Into the Next Dimension

Title of the Lesson

Into the Next Dimension

Brief description of the lesson

Here we are hoping to help students realise the simple relationship between area and volume. The tasks will present the students with a challenge that requires thinking and presentation of ideas to their peers.

Aims of the Lesson

a) **Short-term goals:**
i. For students **t**o discover the volume of a cylinder through the relationship between the area of a 2D shape and the volume of a uniform shape.
ii. For students to understand:
- The relationship between the area of a circle and the volume of a cylinder.
- Estimation is a useful skill.
- To see that as the dimensions change, the volume changes in proportion.
- Be able to comfortably use the volume of a cylinder formula.
- π is a constant and height, h, is a variable in this task.
- Scale factor – the effect of scaling.

b) **Long term goals:** We would like our students:
- to verbally and visually express their own ideas and concepts.
- to be able to work in groups and develop group problem solving skills. (Key Skill: Working With Others)
- to become confident and competent presenters. (Key Skill: Communicating)
- to connect maths with real life problem solving. (Key Skill: Being Creative)
- to become more creative when devising approaches and methods to solve problems and to acknowledge that a problem can have several equally valid solutions. (Key Skill: Managing Information & Thinking: Thinking creatively and critically)
- to experience meaningful mathematics i.e. that they see a need for what they are studying.
- to connect and review the concepts that we have studied already. (Key Skill: Being Numerate: Seeing Patterns, Trends and relationships)

Learning Outcomes:

As a result of studying this topic students will be able to:

- Easily move from a 2D problem on a page to a real life 3D problem.
- Develop an understanding of the relationship between the area of a 2D shape and finding the volume of a 3D shape. (Key Skill: Being Creative)
- See the relationship between the area of a circle and the volume of a cylinder
- Appreciate the value of estimation. (Key Skill: Being Numerate: Estimating, predicting and calculating)
- Find the volume of a cuboid and a cylinder.

Background and Rationale

- The topic of Area and Volume of 2D and 3D shapes are in the Strand of Number. According to the NCCA Project Maths Syllabus for Junior Cert, all students should learn about: "2D and 3D solids, including nets of solids (two-dimensional representations of three-dimensional objects), using nets to analyse figures and to distinguish between surface area and volume."

- The learning outcomes according to the syllabus are that students should be able to:
    - find the volume of rectangular solids and cylinders
    - find the surface area of rectangular solids.
- Some difficulties students might have are: not seeing the relationship between 2D shapes and 3D shapes. By doing this lesson we will try and address this difficulties by helping students to be more creative in their thinking and to the approaches they use. We will also try and help the students become more independent learners by using their prior knowledge to solve the problems.

Research

- Several Junior Cycle Maths textbooks.
- NCCA (2013). Junior Certificate Mathematics Syllabus: Foundation, ordinary and higher level, for examination from 2016. Dublin: DES.
- Maths Development Team Website, www.projectmaths.ie (2013). Insights into Lesson Study - Maths Counts 2013. [ONLINE] Available at: http://www.projectmaths.ie/for-teachers/conferences/maths-counts-insights-into-lesson-study/. [Accessed 17 October 16].
- Maths Development Team Website, www.projectmaths.ie (2015). Maths Counts 2015. [ONLINE] Available at: http://www.projectmaths.ie/forteachers/conferences/maths-counts-2015/. [Accessed 17 October 16]
- Maths Development Team Website, www.projectmaths.ie (2016). Maths Counts 2016. [ONLINE] Available at: http://www.projectmaths.ie/for-teachers/conferences/maths-counts-2016/. [Accessed 17 October 16]
- Maths Development Team Website, www.projectmaths.ie (2014). Teacher Handbook: Second Year (Draft) Based on the 2016 Syllabus. [ONLINE] Available at: http://www.projectmaths.ie/wp-content/uploads/2015/01/2nd-Year-Handbook-July-2016.pdf. [Accessed 17 October 16].

