

Unit and Lesson Plan for Grade 7 (13 and 14 Years Old Students)

March 2-3, 2017

Unit and lesson plan developed by Akihiko Takahashi

The 4th National Mathematics Conference: Maths Counts 2017 at Maynooth University

1. Title of the Unit: Learning geometry through investigation

2. Brief description of the Unit

This unit is designed for students to be able to deepen their understanding of basic geometric shapes through investigation. The series of three lessons employs the open-ended approach, a type of teaching through problem solving developed in Japan, in order to nurture students' problem solving skills.

3. Goals of the Unit:

- To deepen students' understanding of the concept of basic shapes through open-ended problem solving
- To discover three efficient ways of constructing congruent triangles through drawing as a foundation for understanding three conditions for congruency of triangles.
- To help students develop problem solving skills through constructing and examining a variety of basic shapes by using their properties.
 - explore patterns and formulate conjectures
 - explain findings
 - justify conclusions
 - communicate mathematics verbally and in written form
 - apply their knowledge and skills to solve problems

4. Students' prior learning

Grade 7 Strand 2: 2.1 Synthetic geometry

Students should be able to convince themselves through investigation that theorems 1-6 appear to be true

1. Vertically opposite angles are equal in measure.
2. In an isosceles triangle the angles opposite the equal sides are equal. Conversely, if two angles are equal, then the triangle is isosceles.
3. If a transversal makes equal alternate angles on two lines then the lines are parallel, (and converse).
4. The angles in any triangle add to 180° .
5. Two lines are parallel if and only if, for any transversal, the corresponding angles are equal.
6. Each exterior angle of a triangle is equal to the sum of the interior opposite angles.

Students should be able to construct

1. the bisector of a given angle, using only compass and straight edge
2. the perpendicular bisector of a segment, using only compass and straight edge
3. a line perpendicular to a given line l , passing through a given point on l
4. a line parallel to a given line l , through a given point
5. divide a line segment into 2, 3 equal segments, without measuring it



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6. a line segment of given length on a given ray

5. Background and Rationale

This unit is designed for the students to deepen their understanding of geometric shapes using an open-ended approach. The open-ended approach was developed in Japan during 1970s as a result of the research project on assessing higher-order thinking in mathematics. The original book on the open-ended approach was published in 1977 in Japanese, edited by Shigeru Shimada; the English translation of the book was published in 1997 by Jerry P. Becker. One of the unique features of the open-ended approach is the use of an open-ended task, designed to have multiple correct solutions, in order for students to come up their own solution(s). The teacher then facilitates a class discussion, comparing and discussing the students' multiple solutions, in order to teach something new.

Each of three lessons for this mini unit will use one of the open-ended tasks that was developed by the Japanese teachers as part the project in order for the students to deepen their understanding of geometric shapes and to prepare them to learn three conditions for congruency of triangles.

Since early elementary grades, students have learned about a variety of basic geometric shapes. By grade 7 students have learned the names of the basic shapes, such as square and triangle, but the students often do not fully establish the concept of those basic shapes. For example, every student will recognize the equilateral triangle shown in figure 1, but some cannot recognize that the trapezoid in figure 2 can be divided into two triangles with its diagonal because one of the triangles, the shaded part of the figure, does not match their concept of "triangle" because it is "upside down" and slender.

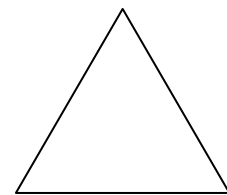


Figure 1

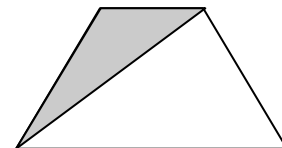


Figure 2

In order to overcome such misconceptions, students should be able to sort a variety of shapes using the definitions of basic shapes rather than simply relying upon visual features of the shapes.

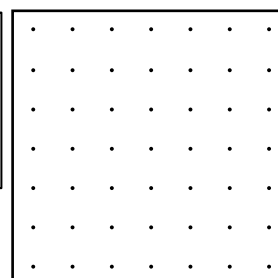
6. About the Unit and the Lesson

The mini unit is designed based on the open-ended approach.

