2nd year Higher Level – Forming a Quadratic Equation

Lesson Plan taught: 6th Feb'17

At O' Carolan College, Nobber, Second Year Higher

Teacher: Clodagh Monaghan

Lesson plan developed by: Clodagh Monaghan (O' Carolan College)

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Nicola Mc Grane (Old Bawn Community School, Tallaght).

1. **Title of the Lesson:** Pragmatic Quadratic

2. **Brief description of the lesson:** The lesson aims to enable students to form a quadratic equation from a word problem with a diagram provided showing dimensions.

3. **Aims of the Lesson:**

   a) Short-term/content goals

- Students will select and use suitable strategies to find the area of a figure made up of combinations of rectangles, recalling prior knowledge of operations, algebra and area.
- Students will find the underlying formula written in words and presented in a diagram.
- Students will communicate to others how they got their equation and why it works.

   b) Long-range/thematic goals:

- To appreciate that mathematics and algebra can be used to solve real world problems.
- To foster students to become independent learners and thinkers and develop enthusiasm for the subject by engaging them with stimulating activities.
- To emphasise to students that a problem can have several equally valid approaches and solutions (Key Skill: Managing Information & Thinking: Thinking creatively and critically).
- To connect and review several concepts across the Strands of the syllabus previously studied.

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- To encourage students to discuss the mathematics they are studying and to learn from each other's viewpoints (Key Skill: Communicating: Listening and expressing myself, discussing and debating).

c) We would like to support our students in developing their literacy and numeracy skills through discussing ideas[1].

4. Learning outcomes:

As a result of studying this topic students will be able to:

- Manage information and recognise prior knowledge, making links across the Strands (i.e. operations, algebra and area) (Key Skill: Managing Information and Thinking).
- Create an equation from a word problem and a diagram.
- Use suitable strategies to form the quadratic equation.
- Explain their findings and communicate their thinking to others (Key Skill: Communicating).

5. Background and Rationale

(a) What the students need to learn according to the syllabus:

(Junior Certificate Mathematics Syllabus, for examination from 2016, page 27).

4.3 Finding formulae	Ways to express a general relationship arising from a pattern or context.	– find the underlying formula written in words from which the data are derived (linear relations) – **find the underlying formula algebraically from which the data are derived (linear, quadratic relations)**

(Syllabus, page 29)

4.8 Synthesis and problem-solving skills	– explore patterns and formulate conjectures – explain findings – justify conclusions – communicate mathematics verbally and in written form – apply their knowledge and skills to solve problems in familiar and unfamiliar contexts – analyse information presented verbally and translate it into mathematical form – devise, select and use appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions.

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[1] This lesson Proposal illustrate a number of strategies to support the implementation of Literacy and Numeracy for Learning and Life: the National Strategy to Improve Literacy and Numeracy among Children and Young People 2011-2020 (Department of Education & Skills 2011).

Also, syllabus page 24.

3.4 Applied measure	Measure and time.	– calculate, interpret and apply units of measure and time
	2D shapes and 3D solids, including nets of solids (two-dimensional representations of three-dimensional objects).	– solve problems that involve calculating average speed, distance and time
		– investigate the nets of rectangular solids
	Using nets to analyse figures and to distinguish between surface area and volume.	– find the volume of rectangular solids and cylinders
	Problems involving perimeter, surface area and volume.	– find the surface area of rectangular solids
	Modelling real-world situations and solve a variety of problems (including multi-step problems) involving surface areas, and volumes of cylinders **and prisms.** The circle and develop an understanding of the relationship between its circumference, diameter and π.	– identify the necessary information to solve a problem – select and use suitable strategies to find length of the perimeter and the area of the following plane figures: disc, triangle, rectangle, square, and figures made from combinations of these

(b) Difficulties students have had in the past with the subject matter:

The reason we chose this topic is because in the past we ourselves as teachers have found that students have difficulty generating equations from word problems. During this lesson we also want students to understand the importance of the equals sign and its role in forming a quadratic equation that can be solved.

Teachers in our group have also experienced the following:

- Students' confusion between perimeter and area.
- Students' confusion regarding the equals sign.
- Students' difficulties with expanding algebraic expressions.
- Students' difficulties with forming quadratic equations from a word problem (diagram included).

We hope this lesson proposal will aid students in these areas.

(c) The thematic focus of this lesson study is for students to be able to appreciate that algebra is a tool for making sense of certain situations and that a problem can have several equally valid approaches and solutions. We hope to address this by providing the students with a

suitable problem that is within their capabilities and explore through the class the practicalities and different possible approaches to forming a quadratic equation. We also hope that by using a word problem involving area we are enabling students to connect concepts they have learnt from other sections of the course.

6. Research

We researched the syllabus and the Project Maths second year handbook to ensure that our lesson linked in with the aims of the curriculum (www.projectmaths.ie). We examined past exam papers (via *www.examinations*.ie) to find a suitable question that we could model our problem on. We then looked at various Maths textbooks to find a question that met the aims and objectives of our lesson.

7. About the Unit and the Lesson

This lesson involves an area problem of an irregular shape that leads to a quadratic equation.
As per the Junior Certificate Syllabus:

- 3.4 Applied measure: Students should learn about problems involving perimeter, surface area and volume.

- 4.3 Finding Formulae: Students should learn about finding formulae ways to express a general relationship arising from a pattern or context. Students should be able to find the underlying formula algebraically from which the data are derived (linear, quadratic relations)

- 4.6 Expressions: using letters to represent quantities that are variable and apply to real life contexts. Students should be able to simplify and expand expressions.

