5th Year Higher Level Maths: Coordinate Geometry of line and Circle

1. Title: The maths way through an ash cloud

For the lesson on 17th
January, 2017
At Adamstown Community College, Lucan, 5th Year
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2. The task:

A plane is travelling along the line x – y = 10, Ahead lies a large cloud of ash from a volcanic eruption that can be represented by the circle $x^2 + y^2 = 52$.

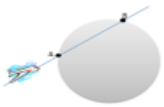
The power point slides:

$x - y = -10$ $x^2 + y^2 = 52$

Task 1

If it is considered unsafe to travel more than $10km$ through such an ash cloud, should the plane alter its course?

Each unit represents $1km$.



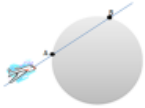
$x - y = -10$ $x^2 + y^2 = 52$

Task 2

It is also considered unsafe to travel further than $1km$ inward from the edge of the cloud.

Should the plane alter its course?

Each unit represents $1km$.



1. Aims of the Lesson: long-range/thematic goals:

- I'd like my students to appreciate that:
- 1) Mathematics can be used to solve real world problems and develop effective communication.
- 2) Algebra is a tool for making sense of certain situations
- 3) I'd like to foster my students to become independent and creative learners when devising approaches and methods to solve problems
- 4) I'd like to emphasise to students that a problem can have several equally valid solutions and
- 5) To experience meaningful mathematics i.e. that they see a need for what they are studying.
- 6) I'd like to build my students' enthusiasm for the subject by engaging them with stimulating activities and
- 7) Help them to connect, review and extend the concepts that we have studied.

2. Learning Outcomes:

As a result of studying this topic student will be able to:

- Draw an accurate graph of a circle and a line
- Use a graphical approach to find points of intersection.
- Use an algebraic approach to find points of intersection.
- Calculate the distance between two points
- Use Pythagoras theorem to calculate length
- Find the midpoint of a line segment.
- Calculate the perpendicular distance between a point and a line.
- Use a variety of approaches to find an unknown distance on a graph.

Background and Rationale

(a) Students will be familiar with the Midpoint, Perpendicular Distance, Graphing lines and circles and using algebra to solve linear/quadratic. According to the LC HL Syllabus regarding coordinate geometry students must be able to:

– recognise that $(x-h)^2 + (y-k)^2 = r^2$ represents the relationship between the x and y co-ordinates of points on a circle with centre (h, k) and radius r
– solve problems involving • the perpendicular distance from a point to a line • the angle between two lines

- recognise that $x^2 + y^2 + 2gx + 2fy + c = 0$ represents the relationship between the x and y co-ordinates of points on a circle with centre (-g,-f) and radius r where $r = g2 + f2 - c$
- – solve problems involving a line and a circle
- – solve problems involving a line and a circle with centre (0, 0).

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(b) As a group of teachers we discussed and agreed that student generally like the coordinate geometry of the line and circle. However, where many problems arise is when the students have to use a variety of different approaches to find the solution. In order to prevent this disengagement from the lesson we would like to reinforce the prior knowledge so that they really understand the topic and use various different approaches competently. The major challenge for students through this lesson proposal is to call upon these topics in a novel situation and to gain confidence in relating them to each other in various paths to solutions.

3. Research

Using our research, we looked at the higher level syllabus for junior and leaving certificate mathematics. Using this information and our own teaching knowledge it would allow us to see what prior knowledge and learning the students have. We have also used information from projectmaths.ie, the internet and text books.

Project maths values the use of practical 'everyday' maths so we decided that we would incorporate this into our lesson plan. This would allow for students to become more engage in the lesson.

4. About the Unit and the Lesson

From the LC HL maths syllabus the students should be able to:

- explore patterns and formulate conjectures
- explain findings
- justify conclusions
- communicate mathematics verbally and in written form
- apply their knowledge and skills to solve problems in familiar and unfamiliar contexts
- analyse information presented verbally and translate it into mathematical form
- devise, select and use appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions

5. Flow of the Unit:

Lesson		# of lesson periods
1	• Equation of the circle (with centre (0,0) Draw a circle. • Equation of a line also	1 x 40 min.
2	• Investigating if points are inside outside or on the circle • Translating the centre of the circle to centre (h,k)	1 x 40 min.
3	• Perpendicular distance between a point and a line	1 x 40 min
4	• The intersection of a line and a circle	2 x 40 min. **(research lesson)**
5	• Finding tangents to the circle	2 x 40 min.
6	• Finding common chords or tangents	1 x 40 min
6	• Follow up and conclusion	1 x 40 min.

