Margaret's (Bandon Group) TICK TOCK!

5th year Higher Level & Area of in-circle of an equilateral triangle.

For the lesson on
[20/01/17]
At Coláiste na Toirbhirte, 5 Maths
Teacher: [Margaret Barrett]
Lesson plan developed by: [Bernice O'Leary, Declan Cronin, Eimear White, Margaret Barrett]

1. Title of the Lesson: 'TICK TOCK'! Finding the Area of in-circle of an equilateral triangle

2. Brief description of the lesson

Given an equilateral triangle pupils will be required to find the area of its in circle using different methods. Using a real-life example of a round clock in a triangular frame pupils will look for methods of finding the area of the clock face.

3. Aims of the Lesson:

1. We would like our students to choose ways to think from various choices and explain them to others.
2. I'd like my students to appreciate that mathematics can be used to solve real world problems.
3. I'd like to foster my students to become creative, independent learners.
4. I'd like to emphasise to students that a problem can have several equally valid solutions using different approaches and methods.
5. I'd like to build my students' enthusiasm for the subject by engaging them with stimulating activities.

4. Learning Outcomes:

As a result of studying this topic students will be able to:

1. Find the area of the Incircle of an Equilateral triangle.
2. Understand more clearly the properties of the equilateral triangle and the incentre.
3. Utilise their previous knowledge of geometrical constructions and the trigonometry of the right-angle triangle.
4. Apply geometrical and trigonometrical reasoning to real life situations.

5. Background and Rationale

1. We recognise that students are challenged by spatial reasoning and particularly by geometry problems in an unusual context.
2. We recognise that students have difficulty applying their knowledge and skills to solve problems in familiar and unfamiliar contexts.
3. We recognise that the students need more understanding and appreciation of how Maths can be applied to real life situations.
4. We recognise the difficulty of word based problems from previous Chief Examiner's reports.
5. We desire our students to become more independent critical thinkers.

6. Research

1. Leaving Certificate project Mathematics Syllabus
2. Leaving Certificate Higher Level Past Examinations Papers
3. Google Searches on Real life applications of the Incentre and Incircle.

4. www.projectmaths.ie
5. Geometry Teaching and Learning Plans on www.projectmaths.ie
6. Chief Examiner’s reports.

7. About the Unit and the Lesson

How will this lesson address the Learning Outcomes?

1. Find the area of the Incircle of an Equilateral triangle

Pupils will find the area of the incircle of a given equilateral triangle using multiple methods such as trigonometric ratios and bisecting angles of the equilateral triangle.

2. Understand more clearly the properties of the equilateral triangles and the incentre

Pupils will measure the triangle sides and angles and use properties of an equilateral triangles to find the radius of the incircle.

3. Utilise their previous knowledge of geometrical constructions and the trigonometry of the right angle triangle

Pupils will use bisector of an angle and conclude the 60 degree angle of the equilateral triangle has been bisected into two 30 degree angles. Radius of the incircle bisects the side of the equilateral triangle. Pupils will implement Pythagoras theorem and apply trigonometric ratios (Sin, Cos and Tan).

4. Apply geometrical and trigonometrical reasoning to real life situations

Pupils will observe a real life context that contains a circle within an equilateral triangle. Pupils will attempt this question and observe multiple methods of finding approximations and solutions from peer work within the lesson.

The Lesson addresses the following learning outcomes from the mathematics syllabus:
Syllabus- Page 24; 2.3Trigonometry

2.3 Trigonometry	Right-angled triangles. Trigonometric ratios.	– apply the result of the theorem of Pythagoras to solve right-angled triangle problems of a simple nature involving heights and distances – use trigonometric ratios to solve real world problems involving angles

Page 24; 2.1 Synthetic Geometry

2.1 Synthetic geometry	Constructions and how to apply these in real-life situations. Dynamic geometry software. The instruments that are used to perform constructions with precision.	– revisit constructions 4,5,10,13 and 15 in real-life contexts – draw a circle of given radius – use the instruments: straight edge, compass, ruler, protractor and set square appropriately when drawing geometric diagrams