- 4.7 Equations & Inequalities: Students should learn about equations and inequalities, selecting and using suitable strategies (graphic, numeric, algebraic, mental) for finding solutions to equations and inequalities. Students should be able to identify the necessary information, represent problems mathematically, making correct use of symbols, words, diagrams, tables and graphs.

- 4.8 Synthesis and Problem Solving Skills: Students should be able to explore patterns and formulate conjectures, explain their findings, justify conclusions, communicate mathematics verbally and in written form, apply their knowledge and skills to solve problems in familiar and unfamiliar contexts, analyse information presented verbally and translate it into mathematical form, devise, select and use appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions.

As per the Project Maths Second Year Handbook:

- Lesson 2.6 (p.14) - Adding and subtracting simple algebraic expressions.
- Lesson 2.7 (p.15) - Use of the distributive law to multiply expressions.
- Lesson 2.9 (p.16) - These lessons will involve the students in investigating & understanding: The concept of equality and what is meant by an equation.
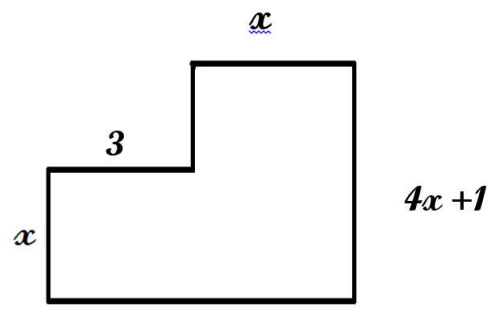- Lesson 2.10 (p.17) - Solve quadratic equations. Solve simple problems leading to quadratic equations.

8. Flow of the Unit

Second Year Teacher Handbook based on the 2016 syllabus.

Lesson		# of lesson periods
1	• Revision and extension of algebraic expressions and simple linear equations from first year (JC 2.6)	3 x 35 min.
2	• Algebraic factors (Use of the distributive law to multiply expressions) (JC 2.7)	3 x 35 min.
3	• Linear equations in one and two variables and linear inequalities in one variable (JC 2.9)	6 x 35 min.
4	• Solve quadratic equations (Solve simple problems leading to quadratic equations) (JC 2.10)	3 x 35 min. **(1 = research lesson)**

9. Flow of the Lesson

Teaching Activity	Points of Consideration
1. Introduction (8 minutes) Learning intention will be shared with the students. "Form an algebraic equation from a word problem accompanied by a diagram."	Teacher will show key works and ask the students to explain and identify examples. Variable Coefficient Like terms

Verbally prior knowledge is tested through questioning of the students. Asking students how they would find the area of a rectangle, how they would add/subtract algebraic terms and how they would multiply out/expand algebraic expressions.	Simplify Constant Solve Equal sign Addition, Subtraction and multiplication (3 methods) of algebraic terms.
2. Posing the Task (2 minutes) A lady wishes to carpet her sitting room. She knows that the area of the room is $24m^2$. Use the given diagram to form an algebraic equation to represent the area of the room. (Worksheet – see appendix 1)	
3. Students Individual work (10 minutes)	Teacher will circulate the room assessing students work using the Record Sheet (see appendix 2) which will help to plan the presentation of students' work on the board and class discussion.
3. Anticipated Student Responses Five anticipated responses starting with the most common anticipated answer ending with the least expected answer.	Teacher will chose students who had tried the 5 anticipated methods to present their work on the board.

Method 1:

Recognises 2 shape with all required sides known
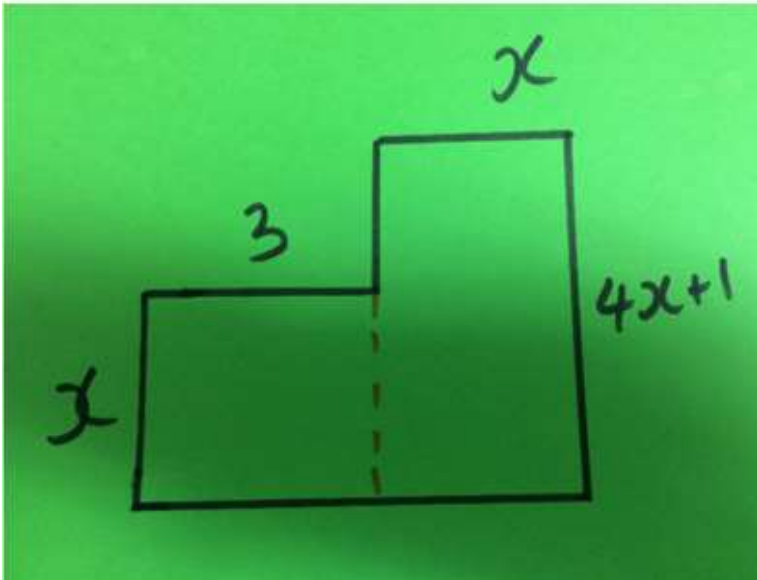
Method 2:

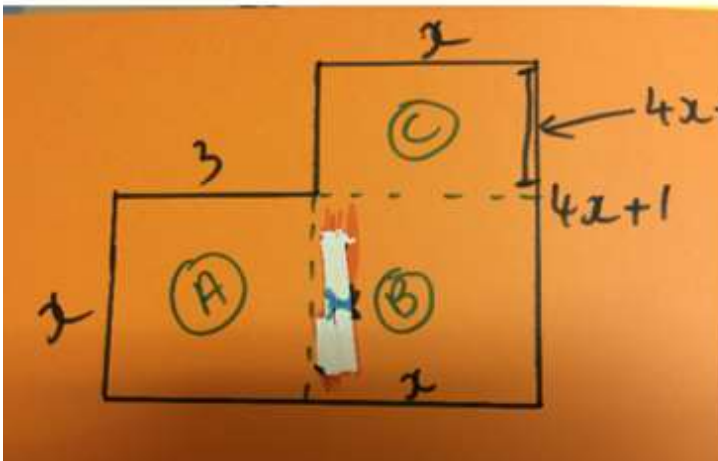Students added a section to make one large rectangle and subtract the extra section.
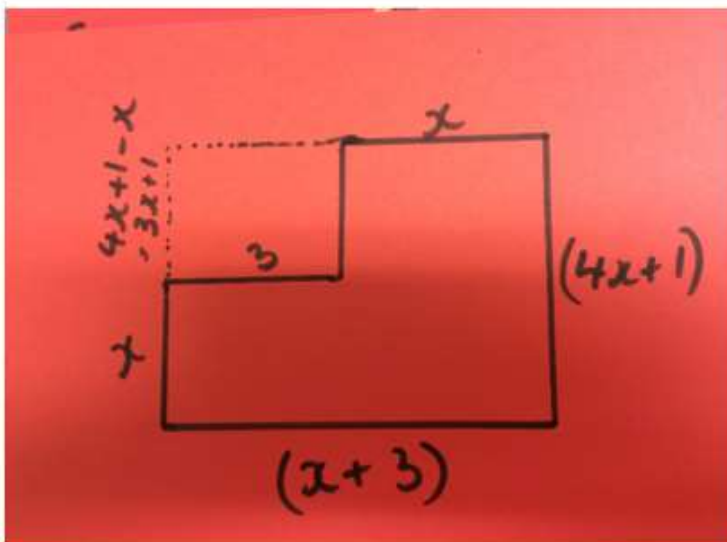
Method 3:

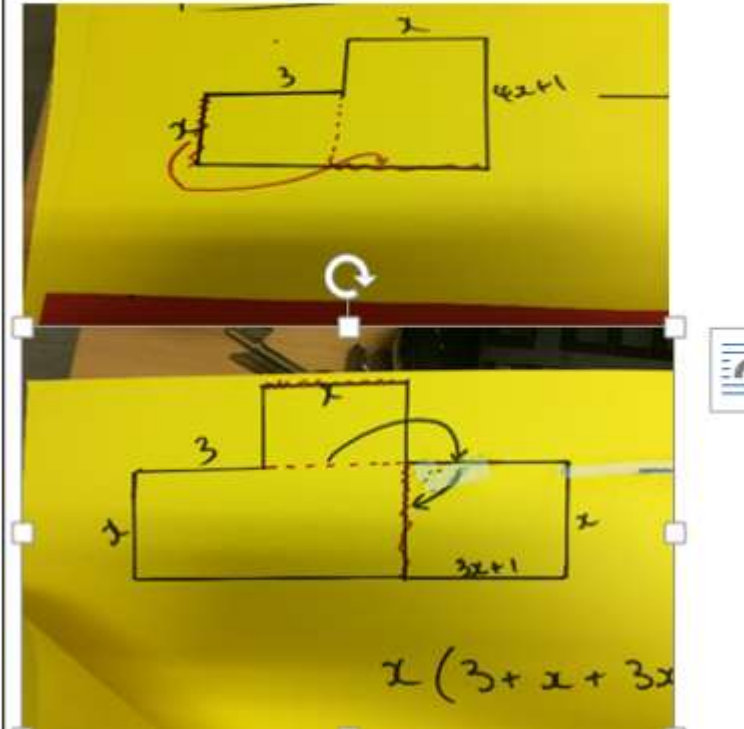Student recognises that there are 2 known sides and calculates the other 2 sides.
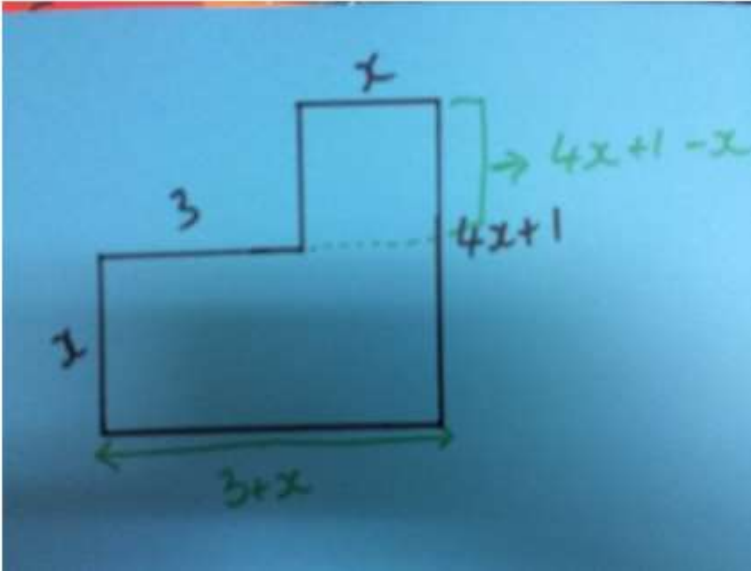
Method 4:

Student sees 3 rectangles and calculates unknown sides to calculate area.

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Method 5:

Student redesigns the shape to create one rectangle.

$$x(3 + x + 3x$$

4. Comparing and Discussing: Cèardaìocht (15 minutes)

For each Anticipated Method above:

- Ask a student to come up to the discussion board (2nd white board in class) and present their method to the class.

- In the event that some above titles are not identified teacher should encourage students to think of any other methods

Showcase responses and ways of forming the quadratic equation in order of difficulty.

