‘Roots and Shoots’ - Factorising a Quadratic Trinomial - 2nd Year Higher Level

For the lesson on 24/1/2017
At Coláiste Pobail Setanta,
Teacher: Kristan Fox
Lesson plan developed by: Dudley O’Donnell, Kristan Fox, Katie Cahill, Mairead O’Rourke

Title of the Lesson: ‘Roots and Shoots’ - Factorising a Quadratic Trinomial

1. **Brief description of the lesson:** Students are given an outdoor space with an area of $(x^2 + 11x + 24)m^2$ and are asked for the length and width of the space in terms of x. The garden is made up of four different sections; a square pool of variable length, a flower garden, a grass lawn and a paved patio. The patio has a fixed area of $24m^2$.

2. **Aims of the Lesson:**

***Long Term Goals*:**

I’d like my students to appreciate that mathematics can be used to solve real world problems.
I’d like my students to appreciate that mathematics can be used to communicate thinking effectively. (Key Skill – Communicating)
I’d like my students to appreciate that algebra is a tool for making sense of certain situations.
I’d like to foster my students to become independent learners.
I’d like my students to become more creative when devising approaches and methods to solve problems. (Key Skill – Being Creative)
I’d like my students to experience meaningful mathematics i.e. that they see a need for what they are studying. (Key Skill – Being Numerate and Managing Information and Thinking)
I’d like to build my students’ enthusiasm for the subject by engaging them with stimulating activities.
I’d like my students to connect and review the concepts that we have studied already.

***Short Term Goals*:**

For students to understand a quadratic trinomial has two binomial factors.
For students to be able to find these two factors with the aid of the array model**.**
For students to express a quadratic trinomial as a product of the two factors.

3. **Learning Outcomes:**

As a result of studying this topic students will be able to:
- Factorise a quadratic where the coefficient of x^2 is one
- Extend this method to quadratics where the coefficient of x^2 is greater than one.

4. **Background and Rationale:**

The content of this lesson relates to *Section 4.6: Expressions* in the Junior Cycle mathematics syllabus. Students learn about transformational activities, with factoring being specified as one of them. It also gives the students an opportunity to engage in an activity which allows them to develop their problem-solving skills, as set out in section *4.8: Synthesis and Problem Solving Skills* As a group we selected factorizing as, in our own experience, we ourselves have found it difficult to give it a context for students. In our experience it has been taught as a procedure without any context. We are hoping that by grounding it in a problem-solving task we can give our students the opportunity to learn the procedure (important!) through a contextualized approach. Our experience tells us that although students often learn the procedure well initially, they often don't see it as a method for solving problems. Hopefully by giving the students the problem first, this disconnect may be overcome.

5. Research:

The first step was a lively and open discussion among the group focused on areas of the syllabus we found difficult to teach or our students found difficult to learn. This was the key starting point in our research. Once we had selected the area we wished to focus our research on, the next step was to select what the goal of the lesson would be. We drew heavily from the syllabus for this. Once the goal was chosen, we then used a variety of textbooks, and referred to the nrich website, to design our task.

Sources:

- NCCA (2013). *Junior Certificate Mathematics Syllabus: Foundation, ordinary and higher level,* for examination from 2016. Dublin: DES.
- PMDT (2015). Maths Counts 2016. [ONLINE] Available at: http://www.projectmaths.ie/forteachers/conferences/maths-counts-2016.
- A variety of Junior Cycle Mathematics textbooks.

6. About the Unit and the Lesson

The unit in which the research lesson is placed is quite a long one, as detailed below. The students will have already learned to expand a quadratic using the array model. They will also have learned to factorise by dividing out the highest common factor.

7. Flow of the Unit:

This lesson will take place in unit two of five units.