Page 25; 2.1 Synthetic Geometry

2.1 Synthetic geometry	– perform constructions **16-21** (see *Geometry for Post-primary School Mathematics*) – use the following terms related to logic and deductive reasoning: theorem, proof, axiom, corollary, converse, implies – investigate theorems 7, 8, 11, 12, 13, 16, 17, 18, 20, 21 and corollary 6 (see Geometry for Post-primary School Mathematics) and use them to solve problems	– perform construction **22** (see *Geometry for Post-primary School Mathematics*) – use the following terms related to logic and deductive reasoning: is equivalent to, if and only if, proof by contradiction – prove theorems 11,12,13, concerning ratios (see Geometry for Post-primary School Mathematics), which lay the proper foundation for the proof of the theorem of Pythagoras studied at junior cycle

Page 25; 2.3 Trigonometry

2.3 Trigonometry	– use of the theorem of Pythagoras to solve problems (2D only) – use trigonometry to calculate the area of a triangle – solve problems using the sine and cosine rules (2D) – define sin θ and cos θ for all values of θ – define tan θ – solve problems involving the area of a sector of a circle and the length of an arc – work with trigonometric ratios in surd form	– use trigonometry to solve problems in 3D – graph the trigonometric functions sine, cosine, tangent – graph trigonometric functions of type • $f(\boldsymbol{\theta})= a+b\text{Sin } c\boldsymbol{\theta}$ • $g(\boldsymbol{\theta}) = a+b\text{Cos } c\boldsymbol{\theta}$ for $a,b,c \in \mathbf{R}$ – solve trigonometric equations such as Sin $n\boldsymbol{\theta}$=0 and Cos $n\boldsymbol{\theta}$= ½ giving all solutions – use the radian measure of angles – derive the trigonometric formulae 1, 2, 3, 4, 5, 6, 7, 9 (see appendix) – apply the trigonometric formulae 1-24 (see appendix)

Page 32; 3.4 Length, Area and Volume

3.4 Length, area and volume	2D shapes and 3D solids, including nets of solids. Using nets to analyse figures and to distinguish between surface area and volume. Problems involving perimeter, surface area and volume. Modelling real-world situations and solving a variety of problems (including multi-step problems) involving surface areas, and volumes of cylinders and rectangular solids. The circle, and develop an understanding of the relationship between its circumference, diameter and π.	– investigate the nets of rectangular solids and cylinders – select and use suitable strategies to find length of the perimeter and the area of the following plane figures: disc, triangle, rectangle, square, and figures made from combinations of these – select and use suitable strategies to estimate the area of a combination of regular and irregular shapes – select and use suitable strategies to find the volume and surface area of rectangular solids, cylinders and spheres – draw and interpret scaled diagrams

8. Flow of the Unit:

Lesson		# of lesson periods
1	• Construction of incircle and Properties of incircle • Revise properties of right angle triangle and trigonometry and types of triangles • Area of Circle and Sector	3 x 35 min.
2	• Introduction to finding and construction of Circumcentre and centroid. (noticing that the centroid cuts the median in the ratio 2:1)	1 x 30 min.
3	• Area of the incircle of an equilateral triangle	2 x 30 min. **(research lesson)**
4	• Correction and discussion on Extension Question on relating incentre with circumcentre • Applying knowledge of Centroid using similar real life contexts • Applying knowledge of Circumcentre using similar real life contexts	3 x 30 min.
5	• Applying knowledge of Orthocentre using similar real life contexts	1 x 30 min.

