Lesson Study Lesson Proposal

1st Years (Mixed Ability) Let’s Think About BIMDAS

For the lesson on 19/12/16
At De La Salle College
Dundalk
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PTA: Liam Doheny

Lesson plan developed by:
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1. **Title of the Lesson:** Let’s Think About BIMDAS

2. **Brief description of the lesson:** During this lesson students will be given three 4’s and are asked to use any of the mathematical operations they know (in the correct order) to come up with as many different answers as possible.

3. **Aims of the Lesson:**

   **Long Range goals:**
   
   I’d like my students to become more creative when devising approaches and methods to solve problems.
   
   I’d like my students to work independently to solve problems.
   
   I’d like to build my students’ enthusiasm for the subject by engaging them with stimulating activities
   
   I’d like my students to connect and review the concepts that we have studied already

   **Short Term Goals:**
   
   For students to be comfortable with the basic mathematical operations
   
   For students to understand BIMDAS, and to appreciate the importance of order of operations when solving problems
   
   For students to understand the different number systems N, Q and Z
   
   For students to apply their knowledge of the properties and order of operations to solve a problem

4. **Learning Outcomes:**

   **By the end of this lesson I’d like my students to be:**
   
   Students will be able to add, subtract, multiply and divide elements of the sets N, Z and Q.
   
   Students will be able to apply the correct order of operations to solve a problem.
   
   Students will be introduced to the associative, commutative and distributive properties.
5. Background and Rationale

In the Junior Certificate Syllabus under Strand 3, Numbers, section 3.1 outlines that the students learn about “the binary operations of addition, subtraction, multiplication and division and the relationships between these operations, beginning with whole numbers and integers” and also “later, they revisit these operations in the context of rational numbers”.

Section 3.1 further states that students should be able to:

- Appreciate the order of operations, including the use of brackets
- Investigate the properties of arithmetic: commutative, associative and distributive laws and the relationships between them
- Investigate models to illustrate the operations of addition, subtraction, multiplication and division in \( \mathbb{Z} \)

Furthermore section 3.5 describes that students learn about “the concept of a set as being a well-defined collection of objects or elements” and that students must be able to “describe the rule that defines a set”.

The Chief Examiners Report of 2015 stresses the importance of the order of operations:

“Teachers should provide frequent opportunities for students to gain competence and accuracy in basic skills of computation and algebraic manipulation. Students should be particularly careful with signs, powers, and the order of operations”.

The report also outlines the importance of mathematical discussion:

“Students should get used to describing, explaining, justifying, giving examples, etc. These are skills that are worth practising, as they will improve understanding, as well as being skills that may be assessed in the examination”.

From our own experiences of teaching the topic we were aware that the students often struggle with Rational numbers. Some students mix up their procedure for adding, subtracting, multiplying and dividing fractions. Furthermore, many students do not understand the basis for a particular procedure and are entirely relying on memorising procedures.

6. Research

NCCA Junior Cycle Mathematics Syllabus


First year Teachers Handbook from Project Maths website

Chief Examiners’ Reports – State Examinations Commission

Active Maths 1, Folens

7. About the Unit and the Lesson

This is a revision topic incorporating the order of operations (BIMDAS) and a review of the number systems \( \mathbb{N}, \mathbb{Q} \) and \( \mathbb{Z} \). In previous lessons students have covered BIMDAS and number systems but will not have covered the commutative, associative and distributive properties yet. For the task students will be given three 4’s and asked to use any of the mathematical operations they know to come up with as many different answers as possible. While going through the task, students will discover that by rearranging brackets the same operations can produce numerous different results. This will emphasise the importance of using BIMDAS when solving any mathematical problem, not just the order of operations chapter in the book!

This lesson will also serve to recap sets (more specifically what constitutes a set) and the different number systems \( \mathbb{N}, \mathbb{Z} \) and \( \mathbb{Q} \). Furthermore through the discussion on the positioning of brackets the lesson is a natural introduction to the commutative, associative and distributive properties.
Questioning:
The use of both higher and lower order questions will be used during the introduction, conclusion and summary to evaluate and deepen student understanding.

Ask questions such as ‘why must we do this first?’ or ‘how could we rewrite this to make it clearer?’ to deepen students understanding of what they’re actually doing when they work through a solution. Follow up with questions such as ‘is this similar to any of the other methods that we’ve used?’ will be utilised as often as possible. By drawing comparisons between similar solutions students will build upon their previous understanding (scaffolding).

Leading questions will also be used in the introduction to prepare the students for the task, and similarly during the conclusion to encourage the students to explore this concept in greater depth for homework.

Class / Peer Discussion:
Students are expected to summarise what they’ve learned by explaining it in their own words.

8. Flow of the Unit:

<table>
<thead>
<tr>
<th>Lesson</th>
<th># of lesson periods</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Natural numbers</td>
</tr>
<tr>
<td>2</td>
<td>Integers (and their application in problems – multiplying by negative numbers etc.)</td>
</tr>
<tr>
<td>3</td>
<td>Rationals (and their application in problems – addition, subtraction, multiplication and division of fractions etc.)</td>
</tr>
<tr>
<td>4</td>
<td>Order of operations (BIMDAS)</td>
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<td>5</td>
<td>Research Lesson</td>
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<td>6</td>
<td>Associative, Commutative and Distributive properties.</td>
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</table>

9. Flow of the Lesson

<table>
<thead>
<tr>
<th>Teaching Activity</th>
<th>Points of Consideration</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Introduction</td>
<td>Students will define and give examples of the elements of the sets ( N, Z ) and ( Q ).</td>
</tr>
<tr>
<td>Revision of number systems. Definition of a set. Define the sets ( N, Z ) and ( Q ). Revision of BIMDAS.</td>
<td></td>
</tr>
<tr>
<td>2. Posing the Task</td>
<td>Students show a few possible solutions on the board:</td>
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<tr>
<td>Students are first given an example of how the task works: Using three 7’s and the mathematical operations explore how many different outcomes you can achieve.</td>
<td>( 77+7=8 )</td>
</tr>
</tbody>
</table>
Student Task:
You have three 4’s, you may use any of the mathematical operations you know to come up with as many different answers as possible.