About the Unit and the Lesson

The content of this lesson relates to Topic 3.4 Applied measure from Strand 3: Number in the current Junior Certificate Syllabus. This syllabus (p. 24, as below) describes the content knowledge and learning outcomes required of our students in this area.

Strand 3: Number Syllabus section 3.4

The learning outcomes set out in the syllabus for Strand 3 are as follows:

In the course of studying this strand the learner will:

- Revisit previous learning on number and number operations
- Develop a meaningful understanding of different number types, their use and properties
- Engage in applications of numeracy to solve real life problems
- Apply set theory as a strategy for solving problems in arithmetic

The learning outcomes of syllabus section 3.4 Applied Measure 2- area and volume (pg. 24)

- Calculate, interpret and apply units of measure and time
- Solve problems that involve calculating average speed, distance and time
- Investigate the nets of rectangular solids
- Find the volume of rectangular solids and cylinders
- Find the surface area of rectangular solids
- Identify the necessary information to solve a problem
- Select and use suitable strategies to find length of the perimeter and the area of the following plane figures: disc, triangle, rectangle, square, and figures made from combinations of these
- Draw and interpret scaled diagrams

Project maths 2nd year handbook; Section 3 (pg. 12)

These lessons will involve the students in investigating and understanding:

- Units of measure and time
- Problems involving perimeter and area of the following plane figures: disc, triangle, rectangle, square and figures made from combinations of these (revision and extension from first year)
- The relationship between the circumference of a circle, its diameter and π
- The nets of rectangular solids (revision from first year)
- The surface area of rectangular solids (revision from first year)
- The volume of rectangular solids and cylinders

Flow of the Unit:

Lesson	Topic	# of periods
1	Problems involving perimeter and area of the following plane figures: disc, triangle, rectangle, square and figures made from combinations of these (revision and extension from first year)	3 x 40 min.
2	The nets of rectangular solids (revision from first year) The surface area of rectangular solids (revision from 1st year)	1 x 40 min.
3	The relationship between the circumference of a circle, its diameter and π	1 x 40 min.
4	The area of a disc	1 x 40 min.
5	The volume of rectangular solids and cylinders	1 x 40 min (research lesson)
6	The volume of rectangular solids and cylinders	2 x 40 min.

Flow of the Lesson

Teaching Activity	Points of Consideration
Introduction *Teacher introduces a rectangle, made up of 6x4 boxes, and asks students what it is and how to find the area.* *Teacher introduces a square, made up of 4x4 boxes, and asks what it is and how to find the area.* *Teacher introduces a disc and asks students what it is and can students identify diameter and radius.*	*Can students identify the shapes etc.?* *Are students able to identify area and calculate area of the rectangle and square?*

Task 1 - Finding The Area Of A Disc	
Teaching Activity	**Points of Consideration**
Posing the Task *Teacher introduces students to a circle, inscribed in a square, circumscribing another square.* *The diagram is on squared paper.* *The students are asked to find the area of the space enclosed by the circle.*	*Are the students using knowledge from previous lesson?* *Are students able to estimate the area?* *Are students able to use the formula?*
Anticipated Student Responses *R1 (a): Area of disc > area of small square.* *R1 (b): Area of disc < area of large square.* *R2: Area of large square > area of disc > area of small square.* *R3: Counting boxes in large square less those not contained within circle.* *R4: Counting boxes in small square plus those contained within circle.*	*Can students compare the different areas?* *Can students use various methods to calculate the area?* *Will students think that the area of the disc is the area of the large square less the area of the small square?*

R5: Using formula πr2	

Comparing and Discussing	
The teacher asks the first student to come to the board to write out and explain R1. *The teacher asks the class if this is the answer they got and the method they used.* *The teacher asks a second student to come to the board and to present R2.* *The teacher asks a third student to come to the board and show the class R3.* *The teacher asks a second student to come to the board and to present R4.* *The teacher asks a third student to come to the board and show the class R5.*	*Allow students to discuss their work so that any misconceptions may be dealt with.* *Do students offer different ways of finding the solution?* *Are students comfortable explaining their thinking?* *Remember to get the students to sign their work at the board.* *Which methods have most students used? Which methods do they think are more efficient?* *Can the students answer the question -* *"What can we learn from these different solutions?"*