9. Flow of the Lesson

Teaching Activity	Points of Consideration
1. Introduction- 5 minutes Key words of Prior knowledge of triangles/circles such as incircle and incentre, equilateral, area of sector and circle. Arc, radius and diameter (premade) trigonometric ratios (Sin, Cos and Tan)	I will use Higher order and lower order (open and closed) questions to refresh pupils' knowledge **<u>Circle</u>** What is the formula for the area of the circle? Where do we find that formula in the log tables? Define the radius of a circle Define the diameter of a circle What is the relationship of radius to diameter? What is an incentre? What are the properties of the incentre? **<u>Constructions</u>** What is the bisector of a line, how can you construct it? What is the bisector of an angle, how can it be constructed? **<u>Triangles</u>** What is an equilateral triangle? Properties of an equilateral triangle What are the properties of a right-angle triangle? What are the ratios associated with right angle

	triangles? What formulas could you use to find the side length of a non-right angled triangle?
2. Posing the Task "A manufacturer wishes to make a clock using a triangular sheet of metal. What is the area of the largest circular clock face that she can make using the measurements in the diagram below?" Timer put on projector counting down 10 minutes	**<u>Verbal clarifications-</u>** 10 minutes; must ask all questions now Must have rulers and other construction equipment Picture is actual size. You should try to come up with as many ways to a solution as possible. I will be asking you to come up to the board to share your solutions. Other Maths teachers will only be observing and will not be able to help you.
3. Anticipated Student Responses	Lesson Note used to identify pupils progress and solutions Teacher will ask pupil to explain their solution enquiring as to any assumptions made
<u>Response 1</u> Approximating from the area of the equilateral triangle using grid paper	**<u>Possible probing questions</u>** What is the area of a grid box? How did you calculate the area of a grid box? How did you deal with partial grid squares? Is this a precise method to find the area of a circle?
<u>Response 2</u> Use a ruler to measure radius (diameter) and use area of circle (πr^2)- approx. length of radius 2.3cm answer = 16.62cm^2 /radius of 2 cm gives 12.57cm^2	**<u>Possible probing questions</u>** How did you measure the radius? What is the formula of a circle? Is this a more precise method than counting grid boxes Is the radius exactly 2.3cm? Is this the most precise method?
<u>Response 3</u> Approximating from the area of equilateral triangle and subtract three 'triangles'	**<u>Possible probing questions</u>** Are the 3 corners actual triangles, why? How did you calculate the area of the triangle? What is the formula for the area of a triangle?
<u>Response 4</u> Using a combination of Pythagoras, trigonometric ratios, the Sine Rule and knowledge of incentre being on the bisector of the angles to find radius and utilise area of circle formula.	**<u>Possible probing questions</u>** How did you construct a right-angled triangle? How do you know it is a right-angled triangle? How did you bisect the line? Is this a more accurate method of finding the radius? Why is this a more accurate method for finding radius than measuring the radius with a ruler?
<u>Response 5</u> The centroid divides each median in ratio 2:1 from the vertex, gives radius as 1/3 of the perpendicular height of the triangle.	**<u>Possible probing questions</u>** What is the centroid? How did you find the centroid? What is the median of a circle? Define a vertex?

Possible incorrect Responses Measuring radius incorrectly using ruler Use of wrong trigonometric ratios in finding radius Use of wrong formula for area of a circle Use of wrong formula for Sine Rule Algebraic slip when calculating hypotenuse using Pythagoras' theorem. Using 3.14 as an approximation for π	
4. Comparing and Discussing 1. Approximation using grid paper 2. Approximation using ruler to measure radius 3. Approximating from the area of equilateral triangle and subtract three 'triangles' 4. Right angle triangle using bisector of the angle or non-right angled triangle using isosceles triangle 5. The centroid divides each median in ratio 2:1 from the vertex, gives radius as 1/3 of the perpendicular height of the triangle.	Pupils will benefit from discussion by: Realisation of different methods of approximation. Understanding all that they have learnt in coming to multiple solutions Participation with peers who are presenting their solution *Questions to Probe Pupil Solutions(if needed)* 1. How did you count the whole grid squares and partial grid squares? 2. Is a ruler method accurate enough, errors with this method, rounding off so not accurate? 3. These 'triangles' are they real triangles; no as one side is curved 4. How did you form the right-angled triangle and how did you know the angle of the equilateral triangle? 5. How did you use your knowledge of the Centroid to find the radius?
5. Summing up Highlighting all knowledge pupils have utilised in finding the area of an incircle of an equilateral triangle.	**Highlighting pupil learning** • Area can be approximated using grid squares. • Measurements can be made and used to approximate the area. • Approximating from the area of equilateral triangle and subtract three 'triangles', using area of triangle formulae. • Applying their knowledge of equilateral triangles and the trigonometry of right angled triangles. • The centroid divides each median in ratio 2:1 from the vertex, gives radius as 1/3 of the perpendicular height of the triangle. **Extension questions** Q1 (i) Construct the triangle ABC, where BC = 6cm, AB = 7cm and AC = 7cm. (ii) Using this triangle, construct the Circumcentre, Incentre, Centroid, and Orthocentre. What is your observation about these four points? Q.2 (i) Construct the triangle ABC, where BC = 6cm, AB = 7cm and AC = 8cm.