The first lesson is designed to familiarize students with the open-ended approach. The students will be given an opportunity to construct variety of squares with different sizes by satisfying a given condition on the worksheet shown below. After coming up several squares with different sizes students are going to be asked to develop a viable argument

Day 1 Problem

Make different sizes of squares by connecting four dots on the worksheet by four lines. Find all the possible squares with different sizes on the worksheet.

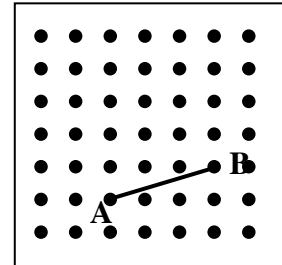


for why the shape is square and why the shape he/she comes up is a different size from other squares. Through examining each square the class is going to come up all the possible squares with different sizes on the worksheet.

The second lesson will use the same worksheet to construct a variety of isosceles triangles.

Day 2 Problem

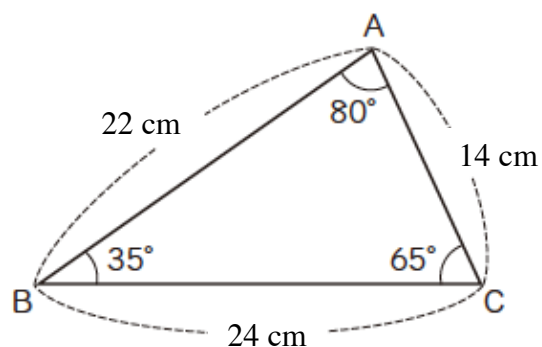
By using a line segment AB as one of the sides, make an isosceles triangle ABC on the worksheet. How many isosceles triangles can you make on the worksheet? Find all the possible isosceles triangles on the worksheet.



There are nine possible points C that can make ABC an isosceles triangle on the worksheet. Since the length of AB is not obvious, students are expected to not only identify possible isosceles triangles but also explain a reason why the triangle is isosceles. In other words, the students need to identify which two sides are equal in length and why these two sides are equal. It is expected that they will be able to enrich their concept of isosceles triangles and understand a way to explain their thoughts through this process. Since this is an important opportunity for students to develop their reasoning skills toward formal proofs, the lesson will focus on providing such opportunity as a major discussion in the lesson. Moreover, students are expected to find more possible Cs by extending the size of the worksheet. By using various isosceles triangles that the students themselves find, they will be given an opportunity to classify their triangles into three categories: triangles with $AB=BC$, with $AB=AC$, and with $AC=BC$ in order to see if they have found all the possible solutions.

At the end of the lesson, dynamic geometry software will be used to see if all the possible Cs were found.

The third lesson is designed for students to construct congruent triangle by drawing. Since the task includes all six measurements, three sides and three angles, there are multiple ways to construct the triangle by drawing. By comparing and contrasting the ways that students find, the class is going to find out three efficient ways of constructing congruent triangles through drawing as a foundation for understanding three conditions for congruency of triangles.



When two figures can fit on top of each other perfectly, we say that they are **congruent**. Draw triangle DEF on your paper so it will be **congruent** to triangle ABC shown above. Come up with an efficient way to draw a congruent triangle ABC.

7. Flow of the Unit

Lesson	Learning objective(s)	
1	Let's find all the possible squares on the worksheet! <ul style="list-style-type: none"> To deepen students' understanding of the concept of basic shapes through open-ended problem solving. To be able to organize their work in order to find all the possible squares. 	30 min
2	Let's find all the possible isosceles triangles on the worksheet! <ul style="list-style-type: none"> To deepen students' understanding of the concept of isosceles triangles through open-ended problem solving To be able to see the relationship among the possible isosceles triangles in order to find all the possible isosceles triangles on the worksheet. 	60 min
3	Let's find ways to draw a congruent triangle! <ul style="list-style-type: none"> To discover three efficient ways of constructing congruent triangles through drawing as a foundation for understanding three conditions for congruency of triangles. 	60 min