Students are then given 5 minutes to come up with as many different outcomes as possible using three 4’s.

After 5 minutes, students are reminded to double check that BIMDAS is being applied.

Furthermore they are also reminded that by positioning the brackets in different places may yield different results

Students are given some more examples focusing on the order of operations and the use of brackets. Students are then allowed to explore some further possibilities for another 5 minutes.

3. Anticipated Student Responses

As there are far too many different answers it would be unpractical to outline all of them. Instead we chosen to focus two key points:
- Potential errors that may occur
- Possible answers that we predict the majority of students may not even consider

The majority of errors that students will make will be as a result forgetting about BIMDAS. Student may say:

R1: $7+7\times7=98$

Students will be reminded that the use of brackets can often make a sum easier to read, whilst also ensuring that BIMDAS is used correctly.

We anticipate that a considerable proportion of the class will only find answers that are Natural numbers. We expect that many may not consider negative results and even fewer will consider answers that are fractions.

The teacher will use best judgement to decide what (if any) level to prompt is necessary to keep the student on task, motivated to keep trying and to ensure that each individual student is learning and benefiting from the lesson.

Students also often forget that if a sum only has addition and subtraction – we just read the sum from left to right. (i.e. addition does not come before subtraction!)

Teacher will use best judgment to decide what (if any) level of prompt is required.

Early finishers will be encouraged to work with and discuss their methods with their neighbors to ensure accuracy and to see can they come up with any further methods.
### 4. Comparing and Discussing
Students are asked to pick one of their outcomes and display it on their show me board. Outcomes are shared among the class and displayed on the class room whiteboard. Students are asked if there are any other outcomes that have not been mentioned by their peers.

Class discussion ensues on the similarities between different methods. Students will discuss the importance of brackets and how their repositioning can completely alter a question.

Students will also be asked can they find any instances where the repositioning of brackets will not change the answer. Further discussion ensues on this topic.

The teacher will have positioned the answers on the whiteboard so that the positive results, the negative result and the fractions are all grouped near each other. *(See boardwork)*

Students are asked if they could make any sets with the answers on the board.

<table>
<thead>
<tr>
<th>Natural introduction to the <strong>associative property</strong></th>
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<tbody>
<tr>
<td>Outcomes are grouped by the number systems (N, Z) and (Q).</td>
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### 5. Summing up
Recap on BIMDAS, the importance of brackets

| All students should have each method recorded in their copy |

### 6. Homework
A worksheet is handed out for the students to complete for homework. Students are encouraged to try some of the more interesting methods discussed.

### 10. Evaluation
Elaine has provided a seating chart which both observers will use to record all noteworthy observations and remarks. This seating chart gives the observers a simple but efficient method of accurately recording observations they have made of students they will be unfamiliar with. This guarantees a greater degree of accuracy for our discussions in our post-lesson reflection.

Student performance will also be evaluated by collecting their worksheets at the end of the lesson. This will allow us to accurately assess each student’s progress through the task and see if they were they able to find links between different methodologies. If given a prompt to assist them with a particular method, were they then able to assess this method and reapply it to their subsequent answers. A camera will be used to photograph the board work to ensure we collect as much info as possible to assist us with our post-lesson reflection.
11. Board Plan
12. Post-lesson reflection
Overall we felt the lesson went very well, that it was pitched to the correct level and that the students benefited from it and appeared to enjoy it. We were particularly pleased to observe active participation by all students during the task. We believe this was a result of the teacher’s clear description of the task and use of an example on the board prior to letting the students begin. It is noteworthy that this was one of the first times that the students experienced structured problem solving and yet they had no issues with it. Students were self-motivated to carry on and get as many answers as possible. This is in stark contrast to some of our own experiences when first introducing structured problem solving to some of our older classes. We believe this highlights the importance of introducing *structured problem solving* as early as possible, that it must be introduced by first year (and ideally should be introduced at primary level). By introducing it early and using it as often as possible, one will give structured problem solving the best opportunity of becoming part of classroom culture in post primary teaching.

The discussion element of the lesson was incredibly positive, we observed very genuine learning with the majority of students making insightful links and showing a good understanding of the topic.

Upon reflection we believe we tried to fit too much into one lesson. We spent slightly too much time revising during the introduction and we tried to cover too much during the discussion. It would have been wiser to omit the links to the associative property during the discussion as realistically this is a different lesson. These links could easily have been made in the subsequent lesson using the answers from the students’ homework. Whilst still showing the usefulness of the task, this method would provide a more natural and sensible means of introducing the associative property.

In conclusion we feel that with some minor adjustments (as outlined above) the lesson will successfully meet its aims and objectives and will be a considerably beneficial lesson for the revision of BIMDAS.