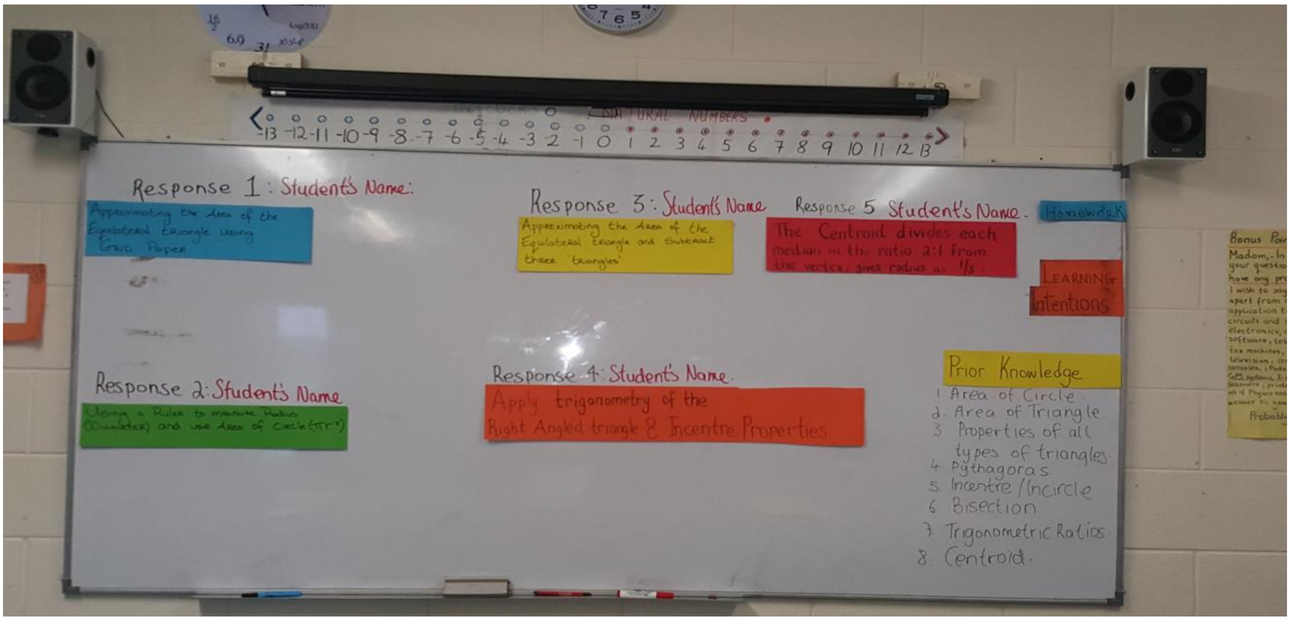
	(ii) Using this triangle, construct the Circumcentre, Incentre, Centroid, and Orthocentre. What is your observation about these four points?

10. Evaluation

Margaret Barrett will be teaching the lesson to a H.L. 5th year class group.
The duration of the lesson will be 45minutes.
Bernice, Eimear, Declan and Iris will observe the students.
The class will be divided into groups of 5/6, for observation.
Data will be recorded using observation sheets and lesson note.
This recording will include

- Grasp of prior Knowledge
- Questions asked
- Misconceptions
- Methods undertaken
- Application of prior knowledge
- Assumptions made
- Engagement with task
- Student perseverance
- Interest in the methods undertaken by their peers
- Student understanding of peer presentations.

