Picture This! Exploring Quadratic Functions Through Images

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St Laurence's College,
3rd year class
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Title of the Lesson: **Exploring Quadratic Functions Through Images.**

To help students understand the connection between the roots of a quadratic graph and the quadratic function itself and model simple quadratic functions

Aims of the Lesson

Students can find it difficult to relate functions as equations to their graphs. The syllabus challenges teachers to develop student understanding of a range of functions and to assist them in making connections between the graphical representations and functions. This lesson and the extension exercises attempts to bridge this gap and build on student prior knowledge using concrete real-life examples. There is an emphasis on the use of roots as the primary method for image modelling. The teacher can control the difficulty of the exercise by altering the scale on the y-axis. Alternative methods for approaching the problems include use of simultaneous equations. Students can check their functions and explore addition properties of functions such as y-intercept.

This lesson helps students view functions as dynamic and useful. There is opportunity for students to work together to construct and deepen their understanding of functions and how they can be applied to real world situations and problems. Students can use their previous knowledge to develop both their questioning and problem solving skills assisting their growth as independent learners.

Learning Outcomes:

Through studying this topic students will be able to:

- Understand the importance of roots.
- Use these roots to formulate an algebraic quadratic function.
- Use quadratic functions to model real world images

Background and Rationale

The students should be able to

- Provide an explanation for the following terms quadratic function, domain, range, quadratic factors and roots
- make use of function notation
- interpret quadratic graphs
- plot points and quadratic curves
- find max and min points

Difficulties that students have had in the past with this matter include

- misunderstanding of function notation
- arithmetic errors in calculating points for graphs
- scaling problems in drawing graphs
- misunderstanding the meaning of graphs
- confusing the relationships between the x-axis and y-axis in real-life situations

The thematic focus of this lesson study is to show students

- Quadratic functions can be used to model real-life structures and shapes all around them in the everyday real world.
- Through this lesson they will develop their independent learning skills.
- Through pair/group work within this lesson, students will learn to work together effectively with others.
- The problems presented in this lesson will hopefully encourage students of all abilities to ask questions and investigate their task through questioning.
- Through independent and group work, students will hopefully present with several ways in which to complete their task and explain their solutions.
- Within this lesson, students should be able to link the problems presented to previous knowledge they have already obtained.

Research

The lesson will be based on 4 quadratic graphs and formulating the function from these graphs.

1 U-shaped graph with two distinct roots and one distinct point.
1 n-shaped graph with two distinct roots and one distinct point.
1 U-shaped graph with 0 as one of the two distinct roots and one distinct point.
1 unlabelled graph for the students to investigate.

The syllabus has been consulted in the designing of this lesson with a clear connection made between the syllabus and aims of the lesson. Past exam papers and mock exam paper questions have been analysed. Text books have been screened for potential examples and questions to aid the lesson. The lack of real world examples of quadratic equations in existing material was deemed an area which needed further work. Quadratic functions from the real world have been sought through the internet. Quadratic functions in real-life contexts have been created using GeoGebra.

About the Unit and the Lesson

This lesson aims to give students an understanding of how the roots of a function on a graph can be used to formulate that function. It is hoped that students will be able to find ways of completing this function using information from the graph.

Flow of the Unit

Lesson		# of lesson periods
1	Factorising Quadratic Expressions (or review of prior knowledge on factorising quadratic equations)	5 x 40 min.
2	Function Notation, definition etc.	5 x 40 min.
3	Graphing Quadratic Functions and Real-Life Problems. Using project maths resources on shifting and scaling http://www.projectmaths.ie/documents/PDF/ActivityOnShiftingAndScalingGraphs.pdf?strand	4 x40min.
4	Identifying a Quadratic Equation from a graph **(research lesson)**	1 x 40 min.
5	• Interpreting Real-Life Graphs (using extension material)	2 x 40 min.

