Lesson Proposal – The Polygon Predicament

Santa Sabina Dominican College,
Sutton,
Dublin 13

Topic: Patterns, Combinations, Derivation of formulae

Year Group: Leaving Certificate Higher Level

For lesson on: 17th January 2017

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Title of the Lesson
The Polygon Predicament.

Brief description of the lesson
Students will derive a formula to ascertain the number of diagonals in any polygon. The task will require students to use prior knowledge to derive a formula as well as test and verify it.

Long-Term Aims
We would like students to

- Gain confidence in choosing appropriate strategies to apply to an unfamiliar problem.
- Make connections between different strands of maths.
- Appreciate the validity of different methods of problem solving.
- Develop confidence deriving and using formulae.
- Communicate their mathematical thinking using mathematical terminology (Senior Cycle Key Skill).
- Be creative – Explore different ways of doing things and of thinking (Senior Cycle Key Skill).
- Check solutions to problems and reach judgement on whether their solution is likely to be correct.

Short-Term Aims
We would like to

- Gain insight into how students solve combinatorial counting problems

We would like students to

- Avoid making common errors such as over-counting.

- Have confidence to move from familiar and comfortable to abstract and uncomfortable.

Learning Outcomes
As a result of studying this topic, students will be able to

- Apply the specifics of one problem to a more general context.
- Choose an appropriate strategy to solve an unfamiliar problem.

Background and Rationale

This unit of study will help students with

1. Strategic Competence – ability to formulate, represent and solve mathematical problems in both familiar and unfamiliar contexts.

2. Adaptive Reasoning – develop capacity for logical thought, reflection, explanation, justification and communication.

Specifically:

- 1.1 Counting – Count the number of ways of selecting r objects from n distinct objects.

- 4.1 – Generating Expressions From Repeating Patterns – Generalise and explain patterns and relationships in words and numbers.

- 4.1C – Finding Formulae – Ways to express a general relationship arising from a pattern or context.

Difficulties Students Have Had In The Past:

1. Identification of appropriate mathematical context.

2. Over-counting.

3. Confusion regarding distinction between circle and polygon (Adjacent points issue).

4. Identification of pattern.

5. Verification of derived formula (testing it).

Research

In preparation of this lesson proposal, the following materials were accessed:

(a) Leaving Certificate syllabus from

http://www.ncca.ie/en/Curriculum_and_Assessment/Post-Primary_Education/Project_Maths/Syllabuses_and_Assessment/LC_Maths_English_2013.pdf

(b) Senior cycle Teachers Handbooks from http://www.projectmaths.ie/documents/handbooks/LCHLHandbook2015.pdf

(c) Selection of textbooks for Leaving Certificate Higher Level Maths

About the Unit and the Lesson:

According to the Senior Cycle syllabus for examination from 2015, students at higher level must be able to

(a) Count the number of ways of selecting r objects from n distinct objects (1.1 Counting, page 17)

(b) Generalise and explain patterns and relationships in words and numbers (4.1 Generating Expressions from Repeating Patterns, page 36)

(c) Find Formulae (Ways to express a general relationship arising from a pattern or context, page 36)

Throughout all units of study, we endeavour to instil the key skills central to teaching and learning at senior cycle (Page 9). This lesson proposal encourages "critical and creative thinking", where learners are encouraged to solve problems in a variety of ways, along with justifying claims and results. "Communicating", also a key skill at senior cycle is developed as students are expected to discuss approaches and solutions, as well as consider and listen to other viewpoints.

The senior cycle syllabus also states that maths "at higher level is geared to the needs of learners who may proceed with their study of mathematics to third level. Moreover, particular emphasis can be placed on the development of powers of abstraction and generalisation and on the idea of rigorous proof". Teachers find this challenging to teach, and learners find it challenging to learn.