Lesson		# of lesson periods
1	• Algebraic Terms; use of keywords, i.e. terms, coefficients and expressions. Collecting like terms, with a starter in this lesson on substitution.	1 x 60 min.
2	• Multiplication of terms to include indices; multiplication of a term by a single bracket. Students to work on multiplying out brackets within an expression and simplifying by collecting like term.	1 x 60 min.
3	• Factorisation; Factorising by Highest Common Factor leading to factorising by grouping.	1 x 60 min.
4	• Multiplying expressions; multiplying expressions using the array model with exposure to the split bracket method.	1 x 60 min.

5	• Factorisation; Students discover factorising a quadratic trinomial in response to an area problem, where the coefficient of x^2 is 1. This will progress towards trinomials where the coefficient of x^2 is greater than 1.	1 x 60 min. (research lesson)
6	• Consolidation of learning from research lesson, removing the area model as a scaffold. Factorising the difference of two squares.	1 x 60 min.

8. Flow of the Lesson

Introduction	
Teaching Activity	**Points of Consideration**
Students are greeted at the door by the teacher and the homework is checked. While this is happening, students who are already seated are doing a warm-up activity. It has been decide that this activity will be: "Find all the factors of 120." Once all students are seated and ready, our lesson can begin.	This is the agreed structure of all mathematics classes in Coláiste Pobail Setanta and so we are keeping to this structure as much as possible.
Prior Knowledge: The area of a square, the area of a rectangle and the expansion of $(x + a)(x + b)$ using the array model. Teacher will ask students "How do we calculate the area of a square?" Correct response will be recorded on the board. Then students will be asked "How do we calculate the area of a rectangle?" and the correct response will be recorded on the board. Then $(x + 5)(x – 3)$ will be expanded using the array model, with input from the students	

<table>
<tr><th colspan="2">Task</th></tr>
<tr><th>Teaching Activity</th><th>Points of consideration</th></tr>
<tr><td>Posing the Task:

Students are introduced to the task which is put up on the board exactly as it appears on their individual worksheets (see Appendix 1). The teacher will read through the question, pointing out the fixed area of the patio ($24m^2$). They will be given 10 minutes to work on the task. If students complete the task successfully within the given time, they will be asked to attempt it in another way.

Task: You have a job for the Summer working with the celebrity gardener Diarmuid Gavin. He has been hired to design an outdoor space for the Setanta Hotel. The space will include a square pool and a patio of fixed area $24m^2$. He also wants to include a flower garden and a grass lawn. Your task is to write the length and width of the outdoor space, in terms of x, given that the area is $x^2 + 11x + 24$


</td><td>Teacher uses Thumb Scaling (thumbs up, thumbs down, thumbs to the side) to assess if students are ready to begin.</td></tr>
<tr><td>Anticipated Student Responses

1. $L \times w = x^2 + 11x + 24$ – students recognize that the area given should be the product of the length and width without any indication of what the length and width are.
2. Factors of 24 – it is anticipated that even the weakest student in the class will manage to write out the factors of 24. They have been given a reasonably strong prompt in the warm up activity.
3. Assigning x^2 as the area of the pool – This shows recognition of x^2 as a square number.
4. Assigning x as the length of the pool –</td><td>2. Factors of 24 If weaker students are struggling to get started, then the teacher may give a slight prompt by asking them what the length and width of the patio area might be.
4. Assigning x as the length of the pool
This is important as it is proof that students understand the pool is of variable length.

5. x and 6, x and 4 (or x and 2, x and 12)
Although this is incorrect, it has educational value for the other members of the class.