Flow of the Lesson

Teaching Activity	Points of Consideration
1. Introduction Review of Terminology. Factorise a Quadratic Equation. Show students a Quadratic Function graphed on the board. Allow students to explore the link between the solved quadratic and the roots.	Check that students understand how to recognise and factorise a quadratic trinomial equation.
2. Posing the Task Students will be given a U-shaped graph and asked to find out as much information about the graph as possible and attempt to find the quadratic equation represented by the graph.	Ask a student to state and domain and range of the function. Ask them to find the factors

See Materials question one. Depending on the ability of the class further examples may be provides to solidify and extend their knowledge.	Then to formulate the equation of the quadratic function using these factors. Is there an alternative method? What is the significance of the y-intercept and can this provide information on our quadratic?
3. Anticipated Student Responses R1: Some students may not be able to start the task. R2: Students may recognize the point and roots and manage to note these down but have no idea what to do with them. R3: Students may want to use points to "sub" into general quadratic equation but cannot remember exactly what this general equation looks like. R4: Students may struggle with a root being 0. R5: Quadratic with no axis or units.	T1: Teacher could suggest to study the domain, range and roots and what they mean comparing to the Quadratic function factorised at the start of class. T2: Factors could be hinted at and suggested again referring to the original quadratic factorised at the beginning of the class. T3: Teacher could try lead these students to ax^2+bx+c=0 again referring to the original quadratic factorised at the beginning of the lesson. Some assistance may be needed in showing them the factor when x=0. T5: Students may need to be led to explore placing axis and units on the quadratic shape in response to the previous completed graphs.
4. Comparing and Discussing Try pick students who • Found roots and/or point • Found factors • Multiplied these factors • Student who subbed in points into general equation • Student who formed two simultaneous equations • Student who solved these equations • Student who wrote quadratic function in full from solved a, b and c.	Focus on the roots, points, x, y values. Check with students how many of them did in similar ways and in different ways.
5. Summing up This should reinforce the connection between the roots of a quadratic graph and the algebraic function itself.	A recap on all the student's ways of completing the task should be incorporated in the summing up of the lesson.

Evaluation

Individual student groups were viewed by the observing teachers. The teachers were particularly interested in the students' ability to extract information from the quadratics provided to them. Teachers looked for evidence that the students could identify the difference between a positive and a negative quadratic, that students could identify roots and that students could use roots to find corresponding factors. The teacher could prompt students to formulate the factors if deemed necessary and the teachers were interested in the level of prompting needed and how this could be reduced in future lessons. The observing teachers looked for evidence of students use of mathematical language. In particular, the terms identified as prior knowledge. Teachers paid attention to student talk looking for evidence of sense making among the groups.

Board Plan

Prior Knowledge

- A review of the terminology of functions; quadratic, domain, range, roots, factors
- Example of factorising (as a prompt to finding the equation form the roots)
- Example of function plotted on the board with table.

Student Problem solving

- Copy of question projected onto the board
- Student understandings of the quadratic explored on the board under the heading used during the matching exercise leading to the formula for the quadratic.