Flow of the Unit:

Lesson		**# of Lesson Periods**
1	Fundamentals Principles of Counting	1
2	Arrangements	1
3	Combinations	1
4	Research Lesson	1
5	Concepts of Probability	1

Advice taken from Lesson Idea LCHL36 from Maths Teacher's Handbook regarding layout of unit.

Flow of the Lesson:

Teaching Activity and Students' Anticipated Responses	Points of Consideration
1. Introduction – 5 minutes Prior knowledge will be activated, in particular the students understanding of what a polygon is and what constitutes a diagonal. Teacher will display a triangle and ask students to write the number of diagonals on mini whiteboards. She will then move on to a quadrilateral and a pentagon. She will highlight the number of sides and the number of diagonals. If students seem to not understand this task, share a few of the students' attempts at expressing their ways of counting the diagonals as examples.	Teacher will use questioning to illicit the prior knowledge from the students. She will use diagrams on the board to illustrate polygons and diagonals. By briefly showing the pentagons one at a time, encourage students to share their ways of counting the number of diagonals.
2. Posing The Task – 5 minutes Teacher will write problem on the board. "How many diagonals are there in an n-sided polygon?" She will ask the class if they want to ask any questions, and give clarification if needed. She will inform the class that they have ten minutes to work on the problem.	Teacher will give opportunity for questions or clarification.
3. Students Individual Work – 10 minutes Students try to solve the problem by using knowledge they have already learned. Students express their idea in a way everyone can understand. Students must think of a precise way to express their ideas.	Using a seating chart note each student's way of counting the diagonals to prepare for organising the whole class discussion - Ceardaíochr assessing students work to plan how to orchestrate the presentation of this work on the board and class discussion. Let each student write his/her way of counting and number of diagonals in an n-sided polygon her copybook. Advice to students: "Let's express it in a different way" "let's think which way of thinking is better". Other members of the research team will observe and record the lesson using Lesson Note.

4. Anticipated Student Responses – 20 minutes

Response 1:

Choosing

 : No diagonals: $\binom{3}{2} = 3 - 3 = 0$

 : 2 diagonals: $\binom{4}{2} = 6 - 4$ sides $= 2$

 : 5 diagonals: $\binom{5}{2} = 10 - 5$ sides $= 5$

 : 9 diagonals: $\binom{6}{2} = 15 - 6$ sides $=9$

N –sided polygon: $\binom{n}{2} - n$

Response 2

Mental Arithmetic

8 points/8 sides (edges)

$n = 8, n - 3 = 5$ optional points for a diagonal.

$\frac{8(8-3)}{2} = \frac{8(5)}{2} = \frac{40}{2} = 20$

repetition

$n = 7, n - 3 = 4$ optional points for a diagonal.

$\frac{7(7-3)}{2} = \frac{7(4)}{2} = \frac{28}{2} = 14$

$\frac{n(n-3)}{2}$

Students who get stuck will be prompted by teacher to find the diagonals in a 4 sided, 5 sided etc shape.

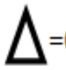
Students who finish early will be asked to find another way to solve the problem.

Anticipated incorrect response include

- Not taking edges away
- Not taking 3 away (may take 2 away for either side but not point itself)
- May forget to divide by 2
- Mathematical errors
- Find quadratic pattern but not general formula
- Unable to tease pattern out into a series

Response 3

Quadratic Sequences

T_1:  =0 T_2:  =2

T_3:  =5 T_4:  =9

No of diagonals: 0 2 5 9

First Difference: 2 3 4

Second Difference: 1 1

$2a = 1$

$a = \frac{1}{2}$ $\qquad T_n = an^2 + bn + c$

$\left(\frac{1}{2}\right)(1)^2 + b(1) + c = 0$ $\qquad T_2 = \left(\frac{1}{2}\right)(2)^2 + b(2) + c = 2$