Students who are finished early will be asked to</td></tr>
</table>

this is an important stage of the factorization as it shows recognition of x^2 as the product of (x)(x) 5. x and 6, x and 4 (or x and 2, x and 12) – some students may assign factors 4 and 6 (or 2 and 12) to the flower garden and grass lawn, without the addition symbol. 6. x and 8, x and 3 – it is anticipated that some students in the class will assign the correct factors to the rose garden and the grass lawn. They may have done this without yet incorporating the addition symbol. 7. Assigning 8x and 3x as the area of the grass area and flower garden – Recognising 8x or 3x as an area gives an indication that students have assigned both a length and a width to these areas, i.e. 8 times x or 3 times x. 8. x + 8, x + 3 – the more able students in the class will incorporate the addition symbol. This, we think, will be their response to calculating the *total* length and the *total* width. 9. $(x+8)(x+3) = x^2 + 11x + 24$ – students recognize that the product of their length and width is the area given in the problem. 10. Verification of the factors – some students may take this a step further and verify their area by multiplying their expression for length by their expression for width. 11. Generalising– students who are attempting a second method may simply write their work from the array out as an expression. i.e. $x^2 +3x + 8x + 24$. 12. Factorising by Grouping - As they have already factorized by grouping, the step above should, we think, lead them naturally into attempting that method.	try another method.
