The Problem with Cats and Dogs!
First Year Mixed Ability: Venn Diagrams

For the lesson on 6th
December 2016
in St Louis Community School, Ms Ryan's class

Lesson plan developed by:
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Brief description of the lesson **:**
This lesson is designed to encourage younger students to use Venn diagrams as a problem solving tool and see that it has a purpose beyond number, they can be used:

- to draw comparisons between sets
- identify differences
- quickly and effectively analyse data

Students will be able to:

- Verbalise their understanding of sets and Venn diagram
- Investigate the results of manipulation within the Venn diagram.
- Recognise that Venn diagrams can be used as a problem solving tool
- Classify items into categories/sets

Aims of the Lesson:

Thematic goals:

I'd like my students
- to appreciate that mathematics can be used to solve real world problems
- to appreciate that mathematics can be used to communicate thinking effectively
- to become more creative when devising approaches and methods to solve problems
- like my students to connect and review the concepts that we have studied already

I'd like
- to emphasise to students that a problem can have several equally valid solutions
- to build my students' enthusiasm for the subject by engaging them with stimulating activities

Short term goals:

As a result of this lesson:

Students should be able to:

1. Comprehend the effect of the manipulation of the intersection on the cardinal numbers of the union and set difference.
2. Recognising the validity of the use of Venn diagrams in solving mathematical problems.
3. Developing their problem solving and communications skills through thorough explanations with appropriate symbols/visual aids.

Learning Outcomes:

i. Use prior knowledge to solve real life problems.
ii. Appreciate the validity and efficiency of Venn diagrams as a method of solving problems.
iii. Make connections logical thoughts and Venn diagrams.
iv. Identify Venn diagrams as a valid method to present data allowing analysis.
v. Enabling students to draw comparisons/similarities. (and perhaps use as a cross curricular tool)

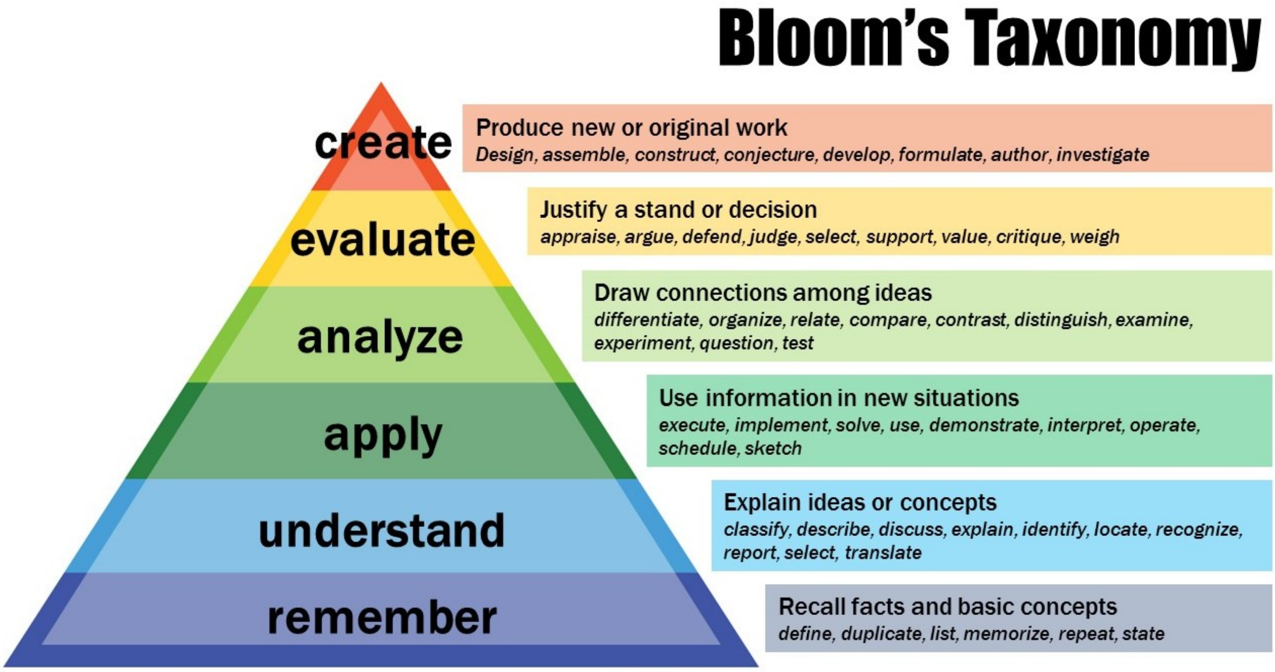
Background and Rationale:

Sets, in particular Venn diagrams is an area in Maths that we rely very heavily in when it comes to Senior Cycle probability. Our students struggle with identifying when they can use Venn diagrams to simplify a problem. To address this issue as a group we went back to the beginning to assess why we teach Venn diagrams in junior cycle, we did this by asking and reflecting on:

1. Why we teach Venn diagrams?
2. How should it benefit our students?

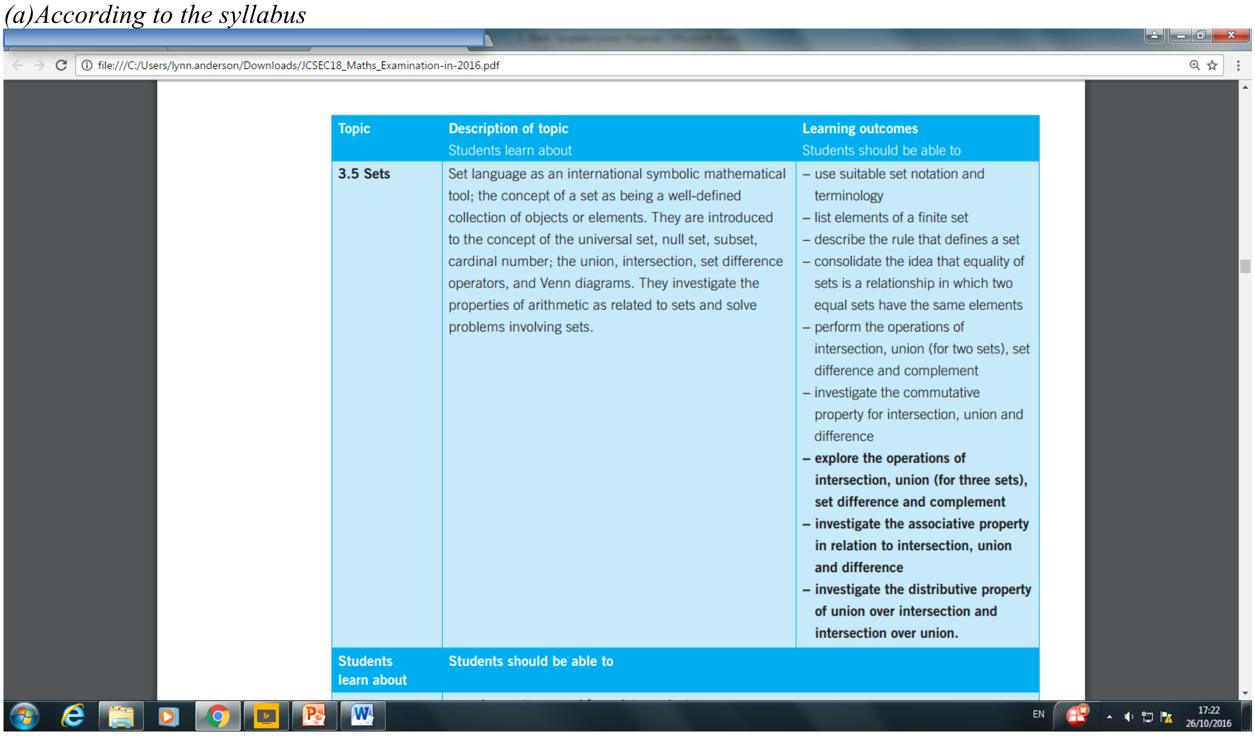
By definition Venn diagrams show all possible logical relations between a finite collection of different sets.
Their purpose is to logical compare/contrast sets and their items. This allows the students to classify objects into certain regions according to specific criteria.
For junior cert students it’s a tool that enables them to access the higher order thinking of blooms taxonomy, thus furthering the development of their cognitive thinking. This problem will encourage the younger student to:

- Analyse the data given
- Use a problem solving technique to evaluate their work
- Create a diagram to display their understanding.



(http://www.vanderbilt.edu/)

(a)According to the syllabus



Topic	Description of topic Students learn about	Learning outcomes Students should be able to
3.5 Sets	Set language as an international symbolic mathematical tool; the concept of a set as being a well-defined collection of objects or elements. They are introduced to the concept of the universal set, null set, subset, cardinal number; the union, intersection, set difference operators, and Venn diagrams. They investigate the properties of arithmetic as related to sets and solve problems involving sets.	– use suitable set notation and terminology – list elements of a finite set – describe the rule that defines a set – consolidate the idea that equality of sets is a relationship in which two equal sets have the same elements – perform the operations of intersection, union (for two sets), set difference and complement – investigate the commutative property for intersection, union and difference **– explore the operations of intersection, union (for three sets), set difference and complement** **– investigate the associative property in relation to intersection, union and difference** **– investigate the distributive property of union over intersection and intersection over union.**
Students learn about	**Students should be able to**	

Through reflecting on our teaching experience and the curriculum, we found several persistently common difficulties were identified

- A considerable amount of students fail to label their Venn diagrams correctly thus hindering their ability to answer questions. Some struggle with symbols.
- They struggle with the concept of subsets in particular improper subsets.

