Lesson Study Lesson Proposal

Fifth Year Mixed Ability & Geometry/Trigonometry

For the lesson on 24th January 2017
At Deele College, Mary Gibbon’s class
Teacher: Fiona Friel

Lesson plan developed by:
Fiona Friel & Bella Glackin (Deele College)
Leanna Ni Bhaoill & Siobhan Uí Shearcaigh (Pobalscoil Ghaoth Dobhair

1. Title of the Lesson: How to take the Perfect Selfie

2. Brief description of the lesson:

By drawing information from a graphic representation, students will be able to identify the various mythologies required to establish the distance between two points and discover its relationship between the associated slope or angle.

3. Aims of the Lesson:
   - To develop independent learners
   - To promote a positive disposition towards problem solving
   - To consolidate previous learning
   - To extend students’ knowledge and engage in new learning experiences
   - To learn to reflect on their strategies and adjust their approaches as they problem solve
   - To develop students’ confidence in identifying relevant mathematical methodologies to solve the problem
   - For students to understand the relationship between two points

4. Learning Outcomes:

As a result of studying this topic, students will be able to:
   - Draw information from a word and graphical problem
   - Convert into relevant mathematical information
   - Utilizing
     - distance formula
     - the slope
     - Pythagoras Theorem
     - Trigonometric ratios
   - Solve a problem

5. Background and Rationale

A task must engage learners and present them with a challenge that requires exploration. Problem solving tasks activate mathematical thinking processes as opposed to imitative thinking processes (Mathematics Leaving Certificate Syllabus, pg. 9/10)

It is within this framework that this lesson was developed and a problem-solving environment created in which students will:
   - Make sense of the problem
   - Make sense of the mathematics they are utilizing
   - Arrive at a correct solution
The solutions to the problem are to be located within Geometry and Trigonometry with the aim of the lesson being that students would discover the relationship between slope and angle, and the equation of the line. Too often students while studying the various elements of Geometry view them as standalone topics and fail to see the relationship between them. For example, they will be able to find the equation of a line but fail to comprehend the information that this same equation imparts. Similarly, how the slope is determined using various pieces of information and how this in turn relates to the angle of a line.

6. Research
In preparation of this lesson, use has been made of the following materials:
- Mathematics syllabus Leaving certificate (for examination from 2015)
- Third Year Teacher’s Handbook (Mata website)
- Leaving Cert. Ordinary Level Teacher’s Handbook
- Geogebra tool

7. About the Unit and the Lesson
There are five key skills identified as central to both teaching and learning across the curriculum at senior cycle
- Information processing
- Critical and creative thinking
- Communicating
- Working with others
- Being personally effective (Mathematics Leaving Certificate Syllabus, pg. 9/10)

We anticipate that students will understand the concept of problem solving using a variety of techniques, that it will help students to recognize the link between Pythagoras Theorem and distance and coordinate geometry, trigonometry, the relationship between slope and angle. Using numerous methodologies and strategies to illustrate these connections, to promote a willingness to explore problems and to instill confidence in students when faced with unfamiliar scenarios.

8. Flow of the Unit:
From Handbooks

<table>
<thead>
<tr>
<th>Lesson</th>
<th># of lesson periods</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Review of Junior Cycle ordinary-level coordinate geometry - coordinating the plane, distance and midpoint formula LCOL.20</td>
</tr>
<tr>
<td></td>
<td>Area of a Triangle LCOL. 22</td>
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<td></td>
<td>Revision of preliminary concepts - Plane and points LCOL.25</td>
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<tr>
<td>2</td>
<td>Trigonometry 1 Pythagorus Theorem LCOL.63</td>
</tr>
<tr>
<td>3</td>
<td>Problem Solve: How to take the perfect selfie</td>
</tr>
<tr>
<td>4</td>
<td>Equation of Line LCOL.23</td>
</tr>
<tr>
<td></td>
<td>Intersection of two lines LCOL.24</td>
</tr>
<tr>
<td></td>
<td>Revision of preliminary concepts - Plane and points LCOL.25</td>
</tr>
<tr>
<td>5</td>
<td>Revision - Angles, Axiom 3, Theorem 1, Constructions 8 &amp; 9 LCOL.26</td>
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</table>
9. Flow of the Lesson
Anticipated student responses