Comparing and Discussing (Ceardaíocht) We will present student work in the order listed above. If any of the solution stages above are not presented by any students in the class we will simply move to the next point.	It will be important to show all the work of value. This is why we feel Point 1, Point 2 above must be included. If a student is being asked to the board to

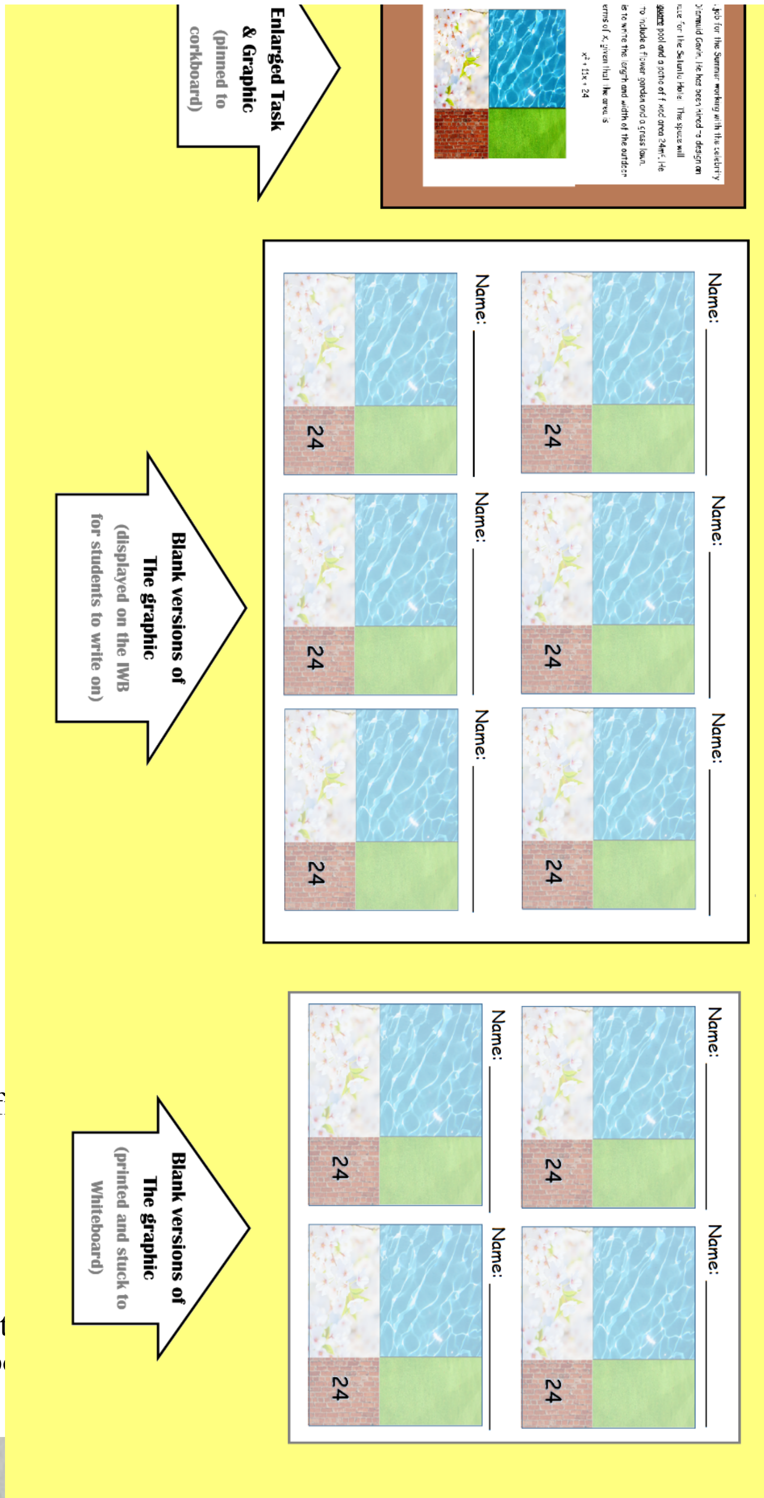
Point 8 – this is the stage that will generate the most discussion. It will be important for the students, with encouragement from the teacher if necessary, to make the connection between the area of each section and the corresponding term in the quadratic trinomial. This point should be emphasized heavily during the comparison and discussion of different students work. All students should be able to see some representation of their efforts on the board, however modest, showing that every attempt has value.	present work as in Point 5 above, then the teacher will have explained to them why they are doing so. The teacher may say something like: "I see you used 4 and 6 initially, but then you changed it. Why did you change it? Would you mind explaining to the class why you changed it? I think that's excellent work, how you corrected your solution." Differentiating between solutions where x and 3, x and 8 are written and x + 3, x + 8 is important as it shows total length. Also, within the array in later steps, it will lead to 3x and 8x correctly being added to form the 11x required. It is anticipated that only the most able students would progress to Points 11 and 12 above. If no students make these steps, then it will not be introduced at this time as finding the factors using the array model is sufficient to achieve the goal of the task they have been set.

