Topic: Fractions
Year Group: 1st Year Mixed Ability

Title of the Lesson:

The Chocolate Challenge at Ballymakenny College!

Brief description of the lesson:

Students are faced with a dilemma. They are given a choice of three tables to sit at.

Each table has a different number of bars of chocolate with different numbers of people sitting at them.

They must decide which table they should sit at to give them the greatest amount of chocolate!

- This challenge is about chocolate. There is a room in Ballymakenny College with three tables in it.
- Table 1 has one bar of chocolate on it, table 2 has two bars of chocolate on it and table 3 has three bars of chocolate on it.

- Unfortunately for you the room is not empty! There are already
  - 3 people at table 1
  - 5 people at table 2
  - 7 people at table 3
Aims of the Lesson:

Short term aims:
- Student will understand how to rank fractions in order of size.
- Students will use their knowledge of equivalent fractions to simplify fractional problems.
- Students will use modelling techniques to aid them in problem solving using fractions.
- Students will call on previous knowledge involving fractions to help them with this problem.

Long term aims:
- I’d like my students to develop their concepts of fractions.
- I’d like my students to develop their curiosity for finding mathematical solutions to practical problems.
- I’d like my students to appreciate that mathematics can be used to communicate thinking effectively.
- I’d like to foster my students to become independent learners.
- I’d like my students to become more creative when devising approaches and methods to solve problems.
- I’d like my students to experience meaningful mathematics i.e. that they see a need for what they are studying.
- I’d like to build my students’ enthusiasm for the subject by engaging them with stimulating activities.
- I’d like my students to connect and review the concepts that we have studied already.
Learning Outcomes:

As a result of studying this topic, students will be able to:

- Understand the concept of ranking fractions according to size.
- Will be able to use equivalent fractions to help them solve the chocolate bar example.
- Will be comfortable using modelling techniques to help them answer the given problem.
- Will call on their previous knowledge to help them to answer the given question.

Background and Rationale

(a) What the students need to learn according to the syllabus.

These lessons will involve the students investigating and understanding:

- The number line to order numbers in Q.
- Models to help think about the operations of addition, subtraction, multiplication and division and the relationships between these operations.
- How to generalise and articulate observations of arithmetic operations.
- How to consolidate the idea that equality is a relationship in which two mathematical expressions hold the same value.
- How to analyse solution strategies to problems.
- The use of decimals.
- The use of percentages.
- The equivalence of fractions, decimals and percentages to compare proportions.
- How to check a result by considering whether it is of the right order of magnitude.
- How to check a result by working a problem backwards.
- How to justify approximations and estimates of calculations.

(b) Difficulties students have had in the past with the subject matter

Students traditionally have difficulty in conceptually understanding what happens when we multiply or divide fractions by each other. Based on the evidence gathered from simple fractional exercises in our school, (see below for example of this) it is clear that students struggle to understand fractional quantities and where they appear on a number line. In our SSE Numeracy testing last year, students scored poorly in the fractional computation sections of the test. This led us, as part of our SSE strategy for the year to take fractions as our focus point. Every subject teacher will dedicate one lesson per term to fractions i.e. they will incorporate fractions into their lesson. We want students to see that fractions are applicable across all subject areas and that they have a very real-life context. We hope that the context of our lesson question will stimulate students’ engagement in the topic as well as enhancing their overall capabilities in tackling fractional problems.

(c) The thematic focus of this lesson study:

- To allow students to develop and understand the concept of fractions.
- To allow students to develop their curiosity for finding mathematical solutions to practical problems.
- To encourage students to become independent learners.
- I’d like my students to become more creative when devising approaches and methods to solve problems.
- I’d like my students to experience meaningful mathematics i.e. that they see a need for what they are studying.
• I’d like to build my students’ enthusiasm for the subject by engaging them with stimulating activities.
• I’d like my students to connect and review the concepts that they have studied already

Research

3. Nrich.maths.org - Copyright of University of Cambridge
4. Maths Development Team Website

About the Unit and the Lesson

This lesson is designed to consolidate the students’ learning throughout the previous five lessons in the unit. The problem posed is designed to consolidate the key concepts of fractions and equivalent fractions but may also link into decimals and percentages, depending on the approach taken by the students. The problem will help students to meet the following learning outcomes:

- Synthesis and problem-solving skills (syllabus page 20). The students will use their prior knowledge of fractions and equivalent fractions and apply this to solve the problem posed.
- Use the equivalence of fractions, decimals and percentages to compare proportions (syllabus page 22). Students will compare the proportions of chocolate at each table to ascertain the best place to sit.
- Justify approximations and estimations of calculations (syllabus page 22). Students may estimate the best place to sit based on the chocolate and number of people at each table but will need to justify these estimations with mathematical proof.
- Investigate models to help think about the operations of addition, subtraction, multiplication and division of rational numbers (syllabus page 22). Students will use chocolate bars as models to help in solving the problem.
- Calculate percentages (syllabus page 22). Students will calculate percentages to compare the proportions at each table.
**Flow of the Unit:**