- Finding the cardinal number of specific regions of a Venn diagram. They struggle to identify the difference between AUB and U. They fail to account for the fact that AnB is part of Set A(or Set B)
- Constructing a Venn diagram given a worded problem.
- In max/min problems, students appear to struggle with the idea that the intersection could be empty(null set)
- Students fail to explore the relationships within Venn diagram
- Students at senior cycle fail to recognise that Venn diagram is a highly beneficial method to analyse data in particular probability

These problems restrict a student's ability to use Venn diagrams as an appropriate problem solving tool, which leads to a restriction for students understanding and their ability to explore relationships and analyse data in this format.

(c) The thematic focus of this lesson study, i.e. larger (see above in number 3 for ideas) goals the team will try to address, and why.

Research

The research involved us analysing this topic from several different perspectives.

As a teacher:

- Why do we teach sets?
- Why is it on the curriculum?
- What skills do our students attain by learning about this topic?
- How do we overcome the problems that students continually meet in this topic?

From a student's perspective:

- What aspects of this topic do students find difficult?
- Do they understand fully the benefit of what Venn diagrams can enable them to do?
- Do they make a connection with this topic and learning skills for other subjects?

Alongside this personal and group reflection, we looked at the resources (Appendix 1) and tried to find a suitable problem that would enable the group of students to achieve the learning goals and recognise the value of venn diagrams for future lessons.

Flow of the Unit:

Lesson		# of lesson periods
1-3	Students will be able to: • list elements of a set • describe the rule that defines a finite set • consolidate the idea that equality of sets is a relationship in which two equal sets have the same elements • use the cardinal number terminology when referring to set membership • perform the operation of intersection, union, set difference (for two sets) • illustrate sets using Venn diagrams	3 x 40 min.

	Based on the CIC and First Year Handbook	
4-5	• investigate the commutative property for intersection and union • Set Difference	2 x 40 min.
Research Lesson	• **Revisit topic and review prior knowledge(Approximately 10 lessons after sets has been covered)** • **Problem Solving Task**	**1 x 40 min**