Flow of the Lesson:

(Note: This was a double class of 80 minutes which explains why the two tasks were given. For shorter classes we would only present the first task and leave the second for homework)

Teaching Activity	Points of Consideration
1. Introduction (10 Mins) 1. Ask the class to come in and take their seats. 2. Check their homework and understanding from the previous class. 3. Take the roll. 4. Introduce the topic via questioning their prior knowledge.	1. The class will come in and will be greeted. They will settle down and take their seat. 2. Students may present their homework or any solutions/questions from the previous class. 3. Students will explain to me orally what they know about the topic on hand or what they think they know about this topic.
2. (a) Posing the Task and work on it (15 mins) 1. Task 1 on the Powerpoint – brief introduction before allowing the students to work. 2. Encourage as many ways to solve the problem as possible. 3. Different materials will also be available for them. (graph paper, rulers , compasses.)	1. Students will look at the slide and try and figure out what the problem could be just by looking at the title of the lesson. 2. Students will take the worksheet and will try and the teacher will go through the worksheet and answer any questions that may arise. This will then tell us if the students understand the task at hand.

(b) Boardwork and discussion for task 1. (20 mins) 4. **Introduce task 2** – question posed on Powerpoint. Students will have **15 minutes** to work through it and **20 mins** for boardwork and discussion.	
3. Anticipated Student Responses Students may have the following approaches: For task 1: Method 1 – Drawing a graph and measuring using a ruler. We may have a worksheet with this for them). Method 2 – Algebra using simultaneous equations to find the point of intersections. Step 1 distance formula/Pythagoras theorem also. For task 2: Method 1: Step 1 – midpoint formula versus translations Step 2 Find OP using distance, Pythagoras or the trigonometric ratios (sin) to find the distance between Q and P Method 2 Using the perpendicular line, find the point of intersection with the circle (Q). Using the perpendicular distance to find PQ	For students who finish early or are unable to solve the problem, the teacher will be on hand to try and push the students in the right direction or give a little bit of help. This should not be too much as we want the students to think for themselves. For students that give incorrect solutions, these will also be noted and explained to the class ad it will allow others to see what the student was thinking. It is importance to reinforce that it doesn't matter if the solution is wrong but they tried different mathematically approaches to come up with some sort of solution.
4. Comparing and Discussing (Ceardaiucht) We think that all of the student's methods should be shared with the class. We will especially look out for novel solutions that we have not thought of in our planning. Where students have taken "wrong" turns with methods that did not work will also be of great interest .These may highlight misconceptions and guide us in engaging the class as to why these didn't work, furthering engagement with the topic and thinking mathematical and critically.	Students will benefit from this as they will figure out that there is not just one way to do this solution but many other ways. This will hopefully give an increased interested to the subject but to also get them thinking more critically too. The ability to see connections is very important.
5. Summing up 1. Go back to the learning intentions and ask the class if they have achieved these. This will probably be done by questioning. 2. Assign the homework to the students and ask them to put in their journal. Note the homework:	1. Students will again look at the learning intentions and will answer questions based on today's work. 2. They will be assessed using the success criteria via questioning and other methods. 3. Students will take down homework in their journal and may ask questions on it. 4. Students may begin homework if there is time.

6.

7. Evaluation

- What is your plan for observing students?

There will be 4 teachers in the room, looking and walking around a designated section of the class as the students complete the lesson. Notes may be taken on what went well, visual and oral clues from the students – observations from the teachers are very important as these aspects will give us valuable feedback for reflections after the class.

- Discuss logistical issues such as who will observe, what will be observed, how to record data, etc.

Other adults will listen to the ideas that the students had in coming up with solving the task. As we will have prepared ideas that might come up, we will be looking out for these methods as well as any others from the students. Ideas that we have not thought of will be looked at in detail. The teaching teacher will take names of students who will later present at the board (and get their agreement to do so) and choose the order. The other teachers, with a seating plan, are noting what students are doing as the lesson proceeds.

- What observational strategies will you use (e.g., notes related to lesson plan, questions they ask,)?

During the preparation, we will have a list of questions that they might have and we will be ready to answer them even if just to help them if they are stuck or have no ideas. Each observer will have a copy of a list of questions such as, "did the student use a ruler?"; "Was the distance formula used?" etc. Aside from the main teacher, the observers will have minimal contact with students but will note if they are asked any questions and what help they subsequently gave.