<table>
<thead>
<tr>
<th>Teaching Activity</th>
<th>Points of Consideration</th>
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<tbody>
<tr>
<td>Distance formula</td>
<td>Ensure in the preceding lessons that the students are comfortable with using geometric and trigonometric formulae, utilising the Formulae and Tables book and using a mathematical set. The teacher explains the problem so there is no confusion.</td>
</tr>
<tr>
<td>Slope formulae</td>
<td>Students should be actively engaged in lesson.</td>
</tr>
<tr>
<td>Pythagoras Theorem</td>
<td></td>
</tr>
<tr>
<td>Trigonometric ratios</td>
<td></td>
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<tr>
<td>The placement of the phone for the best selfie.</td>
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1. Introduction
- Recap Prior Knowledge
- How would you take a selfie? Could we use mathematics to help us take the best selfie?

2. Posing the Task
Kylie Jenner gives advice on taking the best selfie: "Always try to position the camera above you and as far away from your face as your arm will reach."

Find as many different ways as you can to mathematically describe the "perfect selfie" taken by the model above.

Students will be asked to verbalise the problem.

3. Anticipated Student Responses

R1: Go across 14 squares and up 7 squares
(This solution does not address units but will become evident in later discussion)

The teacher and observers will attempt to keep students encouraged and engaged where appropriate.

If partial solutions are given, the teacher will use other partial solutions to build a full solution.

R2: Go across 42cm and up 21cm
(Position camera at a 42cm horizontal distance and a 21cm vertical distance up from eye)
R3: The Coordinates of A(0,0) and B(7,14) – Not Scaled

R4: The Coordinates of A(0,0) and B(21,42) – Scaled

R5: Slope; Rise over run OR Slope Formula

(1) \( m = \frac{\text{rise}}{\text{run}} = \frac{7}{14} = \frac{1}{2} \)

(2) \( m = \frac{\text{rise}}{\text{run}} = \frac{21}{42} = \frac{1}{2} \)

(3) \( m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7 - 0}{14 - 0} = \frac{1}{2} \)

(4) \( m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7}{14} = \frac{1}{2} \)
R6: The radius $|AB| \approx 47$ cm, by construction

R7: Distance $|AB|$ using Distance Formula

A(0,0) to B(7,14)

OR

A(0,0) to B(21,42)

R8: Distance $|AB|$ using Pythagoras Theorem
<table>
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<tr>
<th>R9: The angle of elevation measures ( \approx 26^\circ )</th>
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<tbody>
<tr>
<td>R10: Trigonometric ratios, to find ( \tan^{-1} ), angle of elevation ( = 26.56^\circ )</td>
</tr>
<tr>
<td>R11: The line ( y = \frac{1}{2}x ) contains [AB] (to find slope: ( y = mx + c ))</td>
</tr>
<tr>
<td>R12: Position of camera in terms of angle and distance</td>
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</table>
### R13: Finding |AB| by measuring and scaling

![Image of measuring |AB|](image)

### R14: Misunderstanding the task, think it is a trick question, unsure of information required to solve problem effectively

### R15: Incorrect Solution – equation of a line

<table>
<thead>
<tr>
<th>4. Comparing and Discussing</th>
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<tbody>
<tr>
<td>Teacher chooses students who attempted the problem in the following order</td>
</tr>
<tr>
<td>· Distance</td>
</tr>
<tr>
<td>· Slope</td>
</tr>
<tr>
<td>· Pythagoras Theorem</td>
</tr>
<tr>
<td>· Trigonometric ratios</td>
</tr>
<tr>
<td>· Measurement &amp; Scaling Do we have enough information?</td>
</tr>
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Are students making the connection? 
Are they actively engaged in the problem solve? 
One piece of information is not enough.