<table>
<thead>
<tr>
<th>Lesson</th>
<th># of lesson periods</th>
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<tbody>
<tr>
<td>Use the number line to order numbers in Q</td>
<td>5 x 60 min classes</td>
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<tr>
<td>Models to help think about the operations of addition, subtraction, multiplication and division and the relationships between these operations.</td>
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<tr>
<td>How to generalise and articulate observations of arithmetic operations</td>
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<td>How to consolidate the idea that equality is a relationship in which two mathematical expressions hold the same value</td>
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<td>How to analyse solution strategies to problems</td>
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<tr>
<td>Begin to look at the idea of mathematical proof</td>
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<tr>
<td>Decimals</td>
<td></td>
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<tr>
<td>Percentages</td>
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<tr>
<td>The equivalence of fractions, decimals and percentages to compare proportions</td>
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<td>How to check a result by considering whether it is of the right order of magnitude</td>
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<td>How to justify approximations and estimates of calculations</td>
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<tr>
<td>How to present numerical answers to the degree of accuracy specified</td>
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<tr>
<td>Research Lesson</td>
<td>1 x 60 min class</td>
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Flow of the Lesson

<table>
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<th>Teaching Activity</th>
<th>Points of Consideration</th>
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<tbody>
<tr>
<td><strong>1. Introduction</strong>&lt;br&gt;This lesson is reviewing a topic already covered- Fractions. Specifically, this lesson will involve recall of sub-topics such as equivalent fractions, ordering fractions, comparing fractions &amp; common denominators.</td>
<td>The teacher is not going to mention any specific method, however will refer to a range of mathematical methods that have been explored previously.</td>
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<tr>
<td><strong>2. Posing the Task</strong>&lt;br&gt;If the chocolate on the table I sit at is to be shared out equally when I sit down, which would be the best table to sit at?&lt;br&gt;• Table 1 (1 Bar) = 3 people already sitting&lt;br&gt;• Table 2 (2 Bars) = 5 people already sitting&lt;br&gt;• Table 3 (3 Bars) = 7 people already sitting</td>
<td>• Questioning for understanding.&lt;br&gt;• Diagram of the scenario on the Projector using Apple TV’s and iPads.&lt;br&gt;• Give clear instructions to students</td>
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<tr>
<td><strong>3. Anticipated Student Responses:</strong>&lt;br&gt;• Guess based on total number of bars at each table&lt;br&gt;• Using fraction wall poster from the wall&lt;br&gt;• Cutting, folding, drawing &amp; comparing manipulatives&lt;br&gt;• Using equivalent fractions&lt;br&gt;• Comparing fractions with common denominators and ranking them&lt;br&gt;• Comparing using decimals&lt;br&gt;• Comparing using percentages&lt;br&gt;• Comparing using fractions</td>
<td>• Prepared signage of predicted responses.&lt;br&gt;• Teacher may prompt students to try various methods to calculate the answer.&lt;br&gt;• To help students who get stuck, teacher may encourage students to re-think the question.&lt;br&gt;• Problem will be displayed throughout the lesson and students will be encouraged to continuously reference it.&lt;br&gt;• For students who finish early, they will be encouraged to try other methods and award effort accordingly.</td>
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<td><strong>5. Comparing and Discussing: Céardaocht</strong>&lt;br&gt;• Guess based on total number of bars at each table (1)&lt;br&gt;• Cutting, folding, drawing &amp; comparing manipulatives (2)&lt;br&gt;• Comparing using fractions (3)&lt;br&gt;• Using equivalent fractions (4)&lt;br&gt;• Comparing fractions with common denominators and ranking them (5)&lt;br&gt;• Comparing using decimals (6)&lt;br&gt;• Comparing using percentages (7)&lt;br&gt;• Using fraction wall poster from the wall (8)</td>
<td>• The various methods used by the students.&lt;br&gt;• To identify each individual bar as a whole unit as opposed to seeing the total quantity of chocolate at each table as the whole unit.&lt;br&gt;• To compare and rank different fractional quantities&lt;br&gt;• Through effective questioning, teacher will ascertain understanding.&lt;br&gt;• Students will be able to clearly justify their answer while at the board.&lt;br&gt;• Students will learn alternative approaches from their peers.</td>
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</tbody>
</table>
Students will comment and contribute from the floor.

