4. Learning Outcomes:

As a result of studying this topic students should be able to:
- Use Pythagoras' Theorem to solve a problem.
- Apply prior knowledge of shapes and their area and link this to Pythagoras' Theorem.
- Discuss and justify their solution.
- Understand and apply the connection between sides and area in a right-angle triangle.
- Establish a formula linking formulae for area and Pythagoras' Theorem.

5. Background and Rationale

- Students have difficulties in applying their knowledge of area and shapes to problem solving.
- Spatial awareness poses problems for some students.
- Students have difficulties in connecting mathematical concepts to real-life problem solving.
- Promoting and facilitating independent learning through investigation and discovery by a variety of methods. We would like our students to become more creative when devising approaches and methods to solve problems.
- We would like to build our students' enthusiasm for the subject by engaging them with activities as problem solving is integral to mathematical learning.
- In day-to-day life and in the workplace the ability to problem solve is a highly advantageous skill.
- The use of context-based tasks and a collaborative approach to problem solving can support learners in developing their literacy and numeracy skills.

6. Research

- www.nctm.org/middle
- www.projectmaths.ie
- www.ncca.ie
- The Pythagorean Theorem with Jelly Beans. Mathematics Teaching in the Middle School, Vol. 14, No. 4, November 2008

7. About the Unit and the Lesson

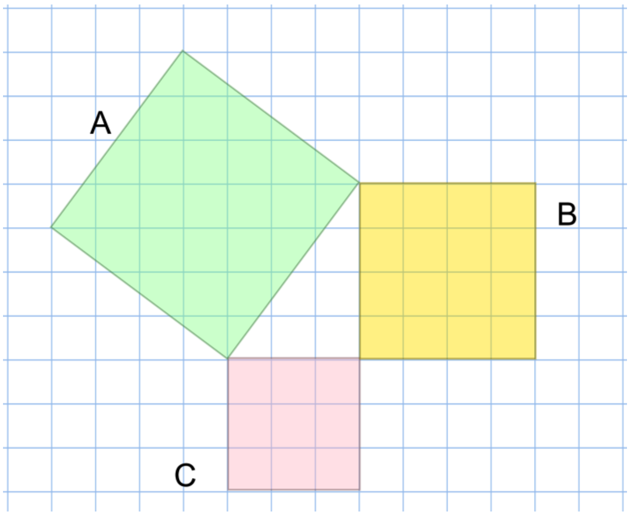
Strand/Topic Description	Learning Outcomes
Strand 2: 2.1 Synthetic Geometry • In a right-angled triangle, the square of the hypotenuse is the sum of the squares of the other two sides. • If the square of one side of a triangle is the sum of the squares of the other two sides, then the angle opposite the first side is a right angle. **2.3 Trigonometry** • Apply the theorem of Pythagoras to solve right-angled triangle problems of a simple nature involving heights and distances. • Solve problems involving right-angled triangles. **2.5 Problem Solving** • Explore patterns and formulate conjectures. • Explain findings. • Justify conclusions. • Communicate mathematics verbally and in written form. • Apply their knowledge and skills to solve problems in familiar and unfamiliar contexts. • Devise, select and use appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions.	· apply the results of all theorems, converses and corollaries to solve problems • apply the theorem of Pythagoras to solve right-angled triangle problems of a simple nature involving heights and distances • solve problems involving right-angled triangles • to develop strategies for dealing with problems. • to develop capacity for logical explanation, justification and communication. • discuss and justify their solution