5. Summing up (5 minutes)

- Refer back to learning intention and check students understanding through questioning.

- Assign homework…

"Can you describe two other ways to answer this question that you saw in today's class?"

10. Evaluation

- A seating plan provided by the teacher

- Three observers. One observer uses the app LessonNote and the other two pen and paper using the Lesson plan.

- Data will then be recorded under the following headings: prior knowledge, posing the problem, problem solving section of the lesson, student feedback, expected outcomes, unexpected outcomes and misconceptions. Any questions the students ask will be recorded also.

- Photos of students work will be taken throughout the lesson.

- Students learning will be assessed through observing their work, their verbal responses to the teacher's questions, questions the students may ask and the various methods they have for finding the area and subsequently forming their quadratic equation.

11. Board Plan

Prior Knowledge:
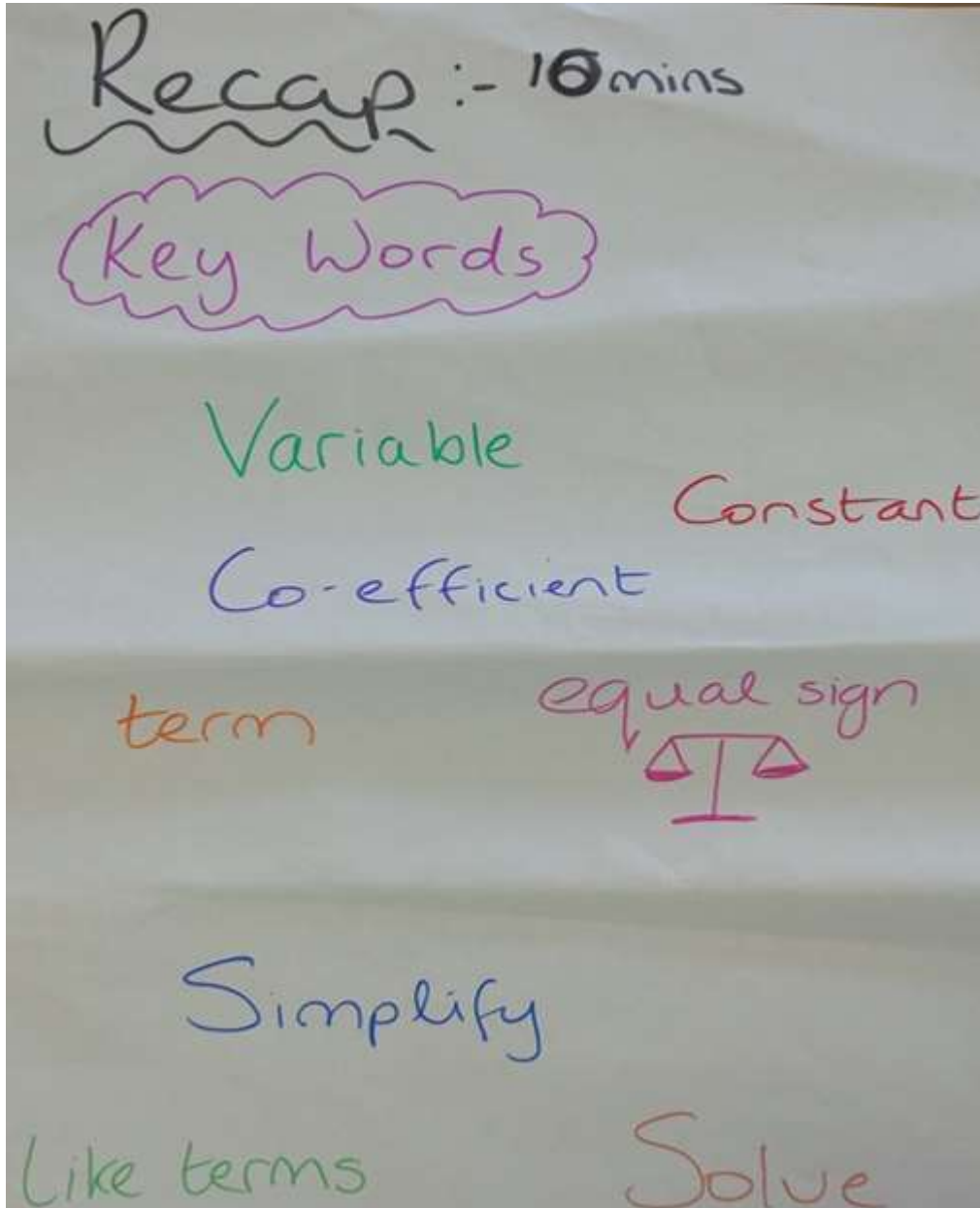
Ⓐ. Adding / Subtracting like terms.

$$2x^2 + 5x - 2 + 3x + 4x^2$$

- Multiplying algebraic terms. (like terms)

$$(2x + 4)(x + 3).$$

- F.O.I.L.

$$(2x + 4)(x + 3)$$

- Array / Area method.

	$2x$	4
x	$2x^2$	$4x$
3	$6x$	12

$$2x^2 + 4x + 6x + 12$$
$$2x^2 + 10x + 12$$

- Distributive method

$$2x(x + 3) + 4(x + 3)$$
$$2x^2 + 6x + 4x + 12$$
$$2x^2 + 10x + 12$$

Learning Intention

Form an algebraic equation from a word problem accompanied by a diagram.

Use the diagram and the dimensions to create an equation including algebraic terms.