Is one piece of information enough in order to take the perfect selfie? What pieces of information do we need? Are they connected? If so, how?

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<tr>
<th>5. Summing up</th>
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<td>Students will be asked to write a few sentences on today’s learning and self-evaluate using KWL</td>
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### 2. Evaluation

**What is your plan for observing students?**

- Personal observation, record notes and photos

**Discuss logistical issues such as who will observe, what will be observed, how to record data, etc.**

- Teacher: Fiona Friel,
- Observers & Note Takers: Leanna Ni Bhaoill, Síobhan Úi Shearcaigh & Bella Glackin

**What observational strategies will you use (e.g., notes related to lesson plan, questions they ask)?**

- Observe how students behave, attempt first, develop solution etc.
What types of student thinking and behaviour will observers focus on?
- Approach to solution, attitude towards problem solve, level of engagement

What additional kinds of evidence will be collected (e.g., student work and performance related to the learning goal)?
- Student notes and oral contribution

3. Board Plan

![Board Plan Image]
4. Post-lesson reflection

What are the major patterns and tendencies in the evidence?

- Many students found the coordinated points first
- Several then developed into Pythagoras Theorem
- Very few measured the angle required or length
- Several of the anticipated partial responses were used
- Students were fearful of being incorrect

What are the key observations or representative examples of student learning and thinking?

- Students were slow to start as they were out of their comfort zone
- When they were assured of multiple solutions, they relaxed considerably and engaged more in problem solve
- They worked mostly together in groups. The security of numbers seemed to allow students to show strengths, share knowledge and provide safety for weaker/unconfident students.
- Very few students worked independently.
What does the evidence suggest about student thinking such as their misconceptions, difficulties, confusion, insights, surprising ideas, etc.?

- The application and manipulation of trigonometric ratios caused difficulties. Students were not comfortable using Tan-1, they knew what tan meant – need for attention to trigonometric ratios.
- Many students knew how to get so far but couldn’t complete solution. They seemed to have difficulty in connecting to the question.
- Manipulation of the square root caused problems.
- Prior Learning is so important.
- Many were confused about what solution should look like.
- Most students persevered in trying to find multiple students.
- Peer observation influenced effort positively.
- 3cm on grid caused confusion – we will consider adjusting measurements on problem.

In what ways did students achieve or not achieve the learning goals?

- They had better understanding of links between various strands of Maths Curriculum.
- They were able to apply the Prior Knowledge but comprehension was not behind it.
- Students actively learned, became critical thinkers.
- Students became comfortable giving contributions.
- Most students were actively engaged.

Based on your analysis, how would you change or revise the lesson?

- Students would need a double class – 10 mins perfect for working it out but need greater time for Cearaíocht, due to the numerous solutions.
- Consider solutions on cards, in terms of less is more.
- Ensure adequate time for self evaluation of student.
- Ensure adequate time to link solutions to learning goal and reality.

What are the implications for teaching in your field?

Reflections on Practice:

- Facilitated the networking and collaboration with other school teachers.
- Gave confidence to teachers in teaching problem solve.
- We often do not give enough credit to students regarding their ability.
- We are often too quick to solve problems for them.
- Emphasises the benefits of working in a team – develops social skills.
- Demonstrates linkages between different strands in curriculum.
- Textbooks do not lend themselves to problem solving.
- “Connect with Maths” or “The Power of Maths” possibly might be considered as good matches for supporting curriculum alignment.
The "perfect selfie" taken by the model above.

Find as many different ways as you can to mathematically describe camera above you and as far away from your face as your arm will reach.

"Always try to position the

The Perfect Selfie