8. Demonstration Lesson Plans

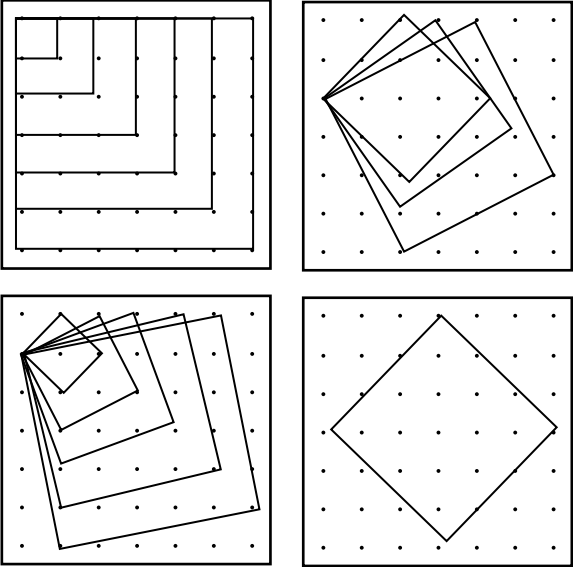
Lesson 1 (30 minutes): Let's find all the possible squares on the worksheet!

(1) Objectives

- To deepen students' understanding of the concept of basic shapes through open-ended problem solving.
- To be able to organize their work in order to find all the possible squares.

(2) Flow of the lesson

Steps, Learning Activities Teacher's Questions and Expected Student Reactions	Teacher's Support	Points of Evaluation																																				
1. Introduction Understand how to construct a square on the worksheet by connecting four dots with four lines. <table border="1" style="display: inline-table; vertical-align: middle;"> <tr><td>.</td><td>.</td><td>.</td><td>.</td><td>.</td><td>.</td></tr> <tr><td>.</td><td>.</td><td>.</td><td>.</td><td>.</td><td>.</td></tr> <tr><td>.</td><td>.</td><td>.</td><td>.</td><td>.</td><td>.</td></tr> <tr><td>.</td><td>.</td><td>.</td><td>.</td><td>.</td><td>.</td></tr> <tr><td>.</td><td>.</td><td>.</td><td>.</td><td>.</td><td>.</td></tr> <tr><td>.</td><td>.</td><td>.</td><td>.</td><td>.</td><td>.</td></tr> </table>	By showing the worksheet, explain what it means to make a square by connecting four dots with four lines. Clarify what is meant by squares of different sizes.	Do students understand the instruction and are ready for solving the problem?
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2. Posing the Problem Give each students several copies of the worksheets and ask the following question. <table border="1" style="display: inline-table; vertical-align: middle;"> <tr> <td> Make different sizes of squares by connecting four dots on the worksheet by four lines. Find all the possible squares with different sizes on the worksheet. </td> </tr> </table>	Make different sizes of squares by connecting four dots on the worksheet by four lines. Find all the possible squares with different sizes on the worksheet.	If students seem to not understand the task, share a few of the students' attempts as examples.	Do students understand that the task is to construct various squares with different sizes?																																			
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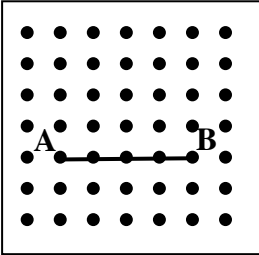
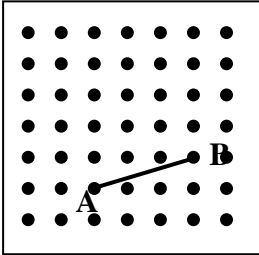
<p>3. Anticipated Student Responses</p> <p>1) Making only 6 squares with different sizes.</p> <p>2) Making more than 6 squares but not be able to find all the possible squares.</p> <p>3) Making all 15 possible squares on the worksheet.</p>	<p>Using a seating chart to note each student's way of coming up squares to prepare for organizing the whole class discussion.</p>	<p>Does each student come up with at least 7 squares with different sizes?</p>
<div style="display: flex; flex-wrap: wrap; justify-content: space-around;">  </div> <p>4. Comparing and Discussing</p> <p>For each proposed square,</p> <ol style="list-style-type: none"> 1) Ask one of the students who came up with a square to show it to the class, 2) Let other students examine if it is a square, and 3) Let other students examine if the square is a different size than others. <p>Repeat the above so students understand a variety of ways to count the number of dots by making equal groups.</p>	<p>By providing an opportunity to examine other students' shapes, help students see a variety of squares with different sizes and orientations.</p> <p>Help them recall their prior learning, such as the definition of square.</p>	<p>Do students use their prior learning to examine other students' work?</p>
<p>5. Summing up</p> <p>Help each student identify the learning from the class and record it in their notes.</p> <ul style="list-style-type: none"> • Variety of squares with different sizes can be constructed by connecting the dots. • Definition can be useful when examining shapes. 		<p>Each student summarizes their learning and records it in their notes.</p>