Task 2 – Finding The Volume Of A Cylinder	
Teaching Activity	**Points of Consideration**
Posing the Task *Teacher introduces a cuboid.* *Teacher introduces a cylinder.* *Teacher introduces a cylinder, containing a cuboid, inserted in a large cuboid. (The bases of these have the same measurements as the shapes in previous task.)* *Students are asked to find the volume of the cylinder.*	*Do students recognize a cuboid?* Do students recognize a cylinder? *Can students understand that there is now a third dimension?*
3. Anticipated Student Responses *R1 (a): Volume of cylinder > volume of small cuboid.* *R1 (b): Volume of cylinder < volume of large cuboid.* *R2: Volume of large cuboid > volume of cylinder > volume of small cuboid.* *R3: Measure of the heights of the cuboid and the cylinder is equal. Volume of cuboid is equal to the area of the base by the height (using measurements from previous task).* *R4: Using formula πr2h*	*Can students compare the different volumes?* *Can students use various methods to calculate the volume?* *Will students think that the volume of the cylinder is the volume of the large cuboid less the volume of the small cuboid?*
4. Comparing and Discussing	

The teacher asks a student to come to the board to write out and explain R1.	*Allow students to discuss their work so that any misconceptions may be dealt with.*
The teacher asks the class if this is the answer they got and the method they used.	*Do students offer different ways of finding the solution?*
The teacher asks a second student to come to the board and to present R2.	
The teacher asks a third student to come to the board and show the class R3.	*Are students comfortable explaining their thinking?*
The teacher asks a second student to come to the board and to present R4.	*Remember to get the students to sign their work at the board.*
	Which methods have most students used? Which methods do they think are more efficient?
	Can the students answer the question - "What can we learn from these different solutions?"

Summing Up	
Teaching Activity	**Points of Consideration**
The teacher asks the students to write a reflection on "what did I learn today?" and "what would be a good name for this lesson?"	*Do students recognise that different approaches to solving a problem are acceptable and that there is more than one way to solve a problem?*
The teacher presents students with homework and explains what they are expected to do.	*What misconceptions are evident?*

Homework Task
Students construct their own cylinder from a sheet of paper and calculate its volume.

Evaluation

Peer observation is planned:
There will be five observers in the lesson along with the teacher.
One teacher will move through the room recording the lesson using Lesson Note App.
Another will take photographs of the students' work.
Three other teachers, at the side of the room, will observe and note students' interaction with the class with an emphasis on:
• How are they engaged during each task?
• How are they learning/successfully completing tasks?
• Student questions and responses.
• Quality of independent written work.
• How are they explaining when at the board?
• Evidence that the stated learning outcomes and aims of the lesson are being achieved?

Teachers will discuss their observations on the students learning during the class at a post-lesson valuation meeting, which will take place directly after the lesson is taught.