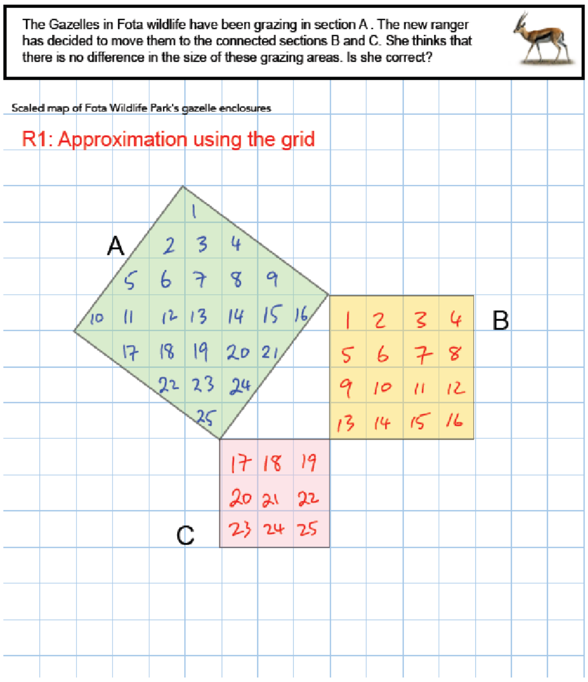
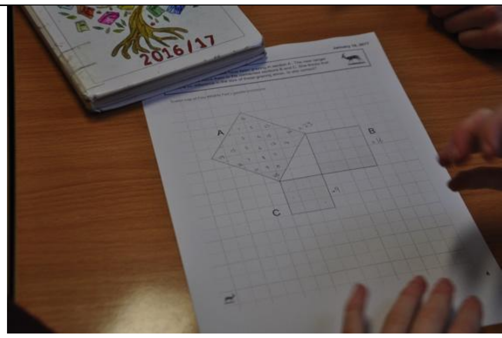
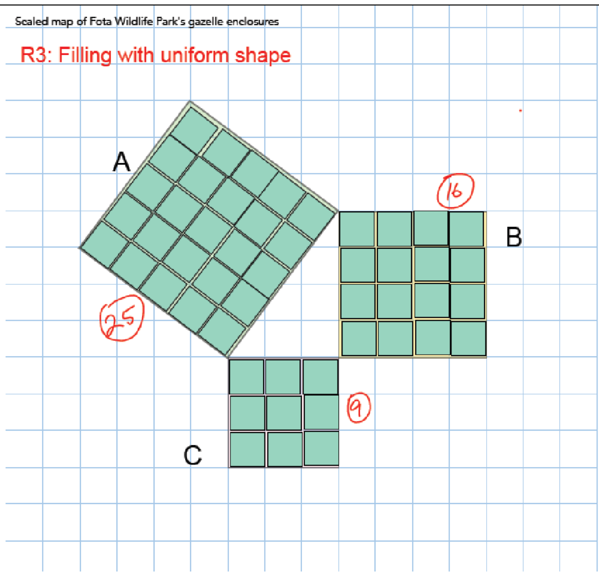
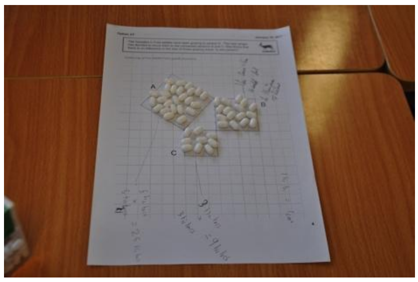
Learning Outcome of the Lesson	Lesson Design
• Use Pythagoras' Theorem to solve a problem	Group work: students in groups of ¾ students will focus on different methods to solving a problem.
• Apply prior knowledge of shapes and their area and link this to Pythagoras' Theorem.	Students will use different methods to calculate area of semi-circles to find a solution to their problem.
• Discuss and justify their solution.	Students will present their different methods on the board.
• Understand the relationship of Pythagoras' Theorem	
• Apply knowledge of area formulae in a problem-solving approach	

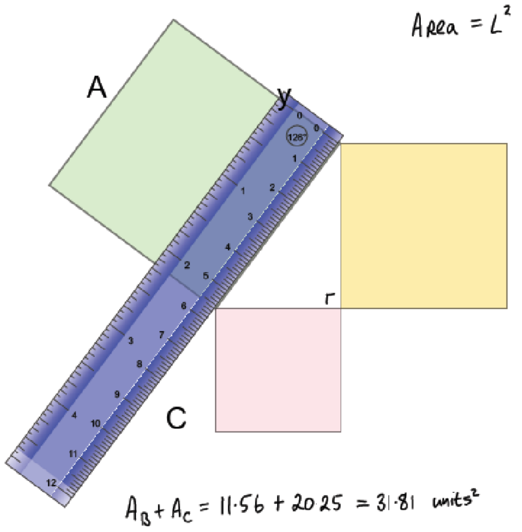
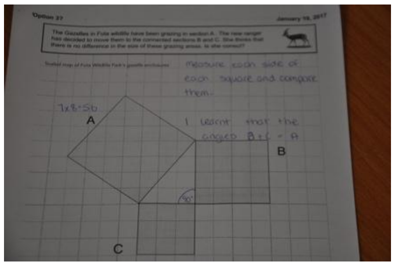
8. Flow of the Unit:

Lesson		# of lesson periods
1	Pythagoras' Theorem through a problem-solving approach	1 x 40 min. **(research lesson)**
2-5	Extension classes on Pythagoras Theorem	4 x 40 min.

9. Flow of the Lesson

Teaching Activity	Points of Consideration
1. Introduction(5mins) Prior Knowledge: Area of a square, recognize a right-angle triangle, estimation of area using grid.	Units of area
2. Posing the Task(2mins) The Gazelles in Fota wildlife have been grazing in section A . The new ranger has decided to move them to the connected sections B and C. She thinks that there is no difference in the size of these grazing areas. Is she correct?  In front of students will be worksheets, ruler, scissors, Tic-tacs	Teacher will ask if problem is clear to all students by focusing on the problem posed on the worksheet. Ask students are they clear about the instructions and do they have any questions. Tell the students they will have 10 mins to solve the problem in as many ways as possible. They will need to present their data on a sheet and will be called to the board. No questioning while problem is being solved. Do you understand the concept of the problem? Are you clear with what is being asked?
3. Anticipated Student Responses (incorrect responses discussed)	Ask students if they understand what they are doing and if they have any questions. Students will present their attempts at solving the problem on the board. We will start with the perceived most basic method and work up towards the more advanced solution.