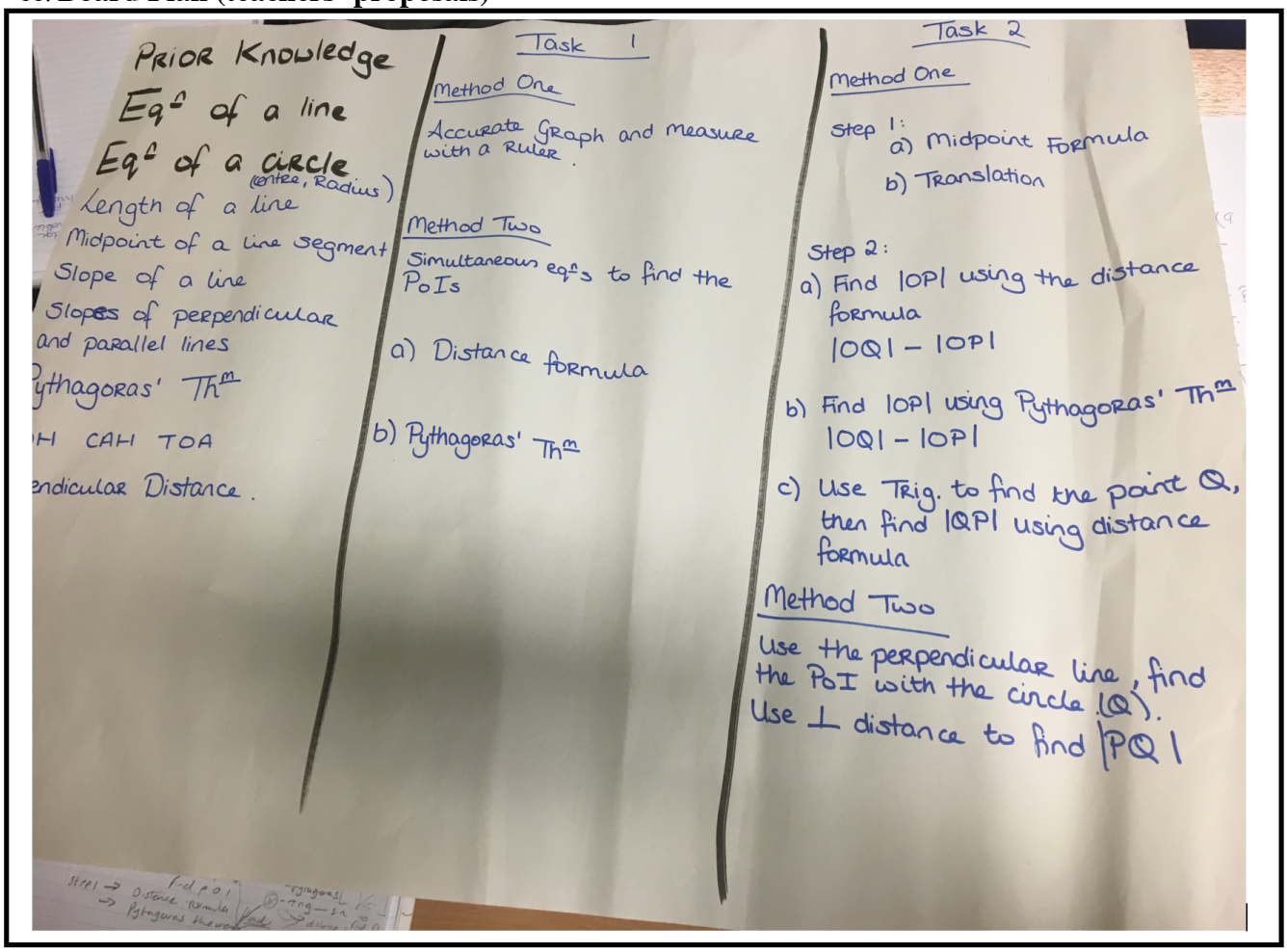
What types of student thinking and behaviour will observers focus on?

Students should be able to think algebraically, visually and using practical mathematics such as measurements using rulers and ratios from previous knowledge. Students will also have to think critically while using their own assumptions and reasoning on whether the plane such divert or not. This also prepares them from rational thinking. Again, the list of questions will be important here.