Summing Up	
Teacher Activity	**Points of consideration.**
The teacher will ask the students to write what they think the learning intention was for the lesson. They will then write a sentence or two explaining what they have learned in the lesson and reflect on whether the intention for the lesson was achieved. (Key Skill – Managing information and thinking)	In Coláiste Pobail Setanta, writing the learning intention on the board at the beginning of the lesson is the agreed procedure among the staff members. However, it was felt that this might give the game away, so to speak. We decided that in this instance it would be a good exercise for the students to write out the learning intention at the end of the lesson, giving them an opportunity to reflect on what they have learned
<u>Setting the Homework Task</u> Students will be given a worksheet with a generic array and asked to factorise $x^2 + 13x + 12$ with an extension activity of $x^2 - 12x + 35$ for the more able students.	Giving an exercise without context is an important step as the procedure itself is not always context driven.

9. Evaluation

We adapted an observation sheet for our particular lesson, and all used this as an evaluation tool. It was agreed that the classroom would be divided into four zones and each observer would cover one zone, taking photographs of students work where appropriate. (Appendix C)

10. Board Plan

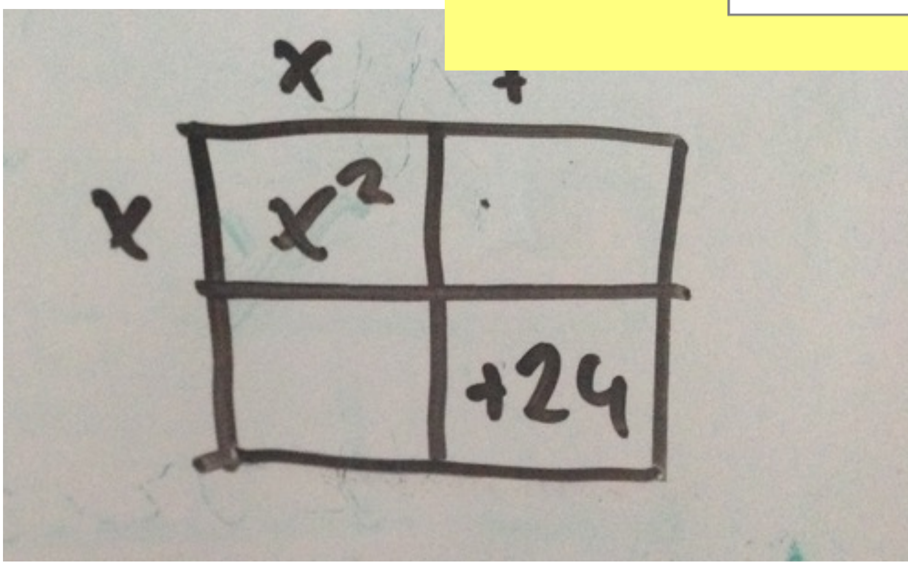


11. Post-lesson reflection

We feel that the goal of the lesson was def
subsequent lessons the students were very
in the assessment of this topic.

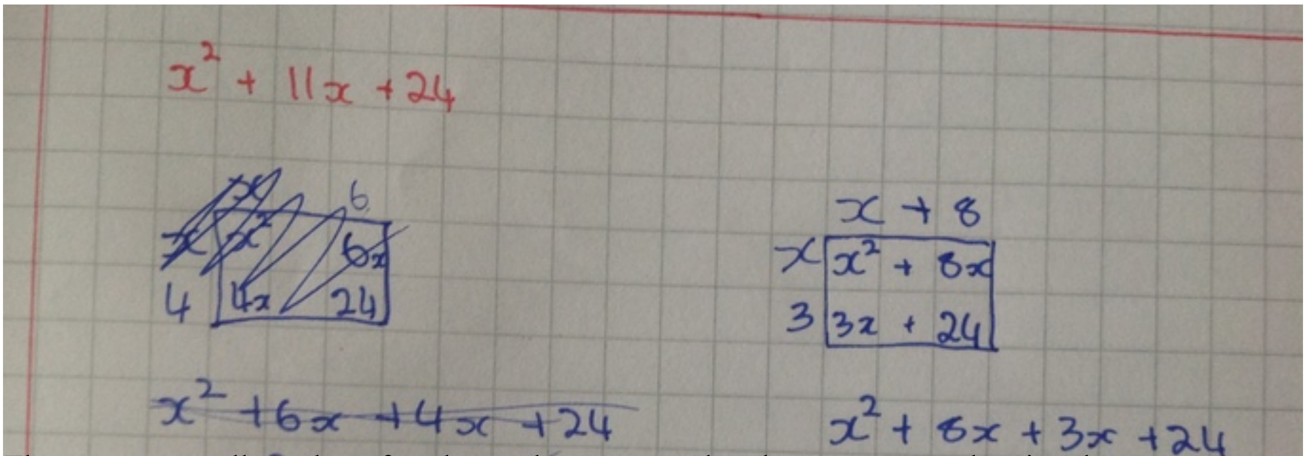
<u>Student Work</u>

In the lesson itself, we saw that the majorit
pool and /or assign x as the length of the p