The Gazelles in Fota wildlife have been grazing in section A . The new ranger has decided to move them to the connected sections B and C. She thinks that there is no difference in the size of these grazing areas. Is she correct?
Scaled map of Fota Wildlife Park's gazelle enclosures
R1: Approximation using the grid
A
B
C	2016/17
Scaled map of Fota Wildlife Park's gazelle enclosures
R3: Filling with uniform shape
A
B
C
25
16
9	

R4: Measuring and using the area formula  	
4. Comparing and Discussing 1. Counting squares 2. Filling in of area with Tic-Tacs 3. Cut out of shapes. 4. Using area of square formula – this will involve finding the relative length of the hypotenuse first.	Ask the students have they come up with a solution. The students will discuss the accuracy of different methods and if there a common element in each method. What as a group do we conclude as the solution(s) to the given problem?
5. Summing up Discussion of whether the gazelles are better in two separate enclosures or if they would be better in one larger enclosure? Discuss any difficulties the students encountered.	Are the gazelles better to be in one larger enclosure or to stay in the smaller two enclosures? Why? Derive $a^2+b^2=c^2$ from the above. Do you think this principle applies to all triangles?

9. Planning the observation

5. Beginning of Lesson:
Observe understanding of prior knowledge and of the task

	Student 1	Student 2	Student 3	Student 4	Student 5	Student 6	Student 7	Student 8
(i) Questions on the formula								
(ii) Misconceptions								
(iii) Wording of the task, e.g. "Width"								
(iv) Do students ask for measurements?								
(v) Questions asked by students								
Other Observations								

6. During Lesson:
Observe student engagement and note progress

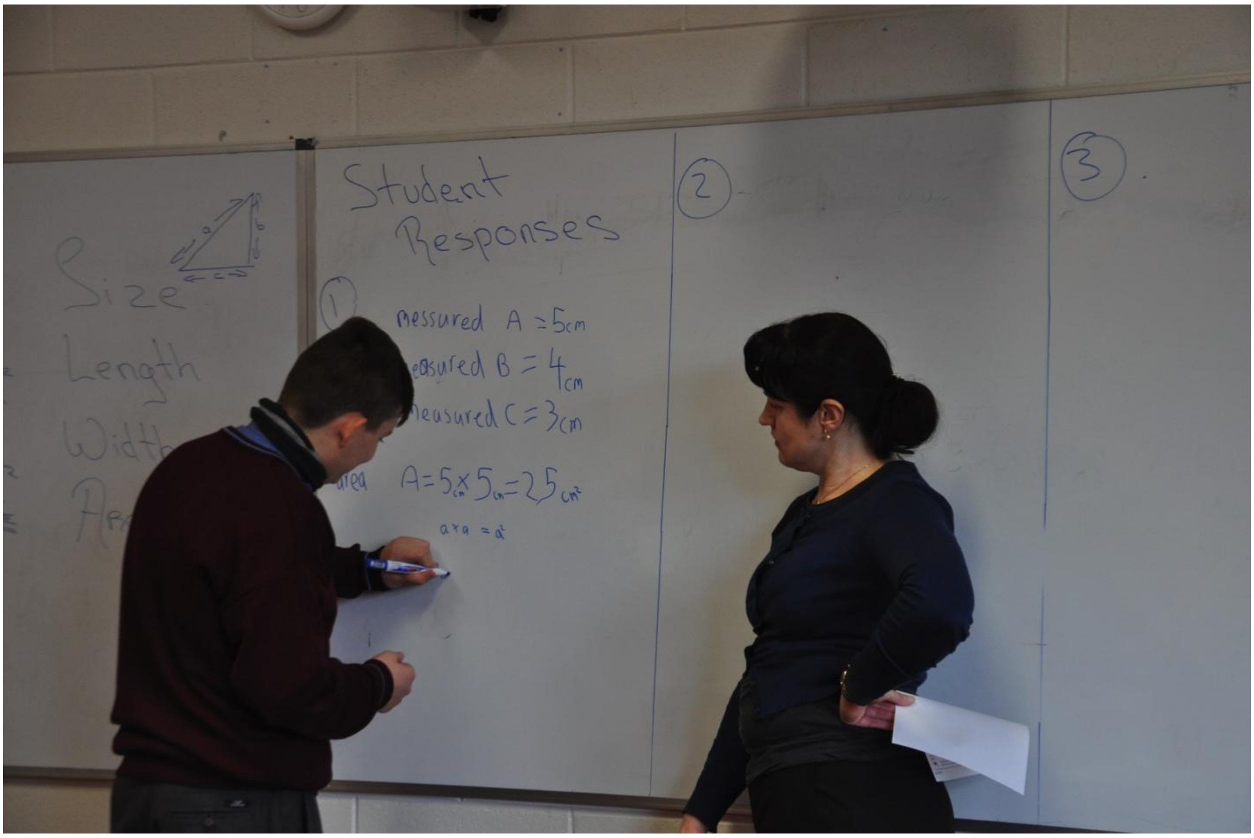
	Student 1	Student 2	Student 3	Student 4	Student 5	Student 6	Student 7	Student 8
(i) Questions asked to teacher								
(ii) Questions asked to other group members								
(iii) Identify if and when a student used a practical approach How long did they spend? Did they go back?								
(iv) Identify if and when a student used a written approach How long did they spend?								
(v) Did students persist with the task? Or give up? Were prompts required?								