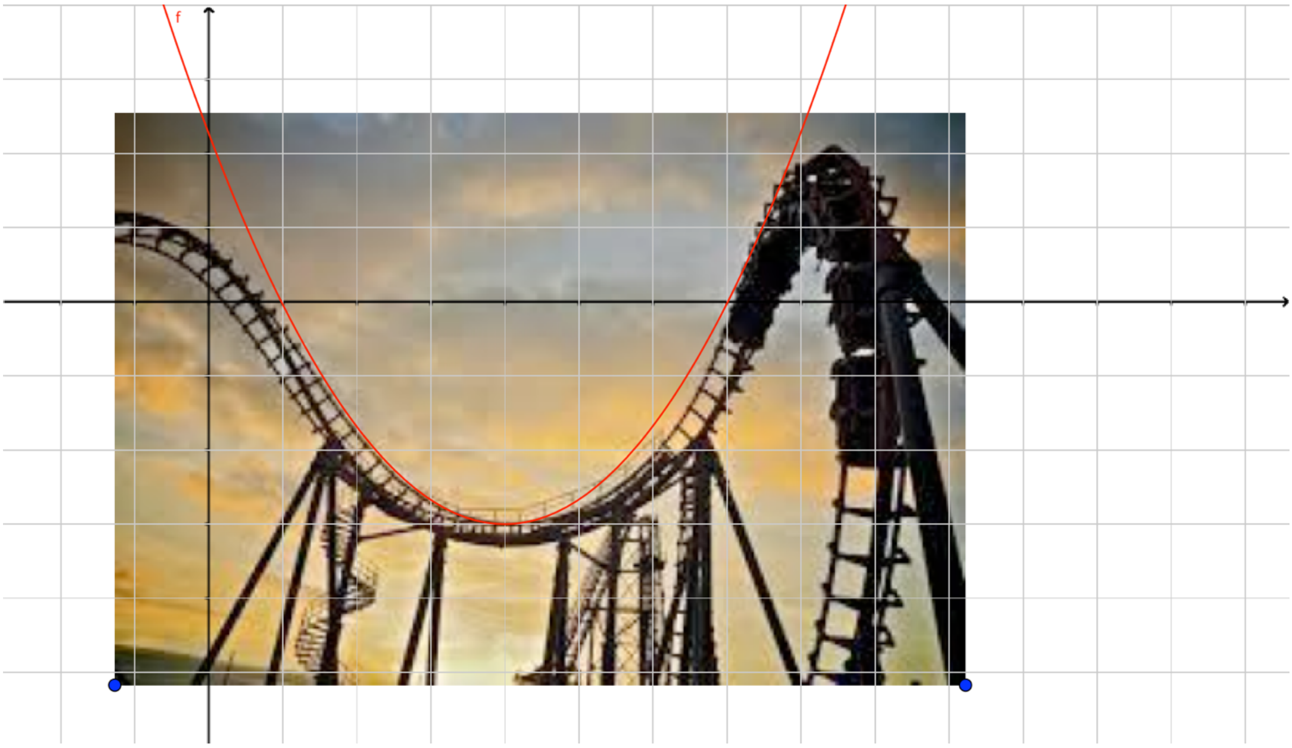
Post-lesson reflection

Students could identify the roots and formulate the factors without teacher prompts. They identified and could use the y-intercept in their attempt to find the function. Some students attempted to use simultaneous equations with varying success, sometimes computational errors slowed down their progress. All students could identify roots and relate them to a potential quadratic solution. There was a tendency among students to leave their function as equal to zero and not put it back as equal to f(x), making checking their equation difficult. More care needs to be taken when teaching roots to point out that f(x)=0 describes the roots but for the function, f(x) is only equal to zero on the x-axis and nowhere else. The students benefitted from completing the project maths scaling and shifting exercises prior to the lesson. The extension exercises three and four where no needed within the 40 minute period of the lesson but where used in later classes. The plenary of the lesson took longer than the usual 10 mins and it is recommended that material be prepared in advance for the board such as the graphed function and table for plotting the function. Overall, the students were engaged by the lesson and where interested in the application of functions. The lesson did help solidify the student's knowledge and understanding of roots, functions and factors of quadratic equations.

Materials

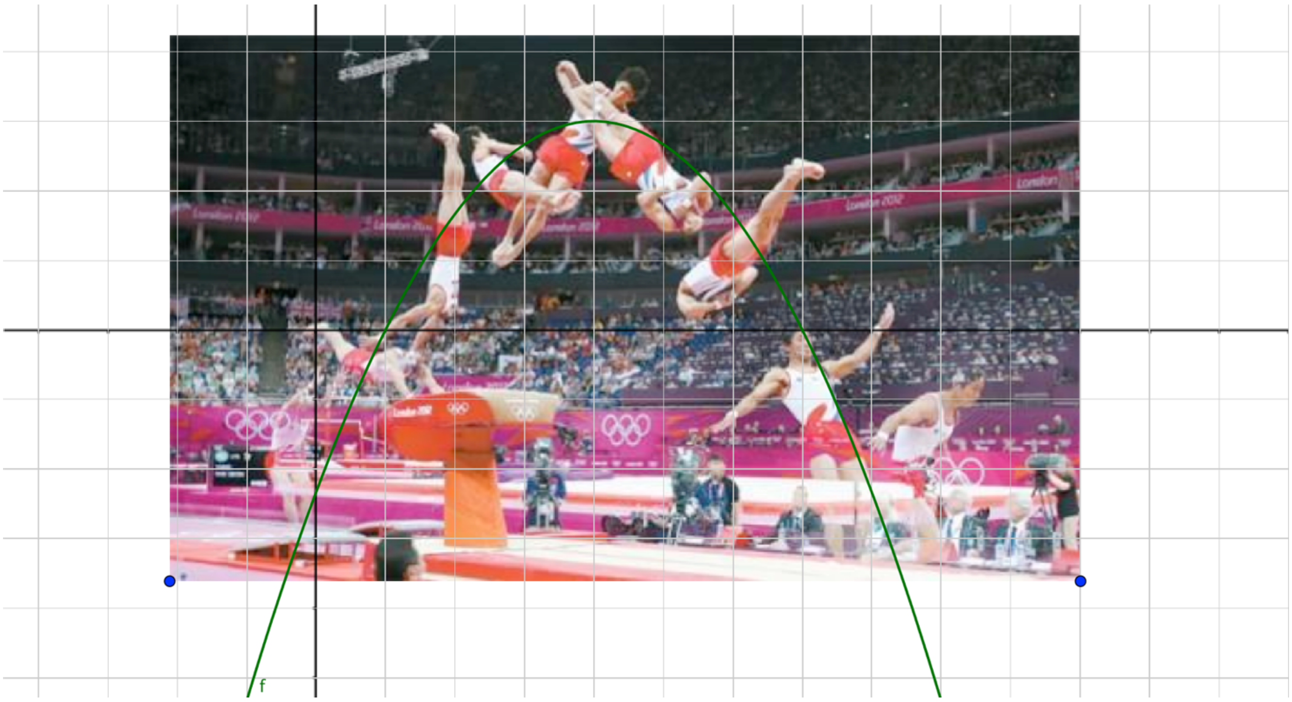
Question One:

Scales need to be added to give a quadratic of (x-1)(x-7), 1 unit per box for the x-axis and 3 units per box of the y-axis.



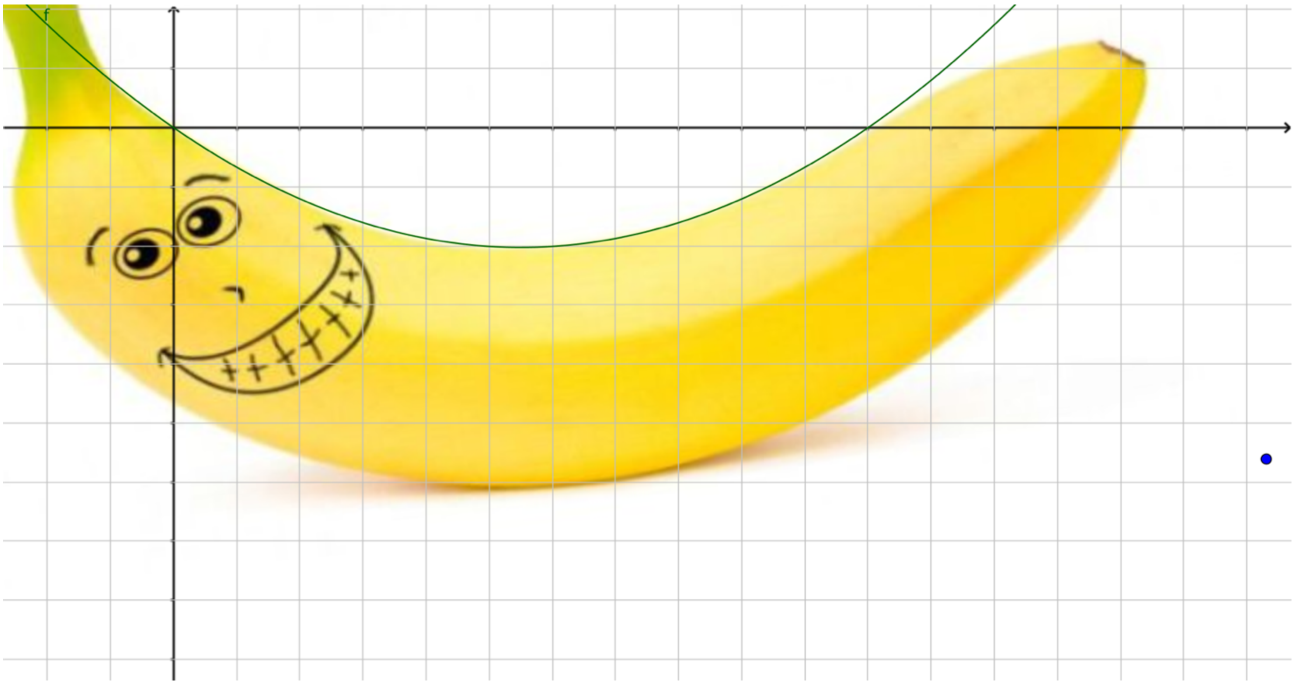
Question 2

Scales need to be added to give a quadratic of -(x-1)(x-7), 1 unit per box for the x-axis and 3 units per box of the y-axis.



Question 3

Using the factors find the equation of the quadratic function, apply appropriate scales



Question Four:

Find an equation to represents Sid's smile and apply appropriate axes (you will need to plot your function to check your function models the smile)