Posing the Task:

Question:-

A lady wishes to buy new carpet for her sitting room.
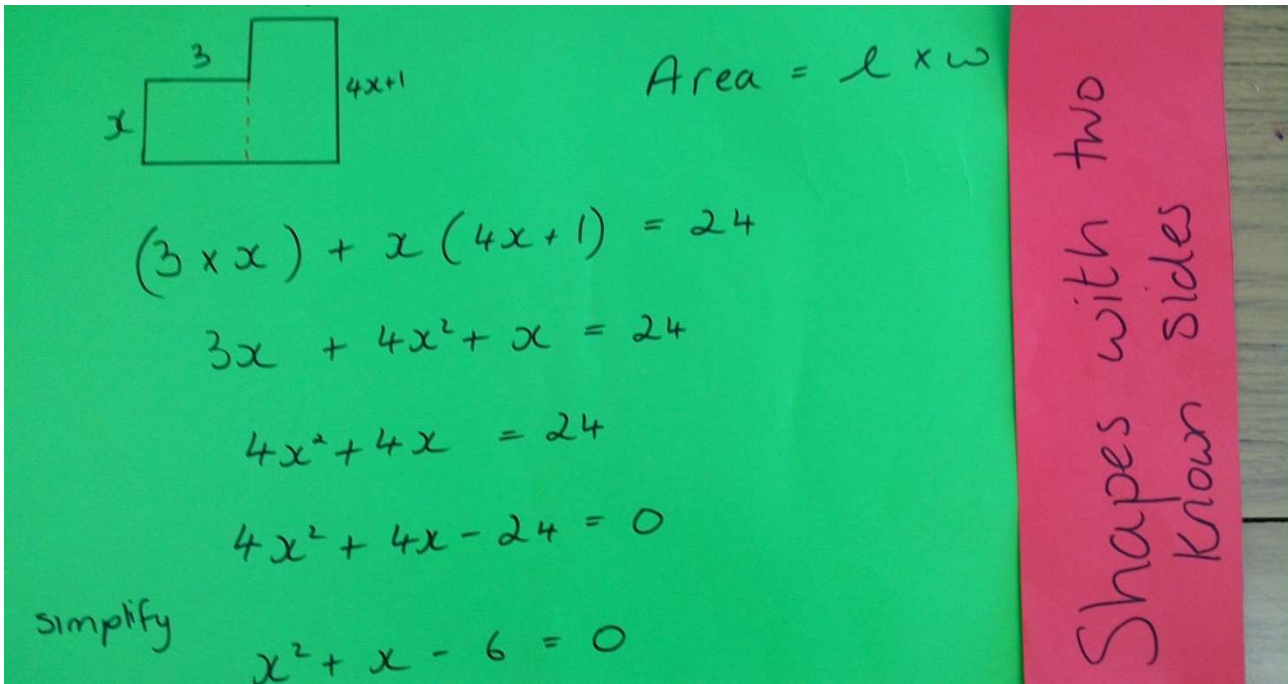
She knows the area of the room is $24 m^2$.

Use the diagram to form an algebraic equation to represent the area of the room.

x

3

$4x+1$

x
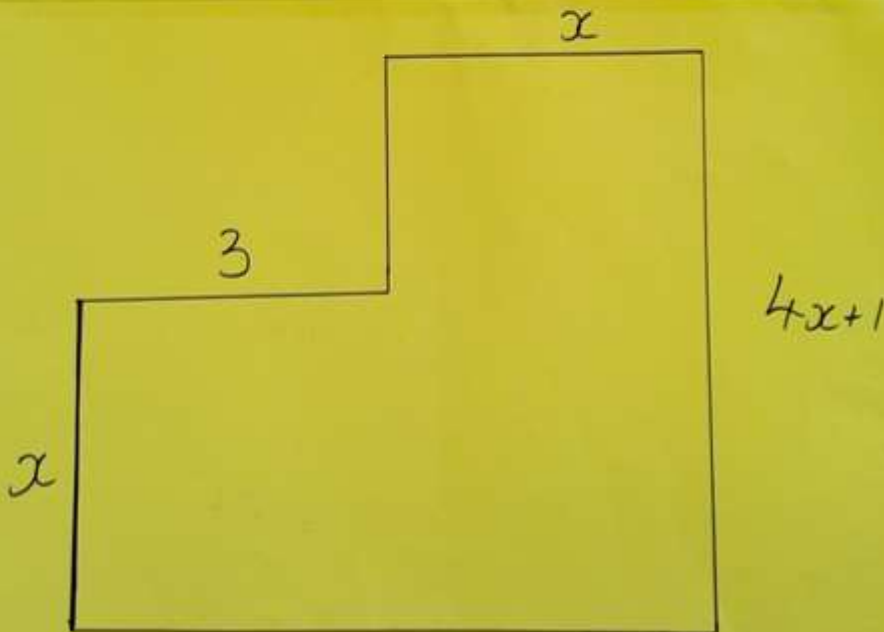
Anticipated responses:

Method 1:

$$Area = l \times w$$

$$(3 \times x) + x(4x+1) = 24$$

$$3x + 4x^2 + x = 24$$

$$4x^2 + 4x = 24$$
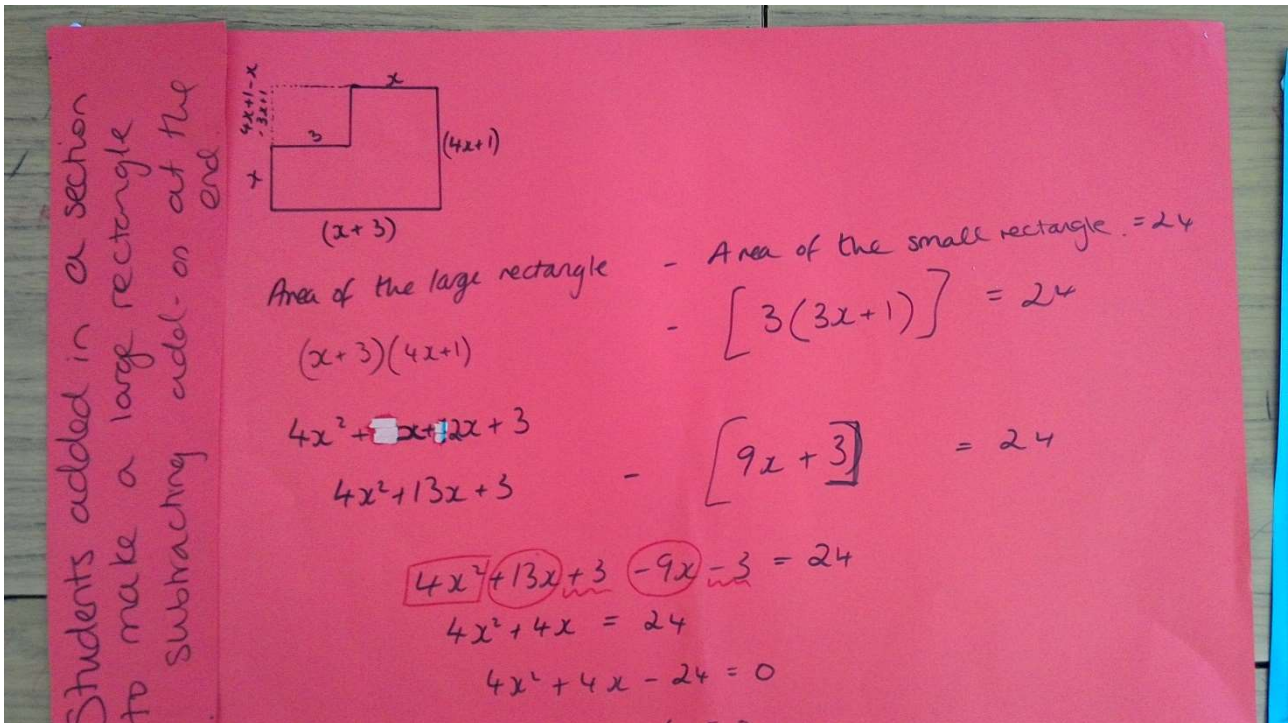
$$4x^2 + 4x - 24 = 0$$

simplify

$$x^2 + x - 6 = 0$$

Shapes with two known sides

Method 2:

Students added in a section to make a large rectangle

Subtracting added-on at the end

Area of the large rectangle − Area of the small rectangle = 24

$$(x+3)(4x+1)$$

$$4x^2 + x + 12x + 3$$

$$4x^2 + 13x + 3$$

$$- [3(3x+1)] = 24$$

$$- [9x+3] = 24$$

$$4x^2 + 13x + 3 - 9x - 3 = 24$$

$$4x^2 + 4x = 24$$

$$4x^2 + 4x - 24 = 0$$

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Method 3:

$$4x+1-x$$
$$= 3x+1$$

$$x(3x) + x(3x+1) = 24$$

$$3x + x^2 + 3x^2 + x = 24$$

$$4x^2 + 4x = 24$$

$$4x^2 + 4x - 24 = 0$$

$$x^2 + x - 6 = 0$$

Method 4:

$$4x+1-x = 3x+1$$

$$4x+1$$

See three rectangles, two of which both sides known. Calculated width of third rectangle.

Ⓐ $3 \times x$ Ⓑ $x \times x$ Ⓒ $x(3x+1)$
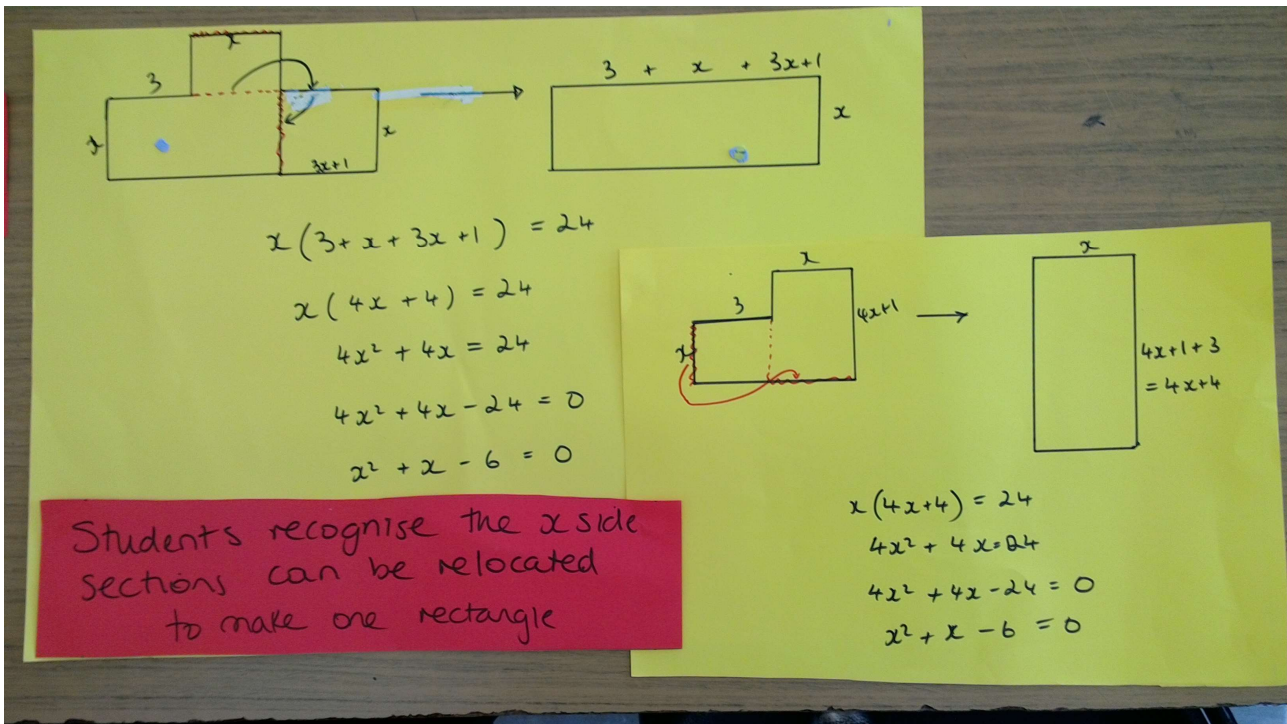
$3x$ x^2 $3x^2 + x$