11. Board Plan



Response 1- Use grid paper to find an approximation of circle area

- Approximating form the area of the equilateral triangle using grid paper

Response 2- Measuring Radius with Ruler and use πr^2

- Use a ruler to measure radius (diameter) and use area of circle (πr^2)- approx. length of radius 2.3cm answer = 16.62cm^2 /radius of 2 cm gives 12.57cm^2

Response 3- Area of Triangle minus area of circle non area of circle

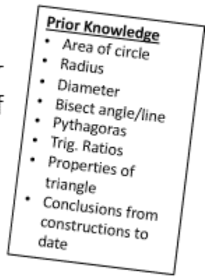
- Approximating form the area of equilateral triangle and subtract three 'triangles'

Response 4- Using trigonometry and constructions to find radius and use πr^2

Use Pythagoras, trigonometric ratios and knowledge of incentre being on the bisector of the angle to find radius and utilize area of circle formula

Response 5- Use properties of centroid to find radius and use πr^2

- The centroid divides each median in ratio 2:1 from the vertex, gives radius as $^1/_3$



12. Post-lesson reflection

Q.1. What are the major patterns and tendencies in the evidence?
From the evidence collected, we found that

- Students tended to spend most of the allocated time constructing both the Incentre and Circumcentre.
- Once a student had found one method of getting the solution, they didn't all attempt trying to find another solution.
- It was obvious during the prior knowledge stage of the lesson that students had a good grasp of the content needed but struggled when it came to applying it to the question.
- Students didn't recognise the applications of trigonometry to the question.

Q.2 What are the key observations or representative examples of student learning and thinking?

- All students worked diligently for allocated time.

- The students’ knowledge of geometry and trigonometry was of a high standard. We overestimated their ability to apply this knowledge to this question.
- The students automatically assumed that construction of both the Circumcentre and Incentre was necessary. Their thinking was that because it was a construction style question that the only approach was to construct.
- We observed that students were afraid to think outside the box and apply their wider knowledge to the problem.

Q.3 What does the evidence suggest about student thinking such as their misconceptions, difficulties, confusion, insights, surprising ideas etc.?

- Students didn’t recognise the radius of the incentre which prevented them from getting to the end solution.
- Students have not currently had enough experience of thinking on their own
- Students have not had enough experience of problem solving.
- Students’ prefer to acquire the knowledge from the teacher instead of investigating their own methods of finding a solution.
- Students didn’t see the value of using the scissors provided as they felt this method of approximation was too basic and not an exam technique.

Q.4 In what ways did students achieve/not achieve the learning goals?

- Students recognised that there were several different ways of arriving at the solution.
- Students were aware of the application of their knowledge to solve real world problems.
- Students enhanced their ability of being independent learners within the class. As the class progressed, they saw the value of collaboration with their peers.
- They learned from their mistakes and the mistakes of others.

Q.5 Based on your analysis how would you change or revise the lesson?

- After the problem, had been handed out, more time could have been given to allow the students some ‘think time’ before they had started.
- We would make sure that students were clear on the wording of the question and instructions of the question.
- We would have emphasised that an approximation for the area of the incircle was also a valid method.
- We would clearly mark the incentre on the diagram and highlight that constructions are not necessary.

Appendices

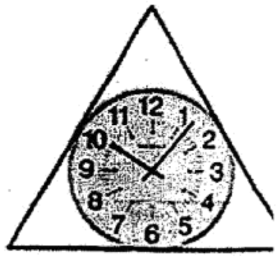
Original Problem

Look at the wall clock in your room, are you sure it shows correct time?

If your answer is yes, that means the manufacturer of clock has used concept of incenter to make sure center of clock coincides exactly with the incenter of the triangle inside which the clock is inscribed.