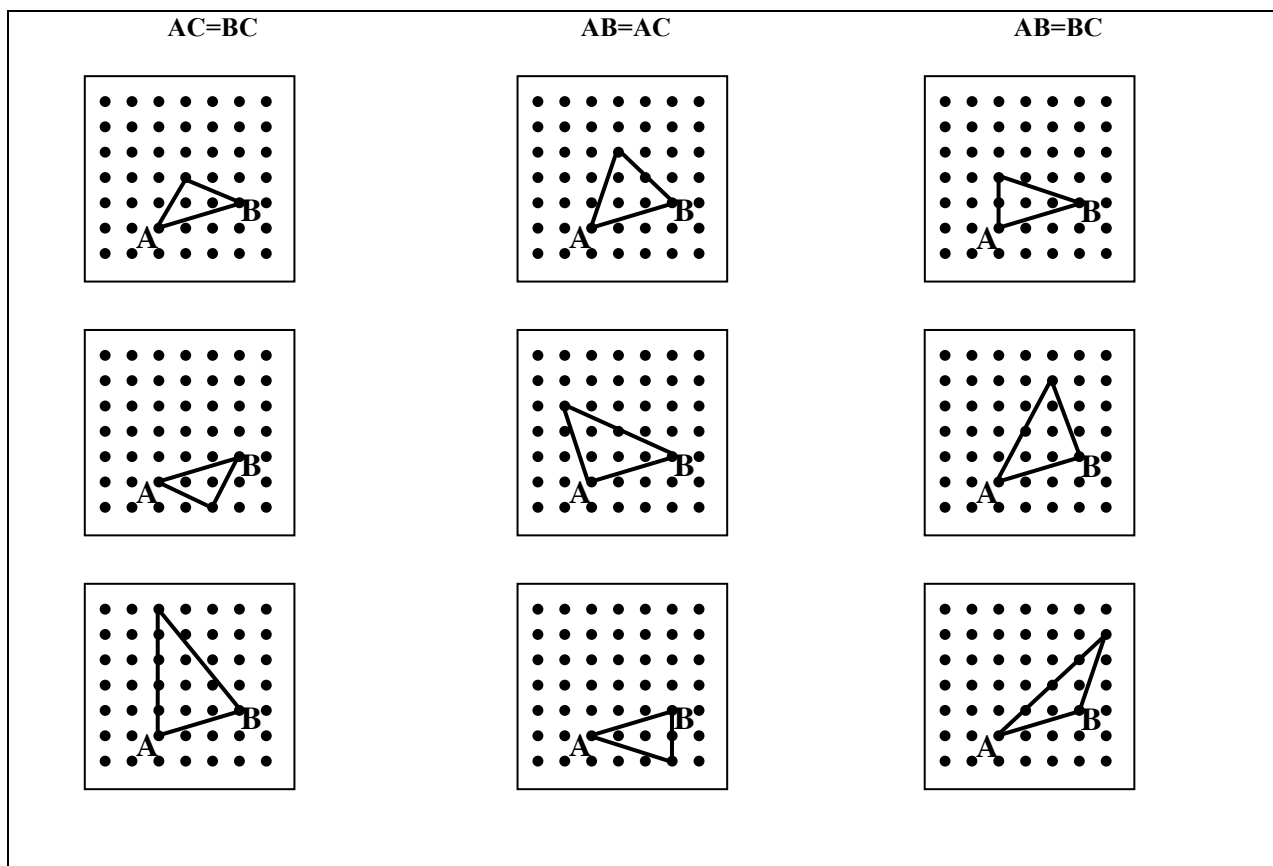
Lesson 2 (60 minutes): Let's find all the possible isosceles triangles on the worksheet!