7. During Discussion:
Observe student engagement.

	Student 1	Student 2	Student 3	Student 4	Student 5	Student 6	Student 7	Student 8
(i) Are the students attentive to what's happening on the board?								
(ii) Presenters: are clarifications or prompts needed to their board work?								
(iii) Did the presenting students' presentation and discussion promote their teaching and learning?								
(iv) During oral questioning at the end of the lesson, was there evidence of understanding and learning?								
Other observations								
Issues that need to be addressed in the next class								
Recommended changes to the lesson plan								

10. Board Plan

Students present anticipated respons leading to conclude that the grazing areas are similar.



11. Post-lesson reflection

Major patterns and tendencies

- It is of huge importance that students are aware of the problem they are trying to solve. This means that the question layout must be clear and students must have a good understanding of what they are being asked to do.
- The major pattern that emerged throughout the lesson was that students used the tic tacs to fill the areas of the squares.
- Little use of measuring lengths of sides was adopted in solving the problem.
- Several students attempted to solve the problem by using perimeter.

Observations and examples of students learning and thinking

- Students were a little nervous at first glance of the problem but quickly began to engage with the problem and attempt to find a solution. Many students began with counting the large square and continuing on to count the other two squares. From this students began to use tic tacs, by placing them in each of the marked squares on the sheet.
- No students used a straight edge to measure the sides of each of the squares.
- There were a few students who, from the offset, understood the problem and counted the squares in each shape and were able to compare the large square to the two smaller squares, see Appendix 2.

Misconceptions and insights

- One of the misconceptions was that students calculated the perimeter of each square in an attempt to solve the problem.
- Students failed to fill all squares with tic tacs and so failed to make any connection between the areas of the shapes.
- Some students had difficulty in understanding the problem to be solved and did not spend any time reading the question asked thoroughly.

Did students achieve or not achieve the learning goals?

- Many of the students achieved the learning goals by understanding that the area on the hypotenuse is equal to the sum of the areas of the other two sides. This was achieved, mainly through placing tic tacs in each square and counting the number of tic tacs in each square. The students then compared and linked the areas.
- On the other hand there were students who did not comprehend the problem at hand and lacked the comprehension of linking Pythagoras to the question.
- It was noted that some of the students did not engage at all with the task, while others counted squares and stopped thereafter. These students did not then achieve any of the learning objectives set out in the lesson.

Lesson revision

- Upon our analysis of the lesson, we would spend more time explaining the problem and the aim of the task to the students. We feel an understanding of the question posed would lead to better outcomes and more student engagement.
- The worksheet used was based on using squares, and their areas, to establish a link to Pythagoras' Theorem. We would not use lettering to identify our shapes but rather use arbitrary names or symbols; for example, the lion's den. This would be done to prevent confusion among the students and to avoid the use of algebra.
- We would like to introduce different shapes, for example semi-circles, to reinforce the concept. This idea would work very well as an extension exercise.

Implications

The benefits of using a problem-solving approach are definitely worthwhile and for the most part engaging to students. It gives a good base for knowledge and allows students to share knowledge and learn from their peers. It lays a good foundation for students from a conceptual viewpoint before building on concepts using a purely numerical approach.
Through planning of the lesson we experienced the importance of formulating a clear and meaningful problem. This allows students to extend their knowledge and share this knowledge with their peers. Team work and communication are central and paramount in this process.

Appendix 1 – The student problem sheet

The Gazelles in Fota wildlife have been grazing in section A . The new ranger has decided to move them to the connected sections B and C. She thinks that there is no difference in the size of these grazing areas. Is she correct?

Scaled map of Fota Wildlife Park's gazelle enclosures