Flow of the Lesson

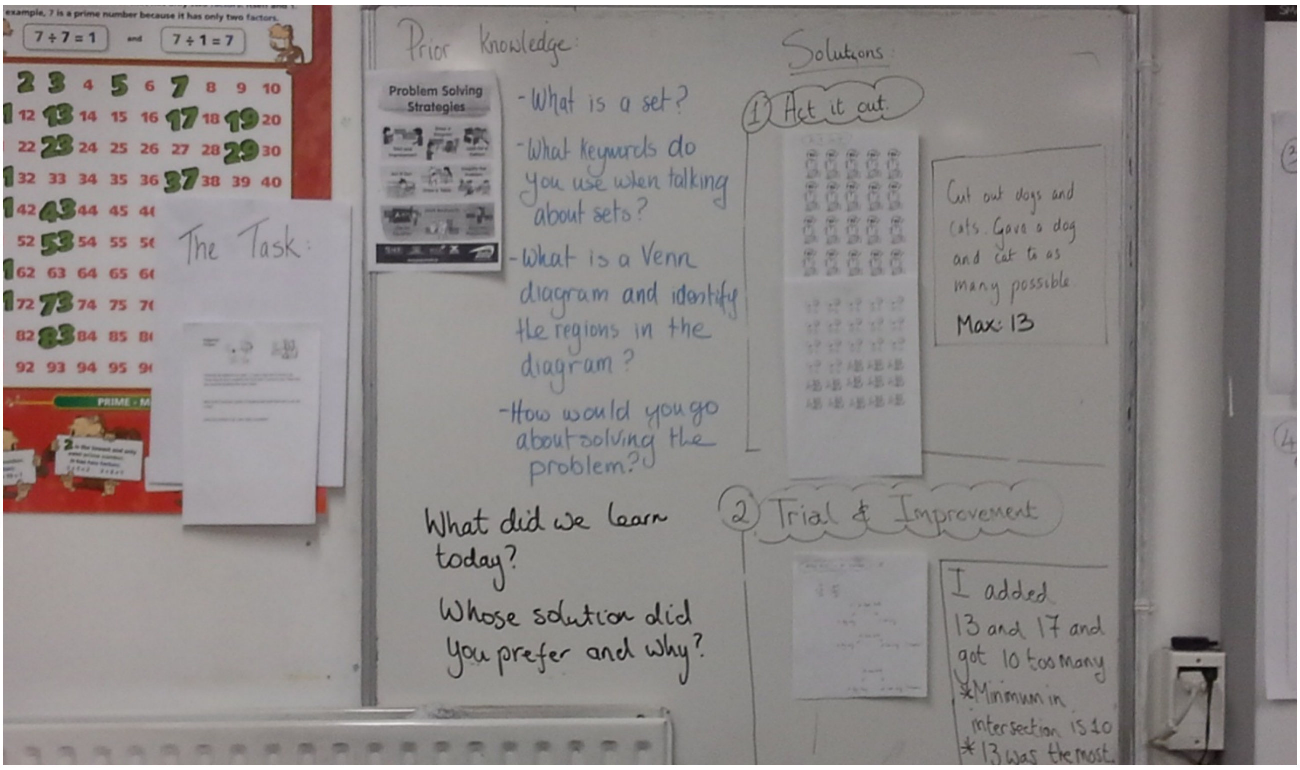
Teaching Activity	Points of Consideration
1. Introduction 7min	Prior Knowledge: • What is a set? • What do we use to display a set? • What is a Venn diagram and identify the regions in the diagram? • How would you go about solving any problem? What approaches would you use?
2. Posing the Task 8 min	There are 20 students in a class. 17 have a dog and 13 have a cat. There may be some students that have both a cat and a dog. There may also be some students that have neither. What is the maximum number of students that could have both a cat and a dog? Solve this problem in as many ways as possible. Read the problem out twice and check for understanding. Check can students define the word maximum Refer to the student pack (Sheet with one class of 20, 17 individual dogs and 13 individual cats)
3. Working on Task 15min	
3. Anticipated Student Responses	R1: Act it Out (see student pack) R2: Trial and Improvement R3: Simplify the problem R4: Look for a pattern

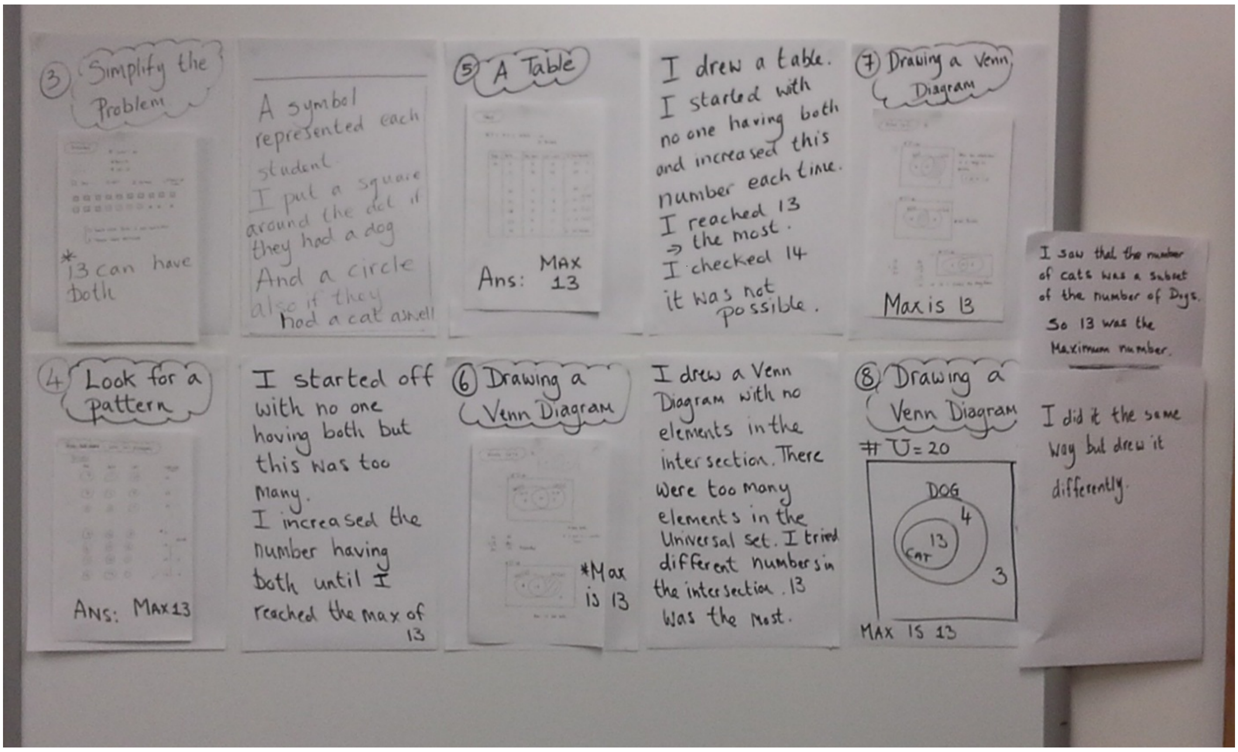
	R5: Draw a table (Not sure if first years will present table form) R6: Draw a diagram : Sets: Identifying the intersection as a null set is not possible. Increasing the intersection to 10 initially as this is the number to make #U=20. Finally getting a max of 13 in the intersection. R7: Draw a diagram with the understanding of subsets. C can be a subset of D. Venn diagram with intersection of two sets with C\D as the null set. R8: Draw a diagram with a contained subset. #D= #C + #D/C(*Unsure if first years will come up with solution) *Will add in responses as per board plan in Appendices
4. Comparing and Discussing: **Board Plan order.**	**R1: Act it Out (scissors and paper provided)** **R2: Trial and Improvement** **R3: Simplify the problem** **R4: Look for a pattern** R5: Draw a table (Not sure if first years will present table form)/ Making couples of cats and dogs **R6: Draw a diagram : Sets: Identifying the intersection as a null set is not possible. Increasing the intersection to 10 initially as this is the number to make #U=20. Finally getting a max of 13 in the intersection.** **R7: Draw a diagram with the understanding of subsets. C can be a subset of D. Venn diagram with intersection of two sets with C\D as the null set.** R8: Draw a diagram with a contained subset. #D= #C + #D/C(*Unsure if first years will come up with solution)
5. Summing up	Teacher will ask students to reflect on what they learned today? Which method did they find most interesting? Student Reflection: 1. Whose method do you prefer and why? 2. Write down one thing that you found interesting about this lesson? Orally: What method would you use if we were talking about a whole school, 1000 cats and 1200 dogs?