(1) Objectives

- To deepen students' understanding of the concept of isosceles triangles through open-ended problem solving
- To be able to see the relationship among the possible isosceles triangles in order to find all the possible isosceles triangles on the worksheet.

(2) Flow of the lesson

Learning Activities Teacher's Questions and Expected Students' Reactions	Teacher's Support	Points of Evaluation
<p>1. Introduction</p> <p>1) Help students recall their previous knowledge about triangles, such as isosceles triangles and equilateral triangles.</p> <p>2) Help students recall the use of worksheet.</p> <ul style="list-style-type: none"> • Ask students make an isosceles triangle on the worksheet by using line segment AB as one of the sides and explain why the triangle is an isosceles triangle. • Anticipated solutions for the task. <ul style="list-style-type: none"> ▪ 6 triangles with $AC=AB$ ▪ 1 triangle with $AB=AC$ ▪ 1 triangle with $AB=BC$ 	<p>Write an informal definition of an isosceles triangle on the blackboard using the students' words such as, "a triangle with a pair of sides with equal length".</p> <p>Give a worksheet to each student.</p> <p>Encourage students to find not only the triangles with $AC=BC$ but also $AB=AC$ and $AB=BC$.</p>	<p>Do students recall what an isosceles triangle is?</p> <p>Do students understand the problem?</p> <p>Do students understand there are 8 isosceles triangles on the worksheet and be ready for solving the problem?</p>
<p>2. Posing the Problem</p> <p>1) Pose the following problem to the students:</p> <div style="border: 1px solid black; padding: 5px; width: fit-content;"> <p>By using line segment AB as one of the sides, make an isosceles triangle ABC on your worksheet. How many isosceles triangles can you make? Find as many isosceles triangles as possible.</p> </div> 	<p>Provide worksheets to keep students' work to use for the class discussion.</p> <p>Pose the problem in written format on the blackboard.</p> <p>Give students enough worksheets so that they can draw each isosceles triangle that they find using a worksheet.</p> <p>Encourage students to talk freely about their ideas when finding isosceles triangles with their partners.</p>	<p>Do students understand the problem?</p>
<p>Students' anticipated solutions:</p> <p>Nine isosceles triangles can be made on a worksheet by using the line segment AB as a side.</p>		

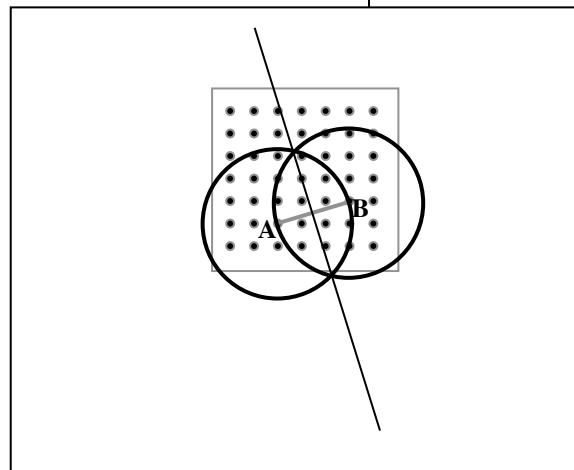


4. Extending the problem

If we have a larger worksheet with more pegs, can we find more isosceles triangles by using line segment AB as one of the sides?