$+c = 0$ $\qquad 2 + 2b + c = 2$

$b + 2c = 0$ $\qquad 2b + c = 0$

$2c = -1$

$2b + 2c = -1$ $\qquad 2b - 1 = 0$

$\underline{-2b - c = 0}$ $\qquad 2b = 1$

$c = -1$ $\qquad b = \frac{1}{2}$

$T_n = \left(\frac{1}{2}\right)n^2 + \frac{1}{2}n - 1$

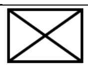
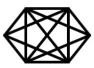
Note: This formula is not the final solution. This is revealed when students check their work (test values). This formula now needs to be adjusted to

$$S_n = \frac{1}{2}(n-2)^2 + \frac{1}{2}(n-2) - 1$$

as the difference between the term number and the number of sides in that shape is two.

Response 4

Arithmetic Sequences

3 sided		0	0 diagonals
4 sided		A:2	2 diagonals
5 sided		A:2 B:2 C:1 D:0 E:0	2+3= 5 diagonals
6 sided		A:3 B:3 C:2 D:1 E:0 F:0	2+3+4= 9 diagonals

General Formula:

2+3+4+5+……+n−2

∴20 sided shape:2+3+4+…+18=170

$$S_n = \frac{n}{2}(2n + (n-1)d)$$

$$S_{17} = \frac{17}{2}(4 + (16)1) = 170$$

5. Comparing and Discussing – Ceardaíocht – Students can use their prior learning such as mental arithmetic, counting, quadratic and arithmetic sequences to express correctly their ways of finding how many diagonals are in an n-sided polygon. For each mathematical expression, 1. Ask one of the students who came up with a Mathematical expression to show it to the class, Ask the questions to this students who is explaining their ideas : “Why is that?” (Asking for evidence) “So?” (Use when you want the student to speak continuously) 2. Let other students infer how the student counted the diagonals by interpreting the mathematical expression, Ask these questions: “Is there anyone who solved it the same way?” “Let’s have another person who solved it the same way explain it”? 3. let the student who came up with the mathematical expression justify if the other student’s inference is correct. 4. Ask another student(s): “Can you understand this way of thinking?” “Can you explain it?” (Read the way of thinking what is on the board) Repeat the above so students understand the variety of ways to count the number of diagonals in an n-sided polygon. Putting together all the ideas: “Is there anything you noticed by looking at these ideas?” “Which way of thinking do you find the easiest to understand?” “Which id the most efficient method?” “Which would you use again?”	Each method will be tagged with the students name and a pre-prepared label identifying the method. By providing an opportunity to infer other students’ way of 1.counting the number of ways of selecting r objects from n distinct objects. 2. generating Expressions From Repeating Patterns 3. Finding Formulae to express a general relationship arising from a pattern or context. help students see that a mathematical expression can communicate a way of thinking.

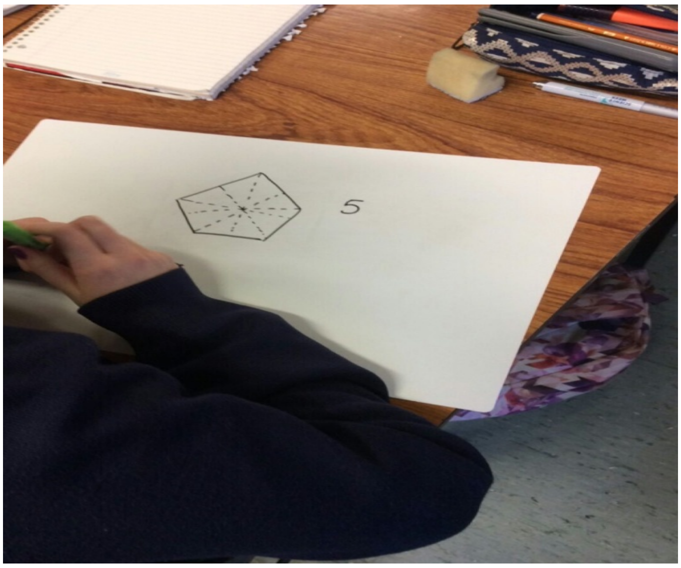
6. Summing Up – 2 minutes Help each student identify the learning from the class and record it in their notes. Students write 1 to 3 points on: What they understood? What they thought of their friend's ideas compared to their own? Questions they have.	Promote discussion about each method, highlighting advantages and disadvantages.