Evaluation Plan

- A seating plan was used and each teacher was given responsibility for a particular group of students.
- For this lesson, it was decided to assign each teacher a particular area of the classroom. There were six students for each teacher, each area was easily accessible to the observer and it was easy to record the work of each student.
- A record of all the observations was to be taken using the observation sheet (Appendix 3)and a digital device.
- As the lesson progressed the student's entire work was noted on the observation forms to look at the individual development and approaches made.
- Each teacher could then note the different solutions found, the similar solutions and the attempted solutions. The purpose to identify if there were any patterns in students thinking/development during the task.
- Teachers were to identify if students had misunderstood the task and if self-correction had occurred.
- All evidence gathered along with the Board Plan and Lesson plan was to be used in post-lesson discussion.

Board Plan: developed through Meitheal Machnaimh





Post-lesson reflection

The reflection began with observations from the teacher who delivered the lesson. From her perspective, the better students performed as expected with several interesting approaches, including a student who used grouping and one that used division. One student's use of a bar chart to compare information and draw conclusions involved excellent reasoning and language skills. One unexpected result was that there were a couple of students who failed to find the solution, especially as they were given the equipment to act out the problem and the teacher encouraged them to use this method. These particular students were not students that were underachievers or presenting with special needs. From this, she concluded that the students did achieve the short term goals of the lesson and this lesson greatly contributed to these students achieving the thematic goals of the lesson and attaining the learning outcomes.

The other teachers agreed with these observations and suggested that some of the students who failed to use the act out, as they thought it was too babyish. One teacher commented that several students initially found the incorrect answer of 10, however when they acted it out they were able to correct their work.

One student misinterpreted the question despite the fact that there was ample time given to the reading and discussing of the task but he did correct himself after 5min on task. The reflections at the end of the lesson were very insightful, students were asked to identify their favourite method and give a reason why. We all agreed that you could see they tended towards methods that were similar to their own but their reasons were quite mature, in that they felt these methods provided the best explanation or were the most efficient.

The lesson itself went extremely well and there were vital components which greatly contributed to the success and enjoyment of this lesson.