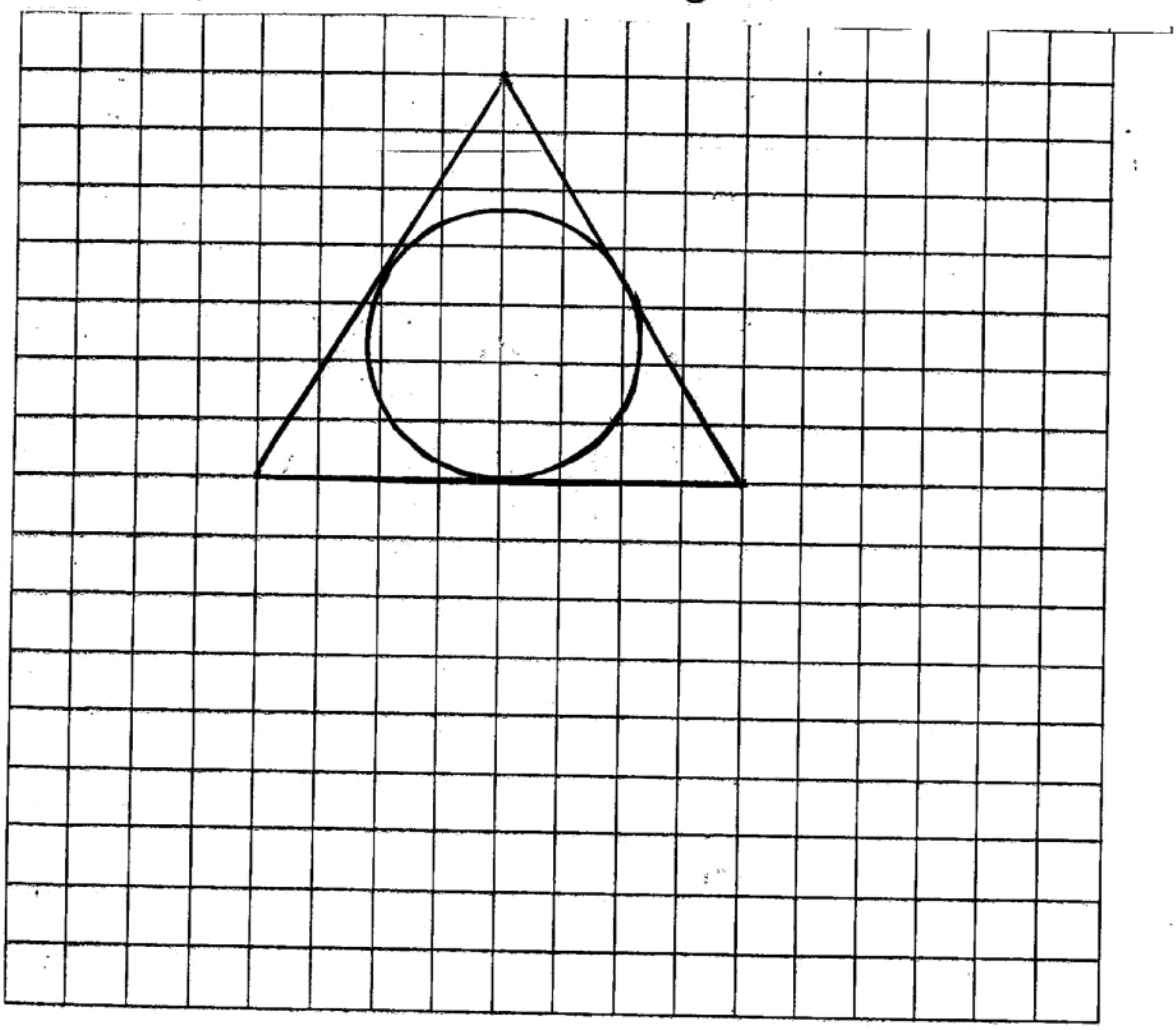
A manufacturer wishes to make a clock using a triangular sheet of metal. What is the area of the largest circular clock face that she can make?