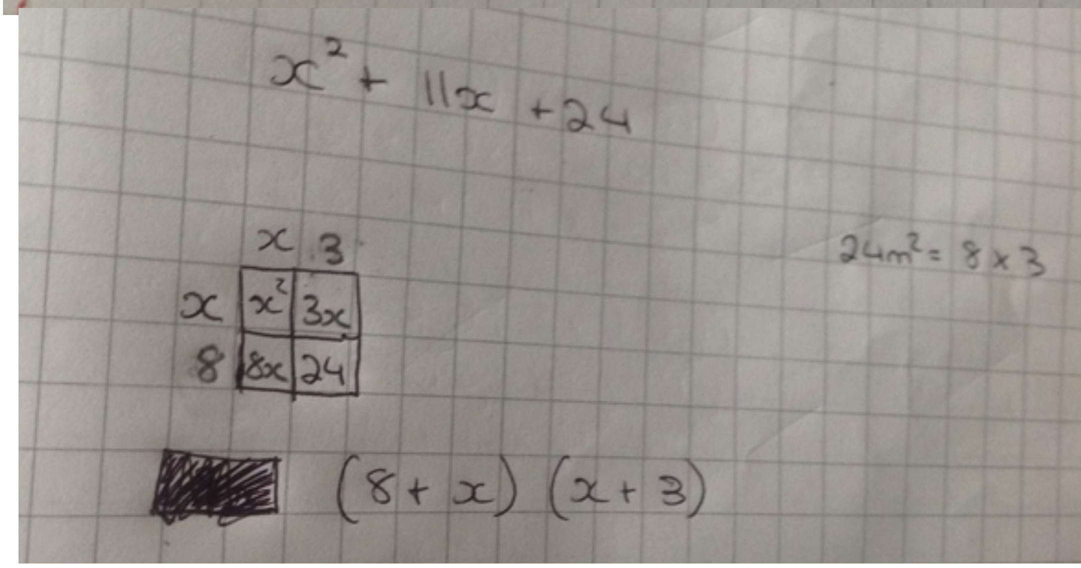


The next stages, assigning 8x or 3x as the areas, or assigning 8 and 3 as the lengths were achieved by 6 students correctly. There was evidence to suggest that students understood that these lengths were factors of 24, as a number of students assigned 6 and 4 as the lengths. We considered that this was very good progress in the task, as they understood what was required. It also gave an excellent opportunity to discuss this misconception during the class discussion.

This misconception was presented to the class by a student who self –corrected and went on to get the correct lengths of 8 and 3. He showed great confidence in explaining his mistake to the class in order to further their understanding.