- Good use of open-ended questions in the prior knowledge
- Expansion of definitions gave students ownership of the terms
- There were some students who were unable to recall the regions of a Venn diagram, so the learning commenced straight away with the clarification of these key terms. Students provided good definitions which were refined in the prior knowledge.
- The language used in the explanation was of a good standard. One key observation was that several students answered the question well but also made a deductive statement which we agreed is an excellent skill for the first years concerned, for example: The max was 13 ……. Therefore 3 had none.
- Students preferred method was split between the use of venn diagrams as you could see the information clearly and the bar chart method as it was also very clear. They all enjoyed the fact there was so many ways to solve this problem.
- It was also observed that this problem came from the higher level course and these students would not encounter this type of problem until third year. With this in mind it is clear that the majority of them coped very well. Few used the cardinal number but could see its validity in the reflection.
- Finally, one observation that is testament to the achieving of the goals of this lesson, at the end of the lesson some of the students who had made mistakes during the lesson were very open about what they had done wrong and what they could have done. This not only shows the confidence within these students but also how their ability to converse Mathematically had taken a positive step forward.

Appendix 1: Resources

- Concise Maths 1 and 2
- Text and Tests 1
- Active Maths 1
- Syllabus for Junior Cycle www.ncca.ie
- www.projectmaths.ie
- Common Introductory Course
- Project Maths First Year Handbook
- http://www.vanderbilt.edu/