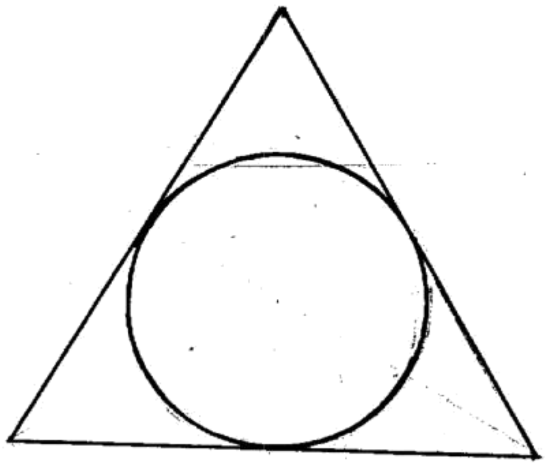
Student Worksheets



Question:

A manufacturer wishes to make a clock using a triangular sheet of metal. What is the area of the largest circular clock face that she can make, using the measurements in the diagram below?





Reflection on the lesson.

What was your preferred method in finding the area of the clock face?	
Why is this your preferred method	
What have you learned from today's lesson	

Extension (Homework) Question.

Q1 (i) Construct the triangle ABC, where BC = 6cm, AB = 7cm and AC = 7cm.

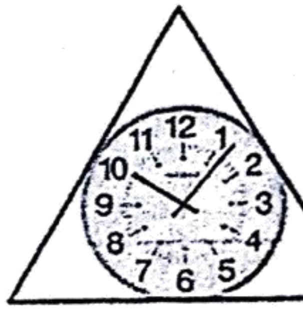
(ii) Using this triangle, construct the Circumcentre, Incentre, Centroid, and Orthocentre.
What is your observation about these four points?

Q.2 (i) Construct the triangle ABC, where BC = 6cm, AB = 7cm and AC = 8cm.

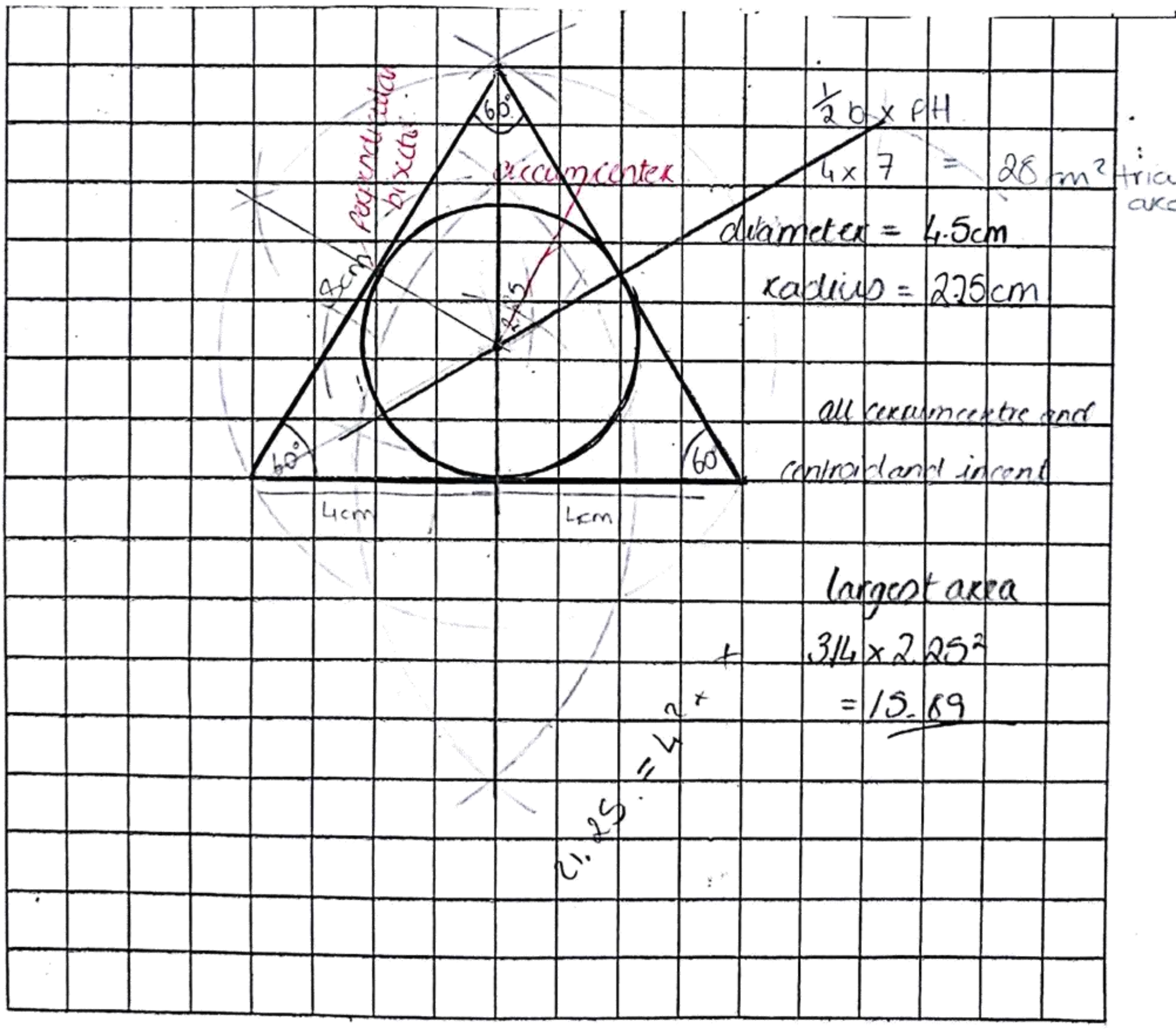
(ii) Using this triangle, construct the Circumcentre, Incentre, Centroid, and Orthocentre.
What is your observation about these four points?