Board Plan
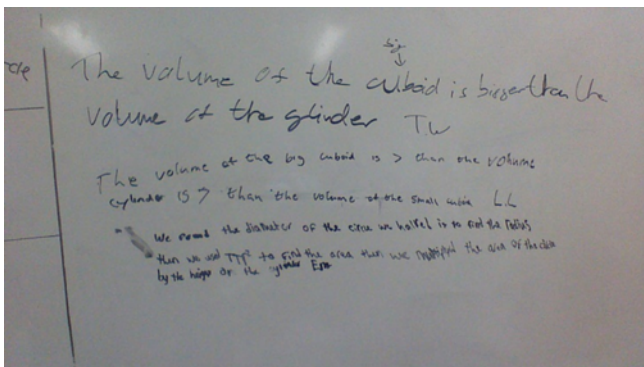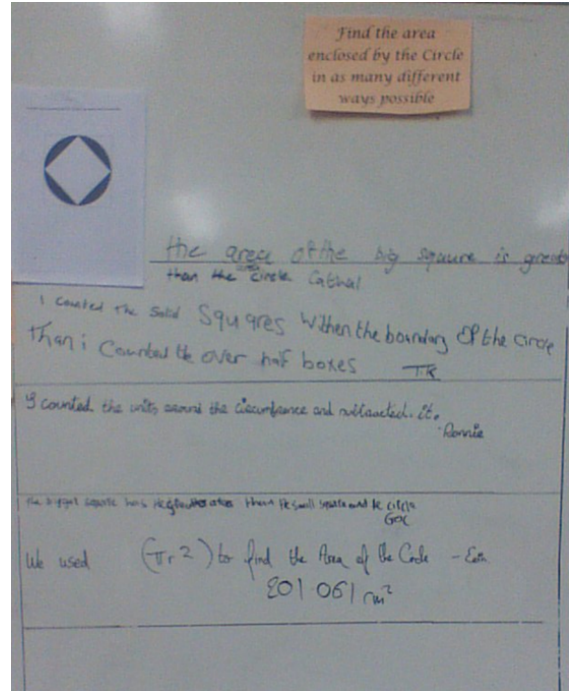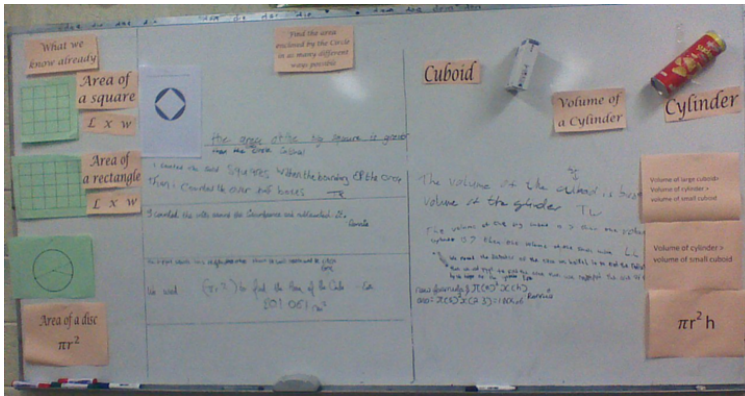
Post-lesson reflection

During the introduction, students were very confident with prior knowledge as expected by the teacher. They were able to recall correctly (using the correct mathematical terminology) the area and perimeter for a rectangle square and disc. This meant that they were ready to progress to the task.

What are the major patterns and tendencies in the evidence?

All students tended to begin by counting the boxes to find the area of the enclosed circle. In most groupings one student was aware of the concept that the area small box < area circle < area large box. Students also seemed at ease with the concept of 2D, identifying the shape and dimension quickly. Students were aware that height made an object 3D in regards to cylinder's and rectangular boxes. All groups worked together to find measurements of the cylinder and rectangular box before discussing possible ways to find volume, showing a confidence in using measurements.
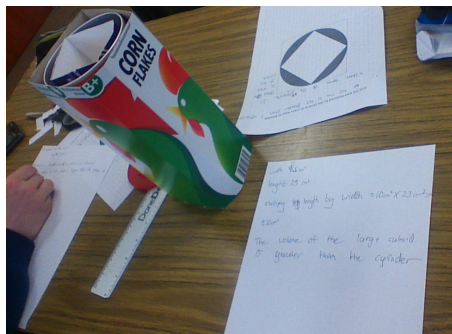

What are the key observations or representative examples of student learning and thinking?

We found that the students involved worked very well in pairs and liked the "hands on" measuring and counting. Students seemed to measure the diameter with a ruler and then calculate the radius.