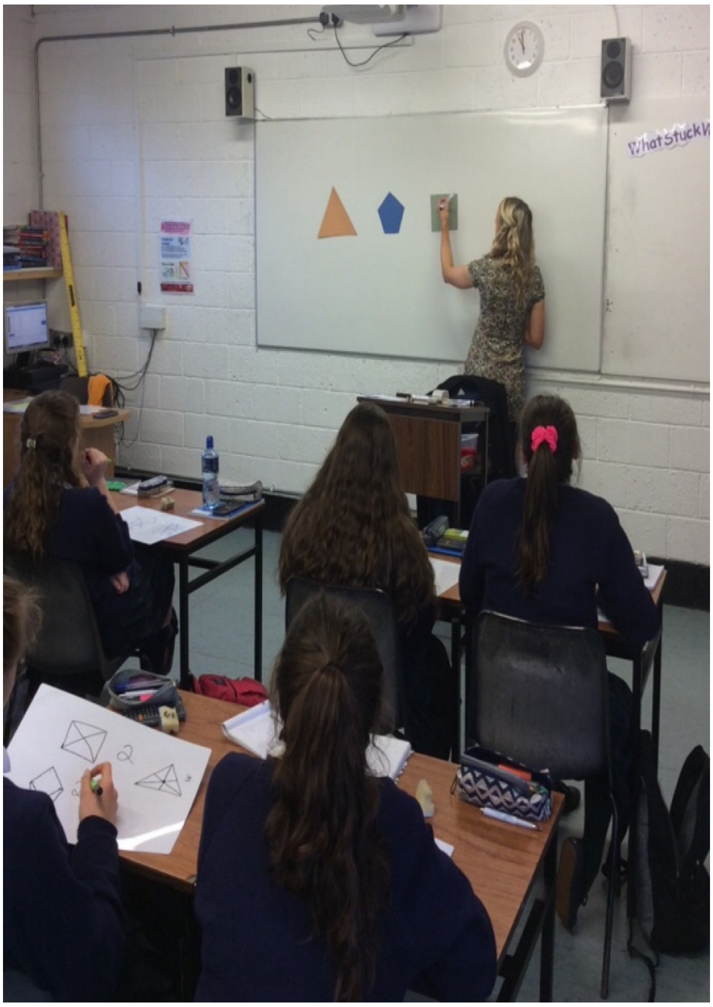
Evaluation – Plan for Observing Students

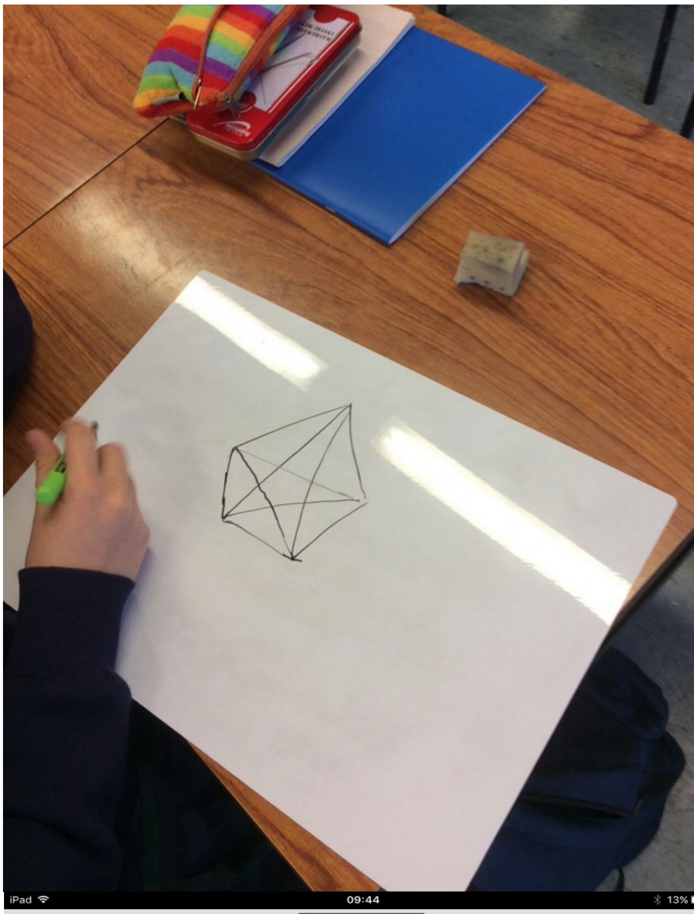
Plan for observing students is as follows:

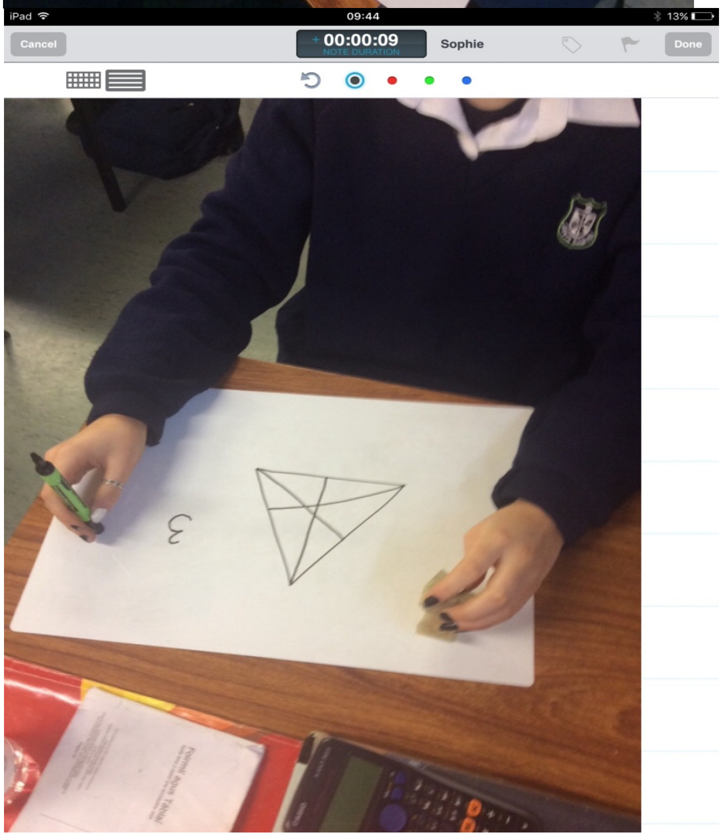
- Teacher to supply classroom layout in advance of research lesson
- Three teachers will observe the 17 students using lesson note app
- Observers will sit for 6 minute prior knowledge part of lesson.
- They will circulate for 10 minute problem solving section of lesson, taking photographs of students approaches to the problem.
- They will be looking for examples of students taking any correct steps to solving the problem.
- They will document which strategies students are using.
- They will take photographs of some students work.
- One teacher will record all questions asked by students.