$$3x + x^2 + 3x^2 + x = 24$$

$$4x^2 + 4x = 24$$

$$4x^2 + 4x - 24 = 0$$

$$x^2 + x - 6 = 0$$

Method 5:

12. Post-lesson reflection

- **What are the major patterns and tendencies in the evidence?**

Students' preferred method was Method 1: Dividing the shape into two rectangles using a vertical line and finding the area of each section separately, as the dimensions required were already on the diagram. A few students completed the rectangle (Method 2) and subtracted the extra carpet not needed.

Students all formed an algebraic equation but failed to simplify.

Throughout the class the language used in the explanation was of a very high standard.

- **What are the key observations or representative examples of student learning and thinking?**

Students were quick to start and handled algebraic multiplication very well, displaying a very good understanding of area.

One student was who unable to answer the question during the time allocated was able to come up with an alternative solution after seeing the three methods presented by their classmates during the Ceardaíocht. A fifth method was then discovered by another student following further probing of the question and the given solutions.

- **What does the evidence suggest about student thinking such as their misconceptions, difficulties, confusion, insights, surprising ideas, etc.?**

Some misconceptions occurred with perimeter and area formulae being confused by some students, another made a calculating error when calculating the dimension of one side…. $3x$ instead of $3 + x$. Also a number of students incorrectly multiplied $4x$ by x as $5x$. This came as a surprise considering they expressed a high level of understanding during the introduction/prior knowledge.

- **In what ways did students achieve or not achieve the learning goals?**

Students achieved their learning goal as every student came up with an equation whether it was written or orally. However, students failed to simplify their equation. This was not specifically stated in the question however and would be a revision for the lesson being taught again.

Students were extremely interested in extending their learning by taking photographs with their phones and sharing on social media (Snapchat Maths Group).

Students calculated the length of on side extremely efficiently. This required them to calculate $(4x + 1) - x$, we were very impressed with this.

- **Based on your analysis, how would you change or revise the lesson**

We would shorten our review of prior knowledge. Not all the vocabulary (key words) necessarily needed to be reviewed and this took up more time at the start of the lesson than was needed.

We would supply scissors and a sheet with a few copies of the shape which the student could cut out if they wished to, this may encourage students to come across method 5 more easily.

We would also have a few larger copies of the shape from the question on the board so the students could demonstrate how they divided up their shape and then they could do their method of forming an equation on the board beside it as the worksheets were quite small for students to see on the board during Céardaíocht.

- **What are the implications for teaching in your field?**

Following on from this lesson, the teacher gave out a short survey to the students regarding the class and their opinions on various aspects of the class (see appendix 3). From the survey it was evident that the students really enjoyed the lesson, with the vast majority of students saying they enjoyed the challenge.

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Students also said they really enjoyed demonstrating their work to their classmates and explaining how they came up with their solution. Students found it interesting to know that there was more than one way to form the equation.

We as a group of teachers definitely recognise the benefits of Lesson Study to our teaching and our professional development.
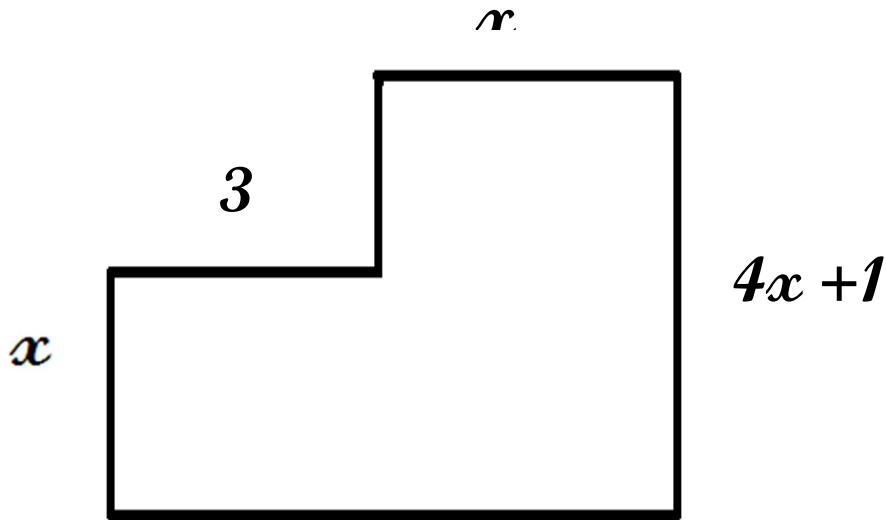
Appendices:

1. Worksheet used by students
2. Record Sheet used by observing teachers
3. Student Survey

Appendix No. 1 - Worksheet

Question:

A lady wishes to buy new carpet for her sitting room. She knows the area of the room is 24m^2. Use the diagram to form an algebraic equation to represent the area of the room.
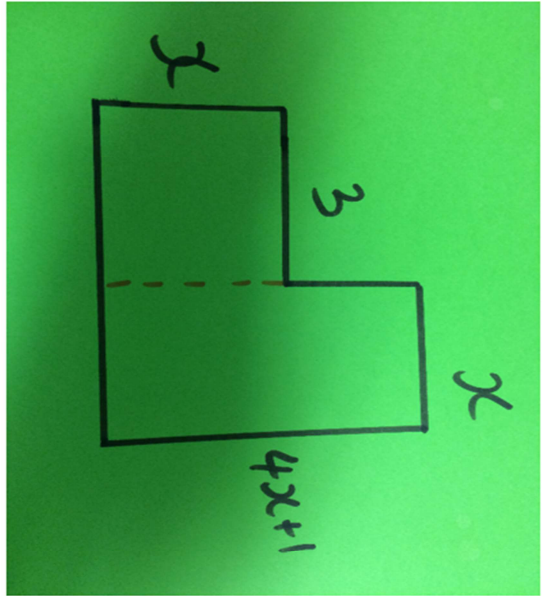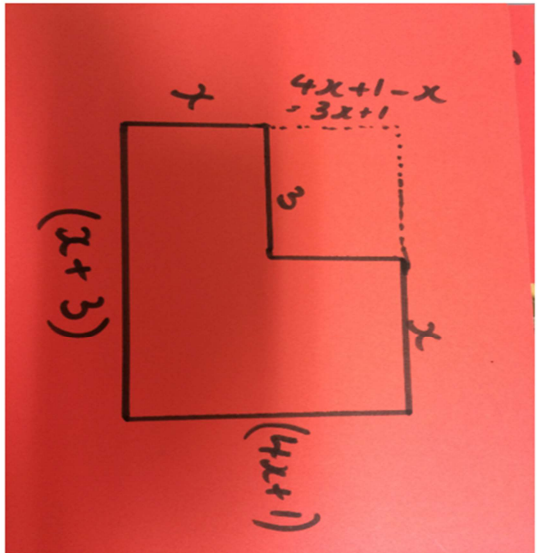

Show all your work

Homework:

Can you describe 2 other ways to answer this question that you saw in today's class?

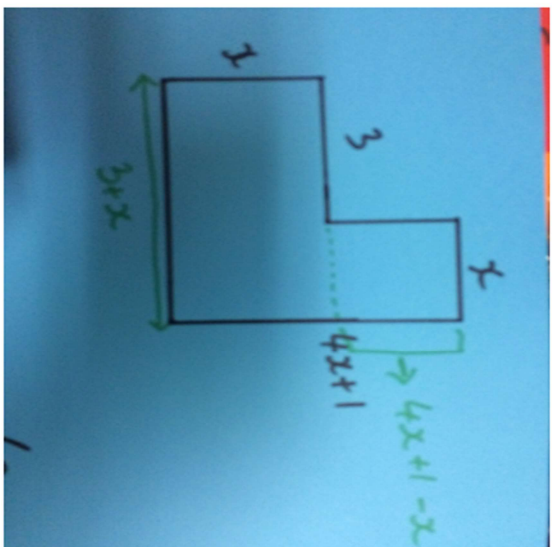**Appendix No.2 - Record Sheet**

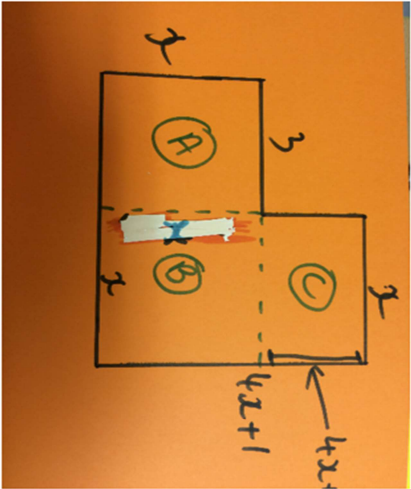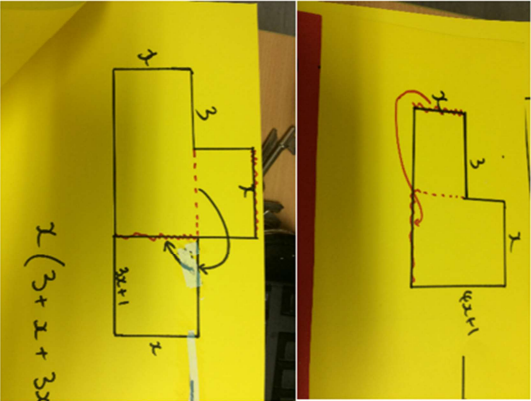Recognises 2 shape with all required sides known

Students added a section to make one large rectangle and subtract the extra section.

Student recognises that there are 2 known sides and calculates the other 2 sides.

Student sees 3 rectangles and calculates unknown sides to calculate area.

Student redesigns the shape to create one rectangle.

Any other methods

Appendix No. 3 – Student Survey

Question 1:

Did you enjoy the lesson?

      Yes   No

Why? _____

Question 2:

Was the recap at the start of the lesson helpful?

      Yes   No

Why? _____

Question 3:

Were the instructions throughout the lesson cleat?

      Yes   No

If No, What could have improved the lesson?

_____

Question 4:

Did you understand the worksheet?

      Yes   No

Question 5:

Is there anything that could have been done differently?

      Yes   No

Why? _____