Students were comfortable with a 3D object having 3 dimensions (l,w,h) and came up with a "new formula" π

We found that students during this lesson reinforced the emphasis of using π correctly.
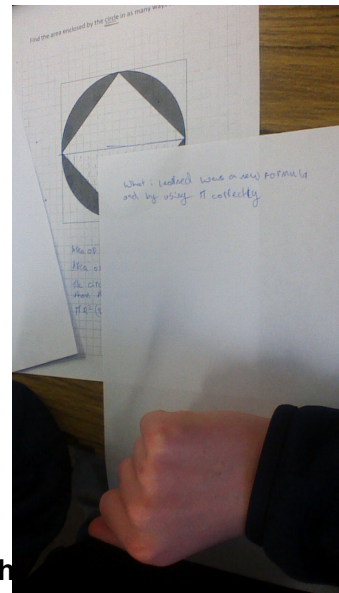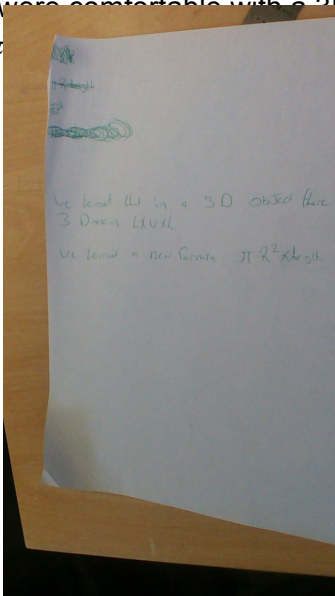
What does the evidence suggest about student thinking such difficulties, confusion, insights, surprising ideas, etc.?

We found that students did not "read the question properly", and several needed clarification of what was being asked. This reminded us to emphasise to students the importance of reading the question thoroughly and not "diving straight into the diagram".

In what ways did students achieve or not achieve the learning goals?

We found the students achieved their goal by moving easily from the area of a 2D shape to finding the volume of a 3D shape. Students linked the relationship between the area of a circle and the volume of a cylinder and developed their problem solving skills. They quickly realised the concept of area – 2D – $cm2$ and Volume – 3D –$cm3$.

It was observed that students showed confidence in their problem solving ability particularly when carrying out calculations. Students came up with all 5 anticipated answers and were able to describe the rationale behind their method of estimation.

Students met the learning objective of the lesson as they discovered the volume of a cylinder, and were confident in expressing their ideas mathematically by estimating, predicting and calculating data.
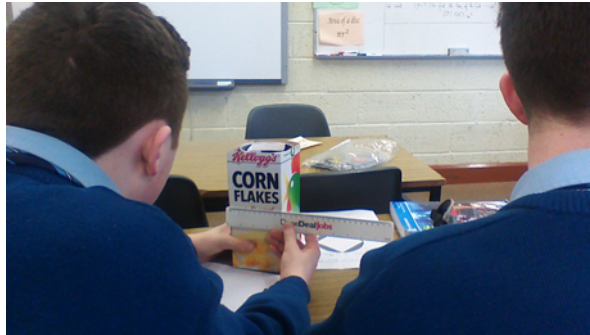
Based on your analysis, how would you change or revise the lesson?

We found that the boys took up a lot of class time writing their answers on the board. Perhaps we would make more of an emphasis on students explaining their answers in more detail to each other. Having every answer pre-prepared would not only remedy this, it would also alleviate the concern of illegible boardwork.
We would recommend that this lesson be taught during a double class.
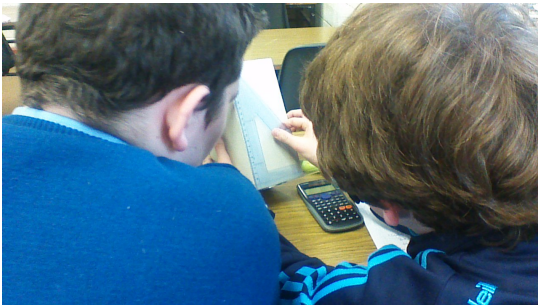
What are the implications for teaching in your field?

Lesson Study provides the opportunity to collaborate with teachers, which was viewed as incredibly valuable. The importance of Lesson Study is evident in the recognition that teachers really do want to facilitate active learning but the constraints of time and resources continue and the collaboration isn't always facilitated. Fostering collegial collaboration through Lesson Study while working together to design, teach and reflect on a structured problem-solving lesson provides teachers with the support and resources to allow them to continue to develop in the role of facilitating students as active learners.