- 1) Let students draw all the isosceles triangles that they have found on their new worksheets.
- 2) Encourage students to find more triangles by using the categories that they used to organize their solutions.
- 3) Let students show the class any new triangles that they have found.
- 3) Help students recognize that all the Cs, which make triangle ABC as isosceles triangles, are in the following geometric figures:
 - All the Cs that make triangle ABC with AC=BC are on the perpendicular bisector of AB
 - All the Cs that makes triangle ABC with AB=AC are on the circumference of a circle with the radius of the length equal to AB and A as its center.
 - All the Cs that makes triangle ABC with AB=BC are on the circumference of a circle with the radius of the length equal to AB and B as its center.

Give students another worksheet with more pegs.



Show above figure to the students by using dynamic geometry software

4. Summing up

- (1) Using the writing on the blackboard, review what students learned through the lesson.
 - Organizing triangles by using lengths of sides, AB=BC, AB=BC, AC=BC, can be useful when examining relationships among the triangles.
- (2) Ask students to write a journal entry about what they learned through the lesson.

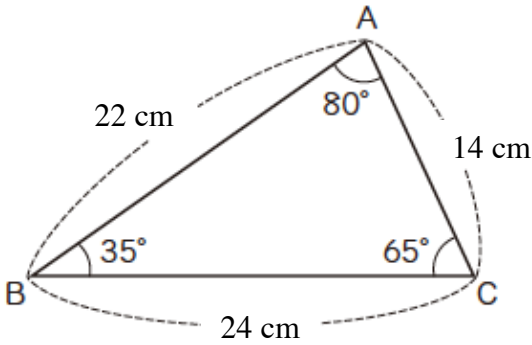
Each student summarizes their learning and records it in their notes.

Lesson 3 (60 minutes): Let's find out ways to draw a congruent triangle!

(1) Objectives

- To discover three efficient ways of constructing congruent triangles through drawing as a foundation for understanding three conditions for congruency of triangles.

(2) Flow of the lesson

Steps, Learning Activities Teacher's Questions and Expected Student Reactions	Teacher's Support	Points of Evaluation
<p>1. Introduction Let a few students read their journal reflections from the Day 2 and help the class to recall what they learned.</p>	<p>Select a few exemplary journal reflections from Day 2 note before the class.</p>	<p>Do students are ready for the new problem?</p>
<p>2. Posing the Problem Show the diagram below using projector and ask the following task.</p>  <div style="border: 1px solid black; padding: 5px; margin-top: 10px;"> <p>When two figures can fit on top of each other perfectly, we say that they are congruent. Draw triangle DEF on your paper so it will be congruent to triangle ABC shown above. Come up with an efficient way to draw a congruent triangle DEF.</p> </div>	<p>If students do not recall the meaning of congruent, demonstrate what does it mean by “fit on top of each other perfectly”.</p> <p>Give a few A4 size papers to each student.</p> <p>Encourage students to come up at least one way to draw a congruent triangle using fewer measurements.</p>	<p>Do students understand the task?</p>
<p>3. Anticipated Student Responses</p> <p>(a) use only three measurements to construct a congruent triangle successfully.</p> <ul style="list-style-type: none"> side-side-side side-angle-side angle-side-angle <p>(b) use 4 or more measurements to construct a congruent triangle successfully.</p> <p>(c) Cannot draw a congruent triangle.</p>	<p>Using seating chart to note each student's way of constructing a square for organizing the whole class discussion.</p>	<p>Does each student come up with at least one way to construct a congruent triangle?</p>

<p>4. Comparing and Discussing By using a drawing of the triangle, ask students to share how to construct the triangle. Discuss which measurements are used for each drawing</p> <p>Help students to realize that not all six measurements are needed for constructing a congruent triangle.</p>	<p>Using the actual size triangle ABC on a transparency to see if the students' DEF is congruent to the triangle ABC.</p>	<p>Do students construct triangle DEF by drawing? Do students understand not all six measurements are needed for constructing a congruent triangle?</p>
<p>5. Summing up Helping each student highlights the learning from the class and records them on their note.</p> <ul style="list-style-type: none"> • Only three measurements are enough to draw a congruent triangle. • Students recognize there may be three different combinations for constructing a congruent triangle by drawing. side-side-side, side-angle-side, angle-side-angle 		<p>Each student summarizes their learning and records it in their notes.</p>