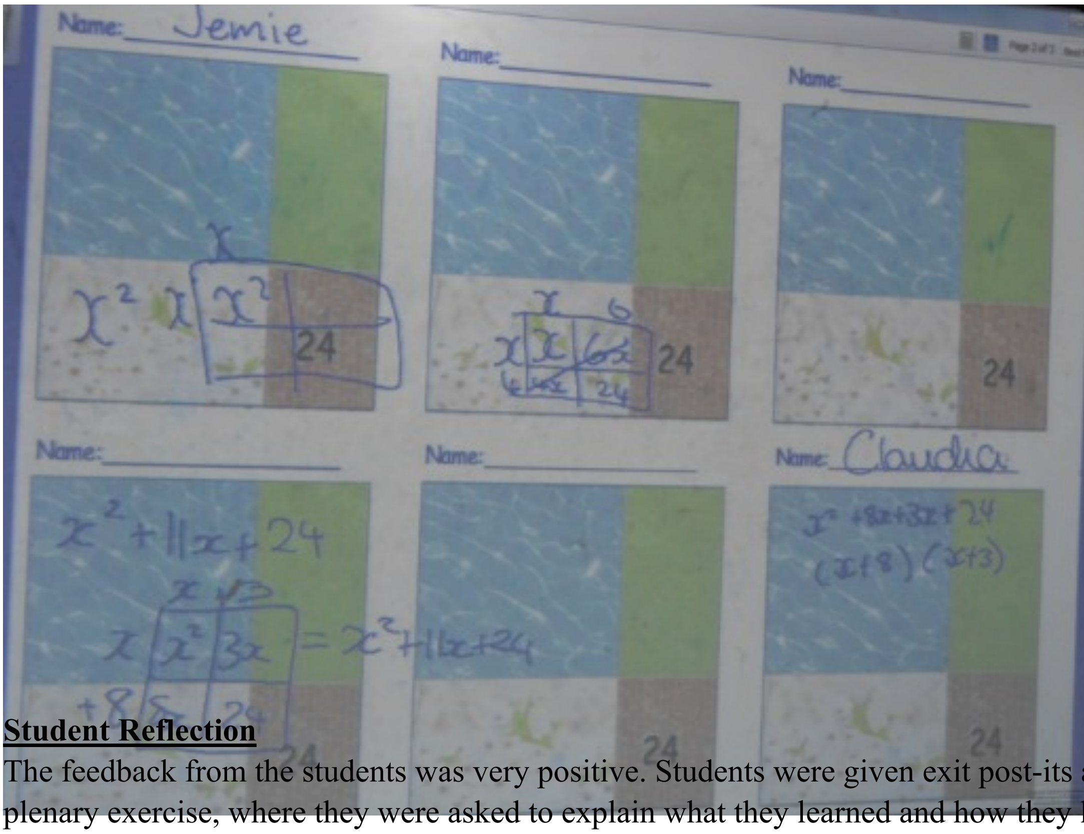


There were a small number of students who progressed to the next stages, showing the area as a product of their total length and width. As a group, we were very pleased to see students making this step and it was highlighted during the whole class discussion.



Class Discussion

What was gained from the whole class discussion surprised even us. Although the entire class had been thoroughly engaged in the task, the level of success was as we had expected. Different students attained different levels of the solution. The most obvious sign of the students increased understanding was the “oohs” and “aahs” and gasps of realisation as each student presented their solution. Despite an engaging class discussion, there was still some time available, in which the class teacher quickly improvised with another quadratic trinomial. The success with the second task was almost universal, and the speed with which they completed the task was phenomenal.



Student Reflection

The feedback from the students was very positive. Students were given exit post-its as a reflective plenary exercise, where they were asked to explain what they learned and how they learned it. The feedback from this exercise was very positive. The majority of students mentioned area, whilst some mentioned that they had learned how to factorise a quadratic trinomial. One student even remarked that they learned how to do it by themselves.

At a parent teacher meeting later that week, one student commented without prompt on how they would like to do more lessons of this nature in future, as she found it a worthwhile and enjoyable learning experience.

Post Lesson Discussion

As stated above, overall we feel that the goal of the lesson was definitely achieved.

The students engaged with the task very well. We feel that the worksheet hooked them, in terms of colour and style. However, on their worksheet, the four smaller arrays were not seen by the students as arrays. Although this did not seem to hold them back in terms of their achievement in the task, not a single student utilised the worksheet as we had imagined. If this lesson were being taught again, we would put borders around the different areas of the outdoor space, to highlight to them the connection to the array. In the discussion, it was suggested that their reluctance to write on the worksheet may be because worksheets are not much used. All students have Surface Pros and mini whiteboards in their journals which are used much more.

There were some students who started the task by measuring the length and width to get the area. We felt that this might be overcome by stating that the picture was not drawn to scale, and emphasising that the length and width should be in terms of x

This was the group's first foray into Lesson Study. The process of problem solving, and the structure provided for a problem solving lesson, was extremely beneficial for the students. As a pedagogical tool, it is something we would very much like to implement more of.
Lesson Study as a process of professional development was also extremely beneficial for us as educators. It facilitated in-depth discussion around teaching methods and different approaches to a problem which, in themselves, were educational and enlightening. Having designed the task, we structured the unit to best facilitate its implementation. This was a great opportunity consider in depth our medium term planning.