Photos From The Research Lesson:









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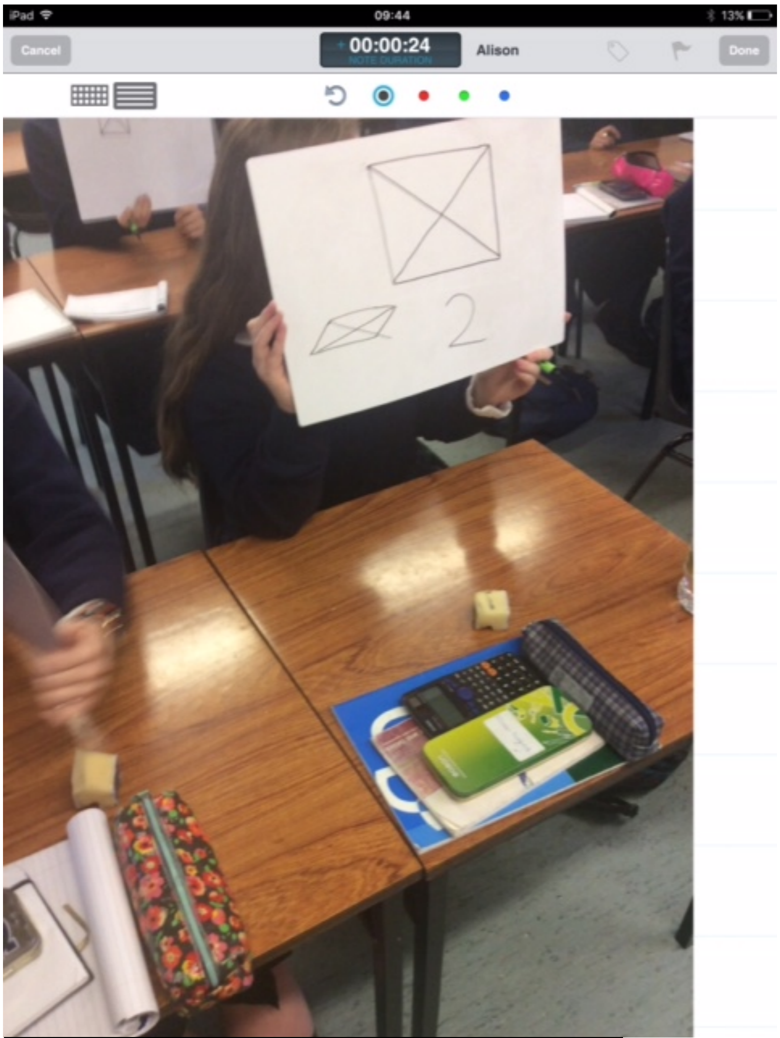
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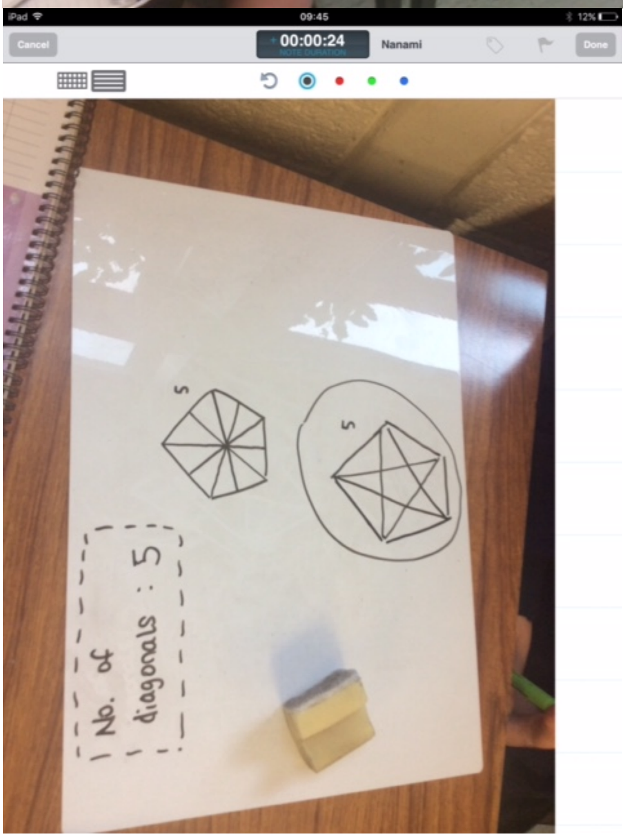
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