5. **Summing up**
- Teacher will recap on the main contributions made on the board, keeping in mind not to prioritise or dwell on any particular method.
- Teacher will set an extension activity to consolidate the learning.

<table>
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<tbody>
<tr>
<td>➢ What is your plan for observing students?</td>
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<td>➢ There will be 4 teachers in the classroom and each teacher will be allocated students to observe.</td>
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<tr>
<td>➢ Teachers will circulate the room around their designated students.</td>
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</tbody>
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| Discuss logistical issues such as who will observe, what will be observed, how to record data, etc. |
| ➢ Barbara, Mike, Caroline & Ginny will observe. |
| ➢ Teachers will observe how the students tackle the problem, the range of strategies used to solve the problem and identify groups to complete the board work, corresponding to our planned sequence (see section 5 above in flow of lesson) |
| ➢ Seating plans and group roles will be pre-determined in advance of the lesson. |
| ➢ Recording templates will be used by each observing teacher to document student strategies, comments and progress. |

| ➢ What observational strategies will you use (e.g., notes related to lesson plan, questions they ask,)? |
| ➢ Seating plans and group roles will be pre-determined in advance of the lesson. |
| ➢ Recording templates will be used by each observing teacher to document student strategies, comments and progress. |

| ➢ What types of student thinking and behaviour will observers focus on? |
| ➢ Any of our anticipated strategies as outlined in section 3 above. |
| ➢ Reluctance to seek alternative methods beyond their initial solution. |
| ➢ Any obstacles that students encounter in developing solutions. |

| ➢ What additional kinds of evidence will be collected (e.g., student work and performance related to the learning goal)? |
| ➢ Alternative strategies used to attempt solving the problem. |
| ➢ Student misconceptions noticed in attempting the problem. |
| ➢ Common problems within the class that may need to be revisited by the class teacher. |
| ➢ Strategies that suited certain students that may help when preparing future lessons. |
Board Plan

Post-lesson reflection

• What are the major patterns and tendencies in the evidence?
All students grasped the nature of the task posed and were quick to engage in the group activity. The students were challenged, engaged and were keen to solve the problem in as many ways as possible. There was a competitive dynamic within the class group as students enthusiastically explored the range of methodologies at their disposal.
Initially, students were concerned that they were not informed as to the number of squares per bar of chocolate. Students needed reassurance that the bars could be divided into any number of squares if they wished.

Students then set about dividing the chocolate using various techniques such as drawing lines, cutting, folding and using the manipulatives provided.

- **What are the key observations or representative examples of student learning and thinking?**
  Students were quick to start the task but were obsessed about the notion that there were a certain number of squares per bar of chocolate.

  We ascertained that 75% of the class were clear about what was required. It was evident that the leaders in the group who understood the task were instinctively explaining this to those who were unsure. This was largely down to the mixed ability groupings as arranged by the teachers.

  Students in general were quick to use the fraction method at the first and second tables in the problem but struggled to apply the same methodology to the third table.

  The majority of groups that started with the fraction method proceeded to convert into decimals or percentages.

- **What does the evidence suggest about student thinking such as their misconceptions, difficulties, confusion, insights, surprising ideas, etc.?**
A common misconception amongst students was their perception of “the unit” and they saw each bar of chocolate as the unit rather than the total amount at each table. This caused problems when trying to determine which table they should sit at to get the most. Two groups did not overcome this obstacle and therefore did not successfully solve the problem.

**In what ways did students achieve or not achieve the learning goals?**

Four of the groups solved the problem and established the correct place to sit to optimise their consumption of chocolate.
Most students understood how to rank fractions in order of size.
Students did not apply their knowledge of equivalent fractions to the extent we had anticipated.
Students modelled the situation using the manipulatives provided.
Students called on their previous knowledge of fractions to help them with this problem.
The pre-prepared posters worked very well in getting all students to follow the flow of the lesson. This problem solving approach worked well as a consolidation of various topics involving fraction, decimals and percentages. It also allowed students to benefit from peer learning and collaboration.

**Based on your analysis, how would you change or revise the lesson?**
The fact that we used bars of chocolate hindered them as they are used to dividing bars of chocolate into eights. It may be more beneficial to use a different concept such as pizzas that are not necessarily associated with a set amount of slices.
• **What are the implications for teaching in your field?**

As a department in Ballymakenny College, we have decided to incorporate this lesson as one of our five core lessons on fractions every year. This lesson will be available to all teachers on SharePoint, our Virtual Learning Environment.

The resources prepared for our lesson study will be available to all teachers in our school.

The approach learned throughout our experiences with lesson study will inform our future teaching of various topics, particularly when introducing real life problems.

Because of choosing fractions for our lesson study, we will prioritise the teaching of fractions within our school.