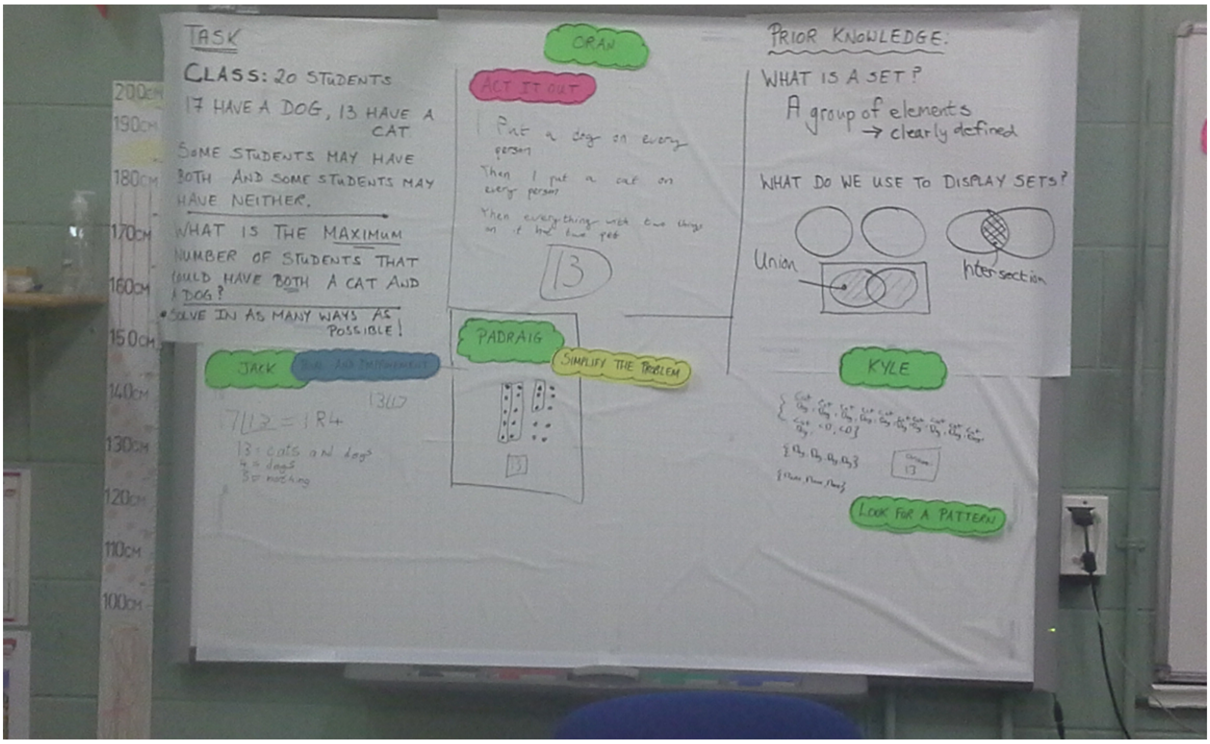
Appendix 2: Lesson Observation

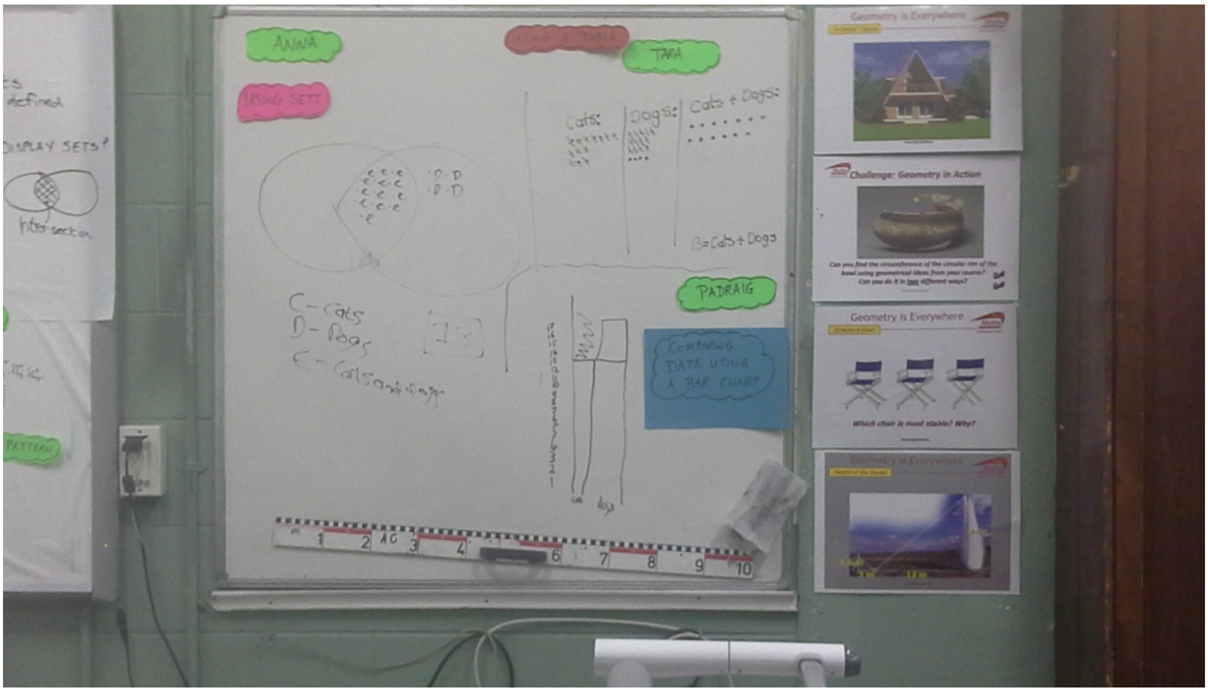
The problem with Cats and Dogs!

Introduction: Prior Knowledge: Are students comfortable with the topic being discussed? Do they have a good understanding of key words and definitions? Any observations regarding the language that is being used to explain their understanding						
Task: Was the goal clear?						
Did the activity support achieving the goal? Refer to long/short term goals.						
On Task: Was the flow of the lesson coherent? How were the students solving the problem? What type of language did they use? What approaches/strategies did they use?						
Ceardaiocht						

Did the students' presentation and discussion promote their thinking and learning? How did students engage with other students work/ did they draw comparisons/differences?						
Conclusion/Summary What did students learn? Was the goal achieved? Evidence of changed thought patterns?						
Any other interesting/unexpected findings?						

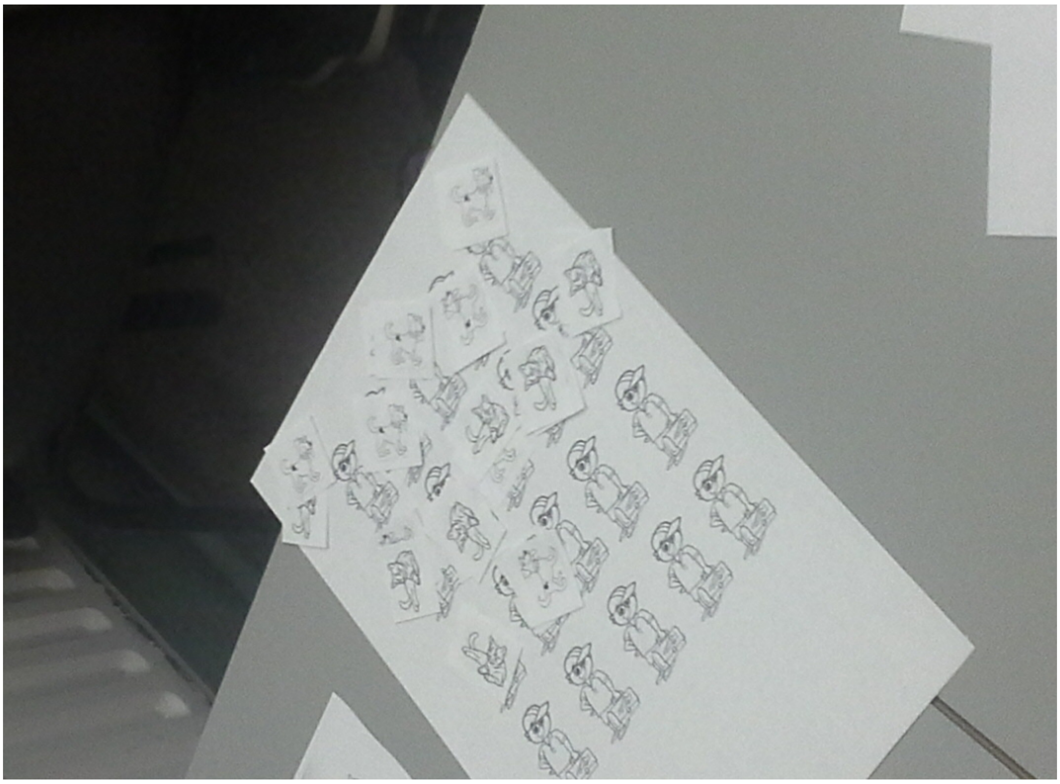
Appendix 3: Students work from Lesson Board Plan