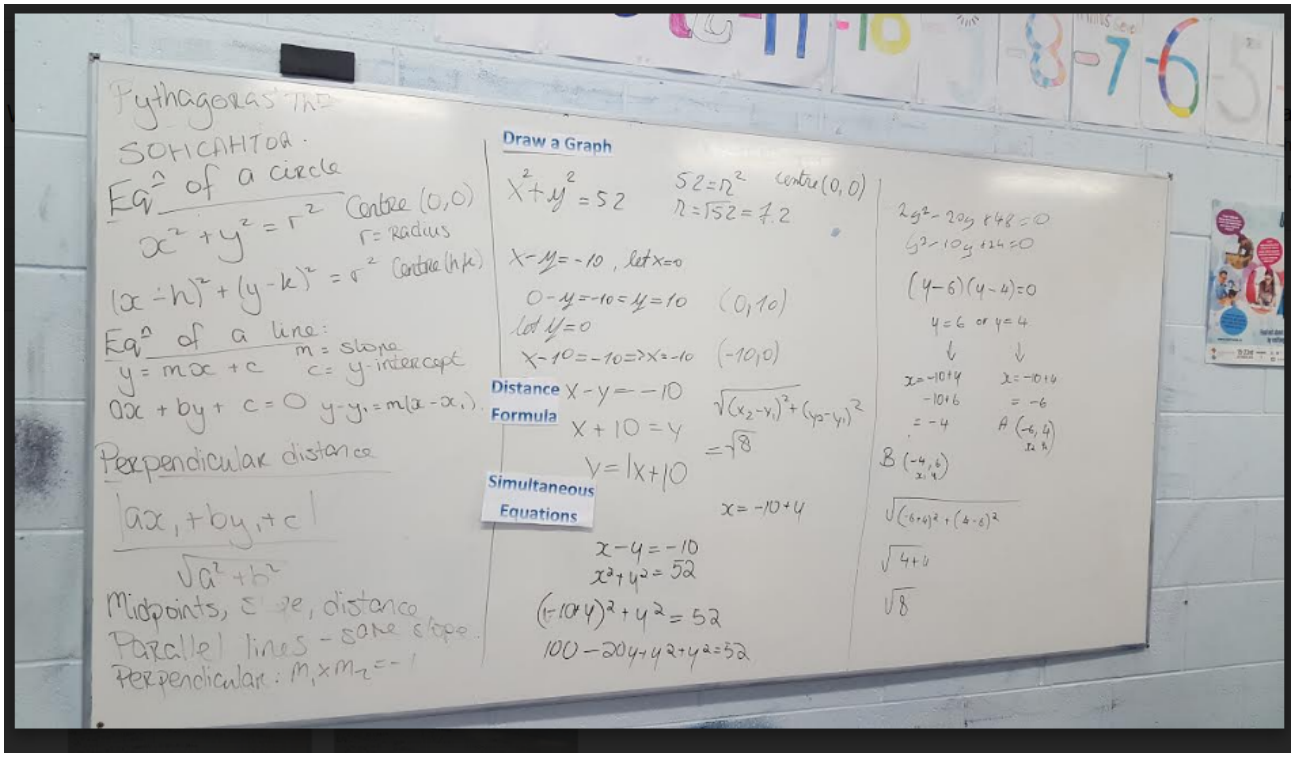
- What additional kinds of evidence will be collected (e.g., student work and performance related to the learning goal)?

Some of the student's work will be collected and displayed on the board. Students ideas will also be noted – ones that we may not have come up with. Reflections on the lesson will also be collected to let us know what we should change in the figure (if any) for this lesson. We will look at what went well, what didn't etc. based on the learning outcomes and if the students achieved these or not.