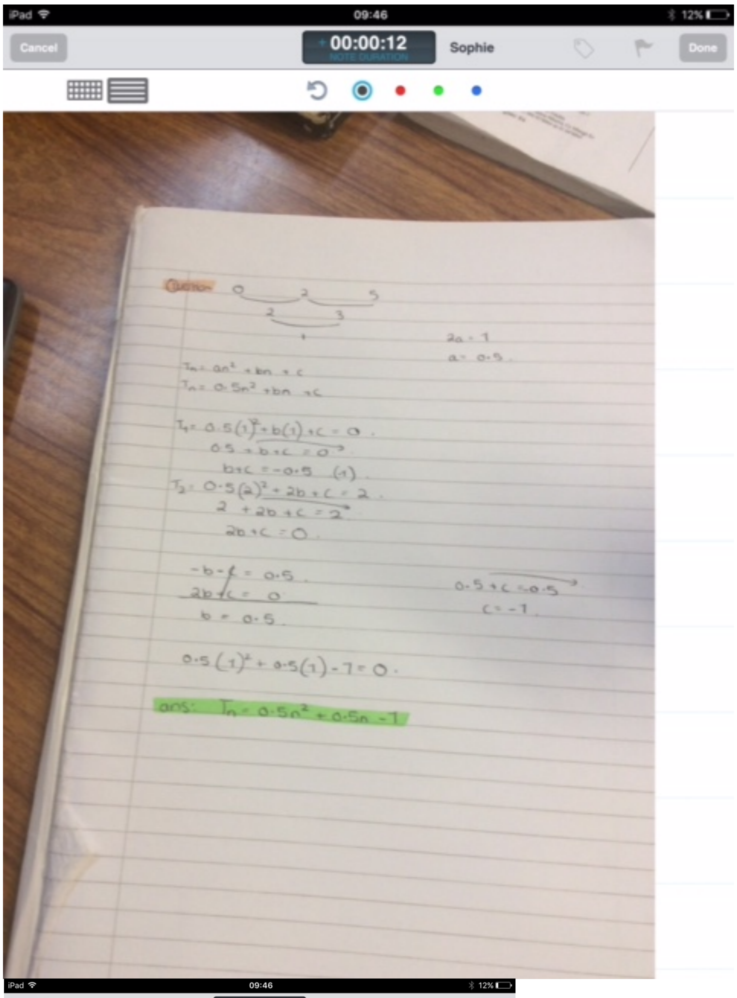
No. of diagonals : 5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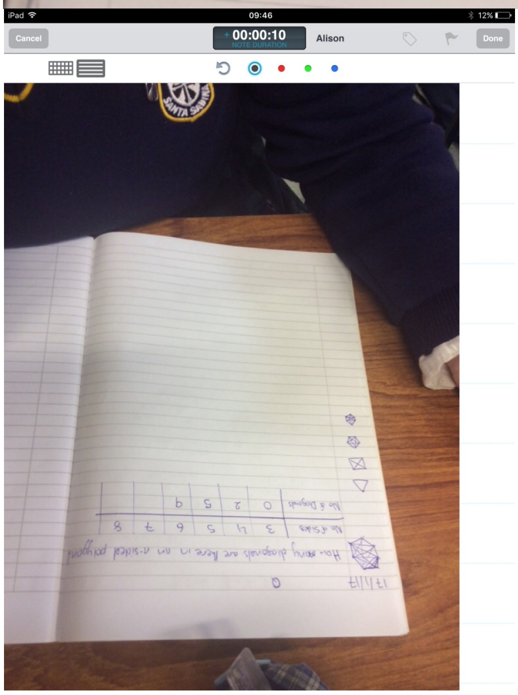
ans: Tn = 0.5n² + 0.5n - 1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Post-Lesson Reflection:

Following the lesson we will examine

<u>Getting Started</u>

- **Could students recall prior knowledge?**
  No! When asked to write the number of diagonals in a triangle, no student wrote 0. Teacher then moved to quadrilateral and had to tease out what exactly a diagonal was as no student knew. Several students knew that it bisected the area of a parallelogram, but though it must go diagonally across the middle. Several other students thought a diagonal was an axis of symmetry.
  Teasing out of prior knowledge took 32 minutes. We thought it would take 10 minutes. We were shocked that they did not know what a diagonal was.
  While they were able to figure out the diagonals in a triangle and quadrilateral, they struggled with the pentagon with instances of overcounting.

- **Were clarifications needed? Take note of questions asked by the students.**
  No questions were asked, although the teacher gave verbal clarification and told the students she wanted a general formula.

- **Could the students get started?**
  Students were able to start no problem as soon as they problem was posed.

Individual Problem Solving Segment

- **Did the teacher give prompts?**
  No, one student led most students down the route of quadratic patterns by spotting it during prior knowledge section.

- **What were the most utilised strategies?**
  Quadratic Patterns

- **Did anyone use multiple strategies?**
  3 students did.

- **Discuss the range (or lack) of strategies used**
  Lack of strategies linked to prior knowledge focus on the pattern

- **Did students succeed?**
  Yes. They showed great confidence and willingness to engage with the problem.

- **Did students who were struggling persist with the problem?**
  They persevered for the whole time allotted.

- **How long did it take them?**
  10 minutes of time was allotted and used by the students

Discussion of Problem

- **Are students attentive to others at the board?**
  Yes, very attentive and engaged. Willing to contribute and eager to hear solutions.

- **Are clarifications needed by those presenting their work?**
  The first student used the formula

$$2a = 1$$

$$a = \frac{1}{2} \qquad\qquad T_n = an^2 + bn + c$$

and another asked where she got the formula from. She was confused as to where the coefficient of n^2 came from. This was explained by the student at the board in a very satisfactory way.

- **Any surprising or unexpected events?**
  We were very surprised by the fact that students did not know what a diagonal was.

  We were delighted to see that following one students presentation, another student noticed that her formula did not work and was able to present to the class why this was so.

- **Were students attentive?**
  Very attentive and engaged. Thoroughly enjoyed lesson as noted on their exit slips.

- **Did lesson promote student learning?**
  Definitely. They gained confidence in problem solving. They learned about diagonals and polygons. They reviewed their solutions and made adjustments. The lesson promoted cooperative learning and was not teacher led.

  The aims of the lesson were reached as students gained confidence in choosing an approach to solve a problem. The lesson also showed multiple solutions to the problem. They appreciated the validity of other student's solutions. They used mathematical terminology in an appropriate and impressive way as they communicated their thinking to their peers.

  The importance of testing their solution was really highlighted and evident.

  Anticipated common errors such as over-counting were highlighted and clarified by students themselves.

  Students successfully moved from the concrete (triangle, quadrilateral and polygon) to the abstract (n-sided polygon).

- **What revisions would we make if teaching the lesson again?**
  We would put a definition of a diagonal on the board when it was established what it was. Students would take that down into their copies.

  We would give up to 20 minutes for the individual student problem solving segment. We feel this may give more variety in student approaches to the problem.

  A possible extension activity might be to pose the problem for an irregular polygon, rather than a regular.

- **What are the implications of this research on our teaching?**
  We will never assume that student's knowledge of basic terms such as a diagonal is what we expect. We will explicitly teach and place greater emphasis on diagonals when they first emerge in second year.

We will use ‘bounce questioning’ a lot more as students gain a greater understanding of each others’ thinking.

We appreciate the importance of setting the problem and standing back to give the students every opportunity to demonstrate their thinking and problem-solving capabilities.