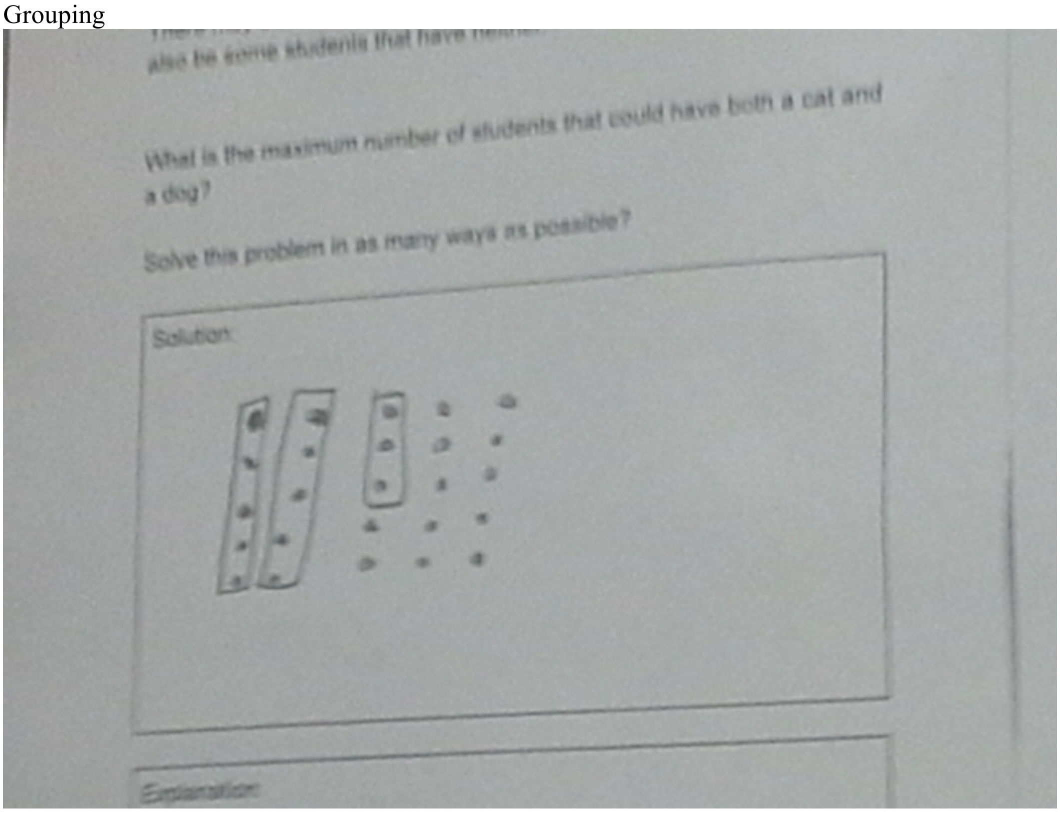


Samples of Students Work

Act it out.



Grouping



Logic (Trial and Error)

Solution: There are 13 people who have cats and 17 who have dogs

$13 \overline{)17} = 1R4$ 13 people have both 4 have dogs and another 3 have nothing

Explanation: divide 13 into 17 and you get 1 R4 which equal to 13 people who have both and 4 just have dogs and another 3 have

Using Sets

Solution: {Cat Dog, Cat Dog, Cat Dog, Cat Dog, Cat Dog, Cat Dog, Cat Dog, Cat Dog, Cat Dog, Cat Dog, Cat Dog, Cat Dog, Cat Dog}

{Dog, Dog, Dog, Dog}

{no Pet, no Pet, no Pet}

nation: I found this answer by culating the information and splaying it with curly brackets.

Using Venn diagrams