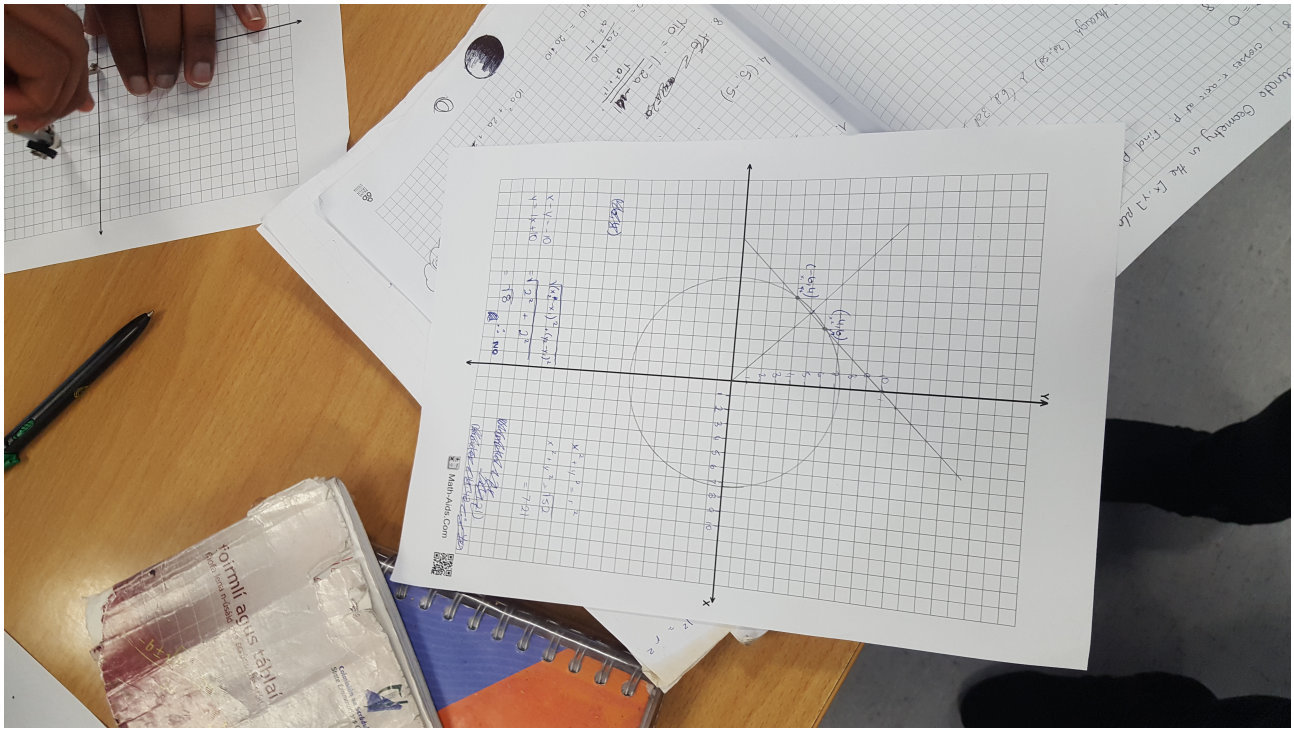
11. **Board Plan (teachers' proposals)**



Student work



The intention with the board plan is to have all the work presented by the selected students on display by the end of the lesson, along with the teachers' pre-planned headings. Any novel methods will be also highlighted as such.



8. Post-lesson reflection

- **What are the major patterns and tendencies in the evidence?**

Most students took some time to gain a full understanding of the problem.
Many were unsure where the line was in relation to the circle and quite a few did not draw the graph initially.
Many of them started by drawing a picture rather than correctly plotting a graph. They realised their mistake when they saw what others were doing.
Nobody measured it with a ruler – one student tried to estimate it using the squares.
Once they had the abstract graph they forgot about the initial problem, no longer looked to answer the problem about the ash cloud.

- **What does the evidence suggest about student thinking such as their misconceptions, difficulties, confusion, insights, surprising ideas, etc.?**

Quite a few students found the perpendicular distance from the midpoint to the line itself and were surprised to get zero.
There was a lot of dialogue and discussion – students were discussing the merits of

various methods.
Nobody used translations.
Nobody used Trigonometry angle – one student did draw the triangles but couldn't recognise that a slope of -1 created an angle of 45°.