You have a job for the Summer working with the celebrity gardener D

Gavin. He has been hired to design an outdoor space for the Setanta I

space will include a ***square*** pool and a patio of fixed area $24m^2$. He als

include a flower garden and a grass lawn. Your task is to write the len

width of the outdoor space, in terms of x, given that the area is

$$x^2 + 11x + 24$$

Homework Question:

Factorise the following quadratic:

$$x^2 + 13x + 12$$

Homework Question: Extension

Factorise the following quadratic:

$x^2 - 12x + 35$

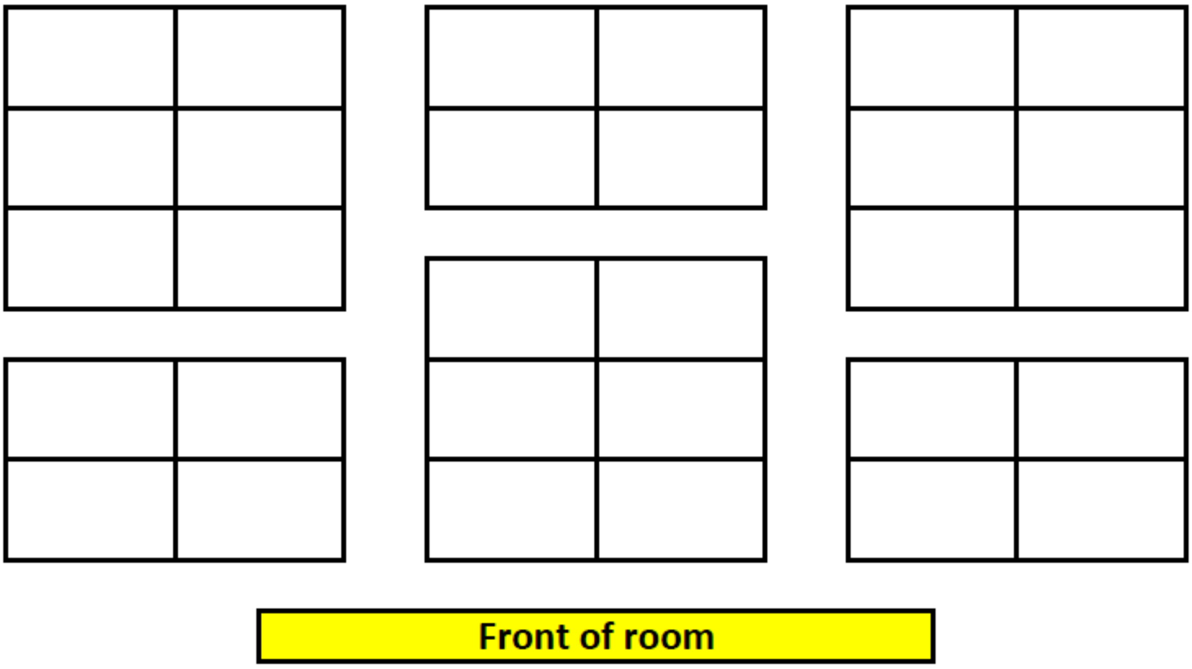
Appendix C – Observation Sheet

Lesson Observation

Observation of student responses	Student 1	Student 2	Student 3	Student 4	Student 5	Student 6	Student 7	Student 8
Did the student understand what was being asked of him/her?								
Student Outcomes:								
1. $L \times w = x^2 + 11x + 24$								
2. Factors of 24								
3. Assigning x^2 as the area of the pool								
4. Assigning x as the length of the pool								
5. x and 6, x and 4 (or x and 2, x and 12)								
6. x and 8, x and 3								
7. Assigning 8x and 3x as the area of the grass area and flower garden								
8. $x + 8$, $x + 3$								
9. $(x+8)(x+3) = x^2 + 11x + 24$								
10. Verification of the factors – multiply expression for length by expression for width								
11. Generalising– i.e. $x^2 + 3x + 8x + 24$.								
12. Factorising by Grouping								
Did the student need a prompt throughout the lesson?								

Were the students engaged throughout the lesson? 1-Very little engagement 2-Some engagement 3-Engaged Comment on any other observations	Student 1	Student 2	Student 3	Student 4	Student 5	Student 6	Student 7	Student 8
What changes do you think need to be made to the lesson plan?								
Did you notice any problems that could possibly arise again in future classes that may limit understanding of the topic?								

Appendix D – Classroom Plan


Front of room