- **In what ways did students achieve or not achieve the learning goals?**

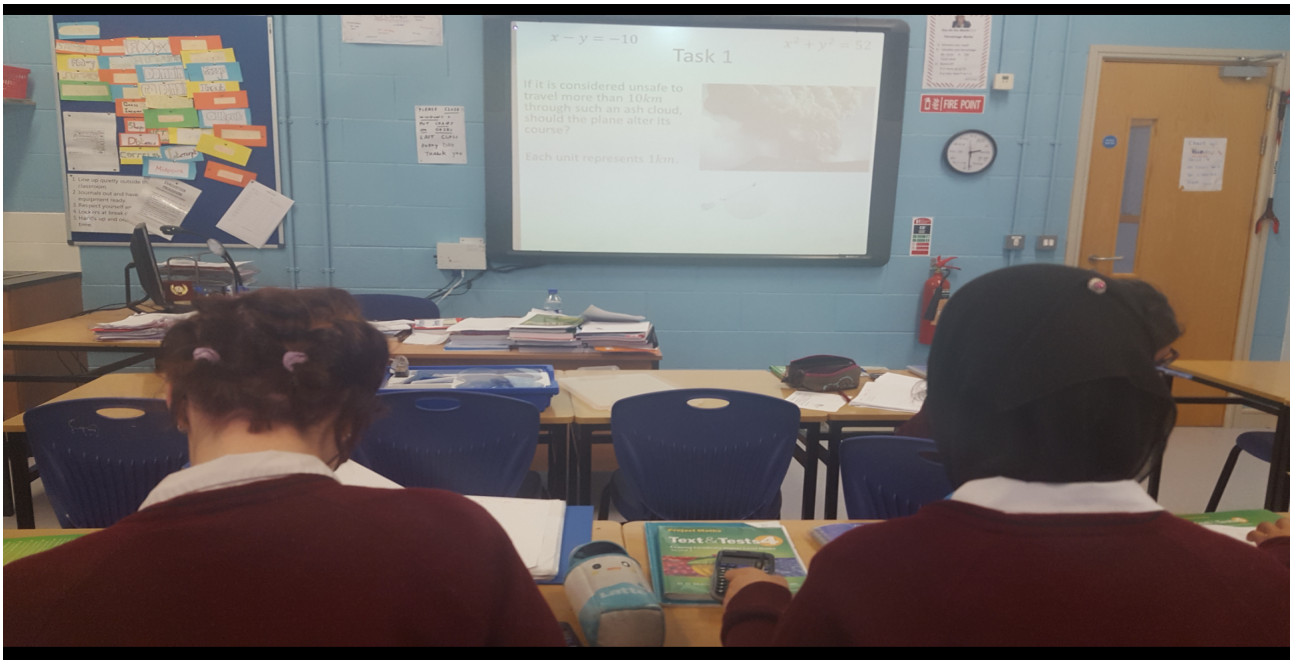
Most students achieved the learning goals – they can find the point of intersection of a line and a circle.
They can also find the length of a line using a variety of approaches.
One student kept redrawing his graph without any calculations – was not gaining anything from each graph.
They coped better with the second task – it was a more complex task but they seemed to have more confidence approaching it.

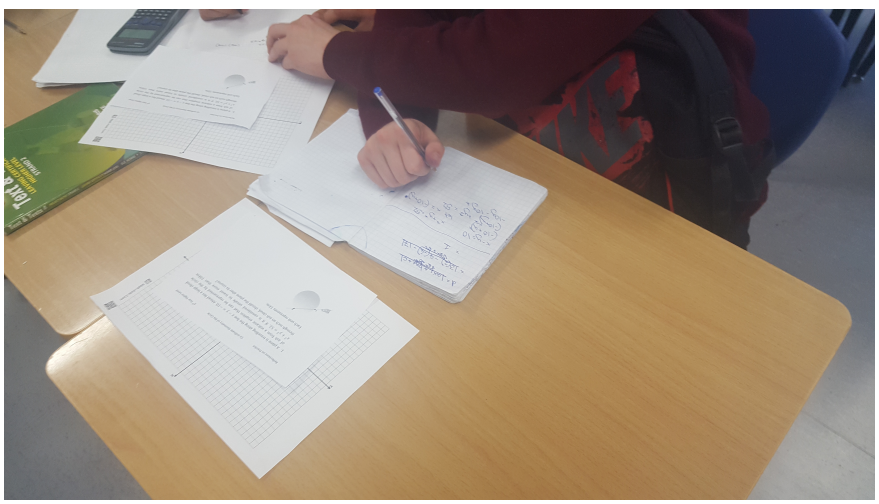
- **Based on your analysis, how would you change or revise the lesson?**

If teaching the lesson again it would be beneficial to spend more time on the introduction to task 1. It is important to ensure students have fully understood the scenario we have created before they start work on the task.

- **What are the implications for teaching in your field?**
  While it is not possible to use a problem solving approach to every topic as there are often severe time restrictions it is very beneficial to use it from time to time to reinforce students' learning. They benefit from greater time spent on a topic and often learn more from each other and the variety of approaches they take. Using a problem to help students to think about their mathematical knowledge and apply it to a different situation reinforces their understanding of a topic and helps them to gain a sense of its real world application

Students at work:





ash from a volcanic eruption that can be represented by the circle $x^2 + y^2 = 52$.