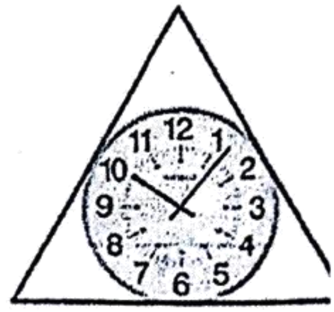
Student Responses

Question:



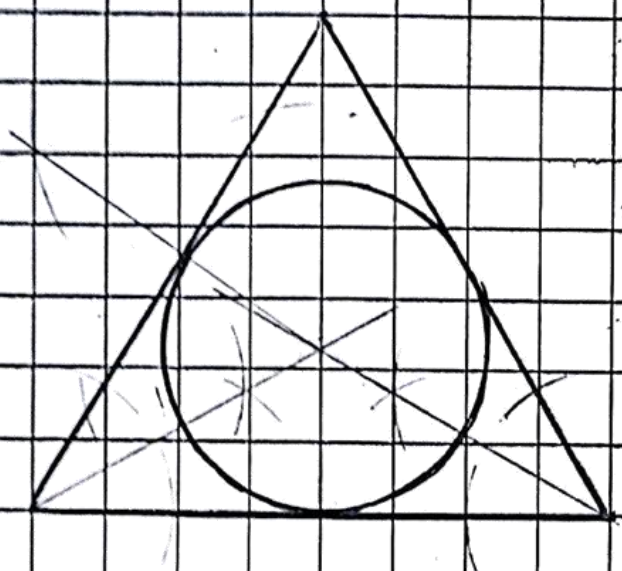
A manufacturer wishes to make a clock using a triangular sheet of metal. What is the area of the largest circular clock face that she can make, using the measurements in the diagram below?





Question:

A manufacturer wishes to make a clock using a triangular sheet of metal. What is the area of the largest circular clock face that she can make, using the measurements in the diagram below?



Radius = 2.3cm

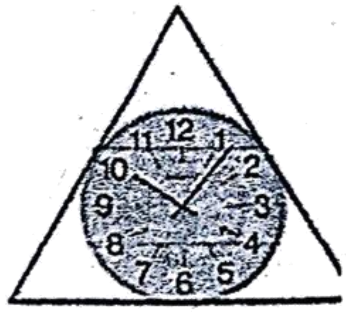
Area of circle = πr^2

$\pi\, 2.3^2$

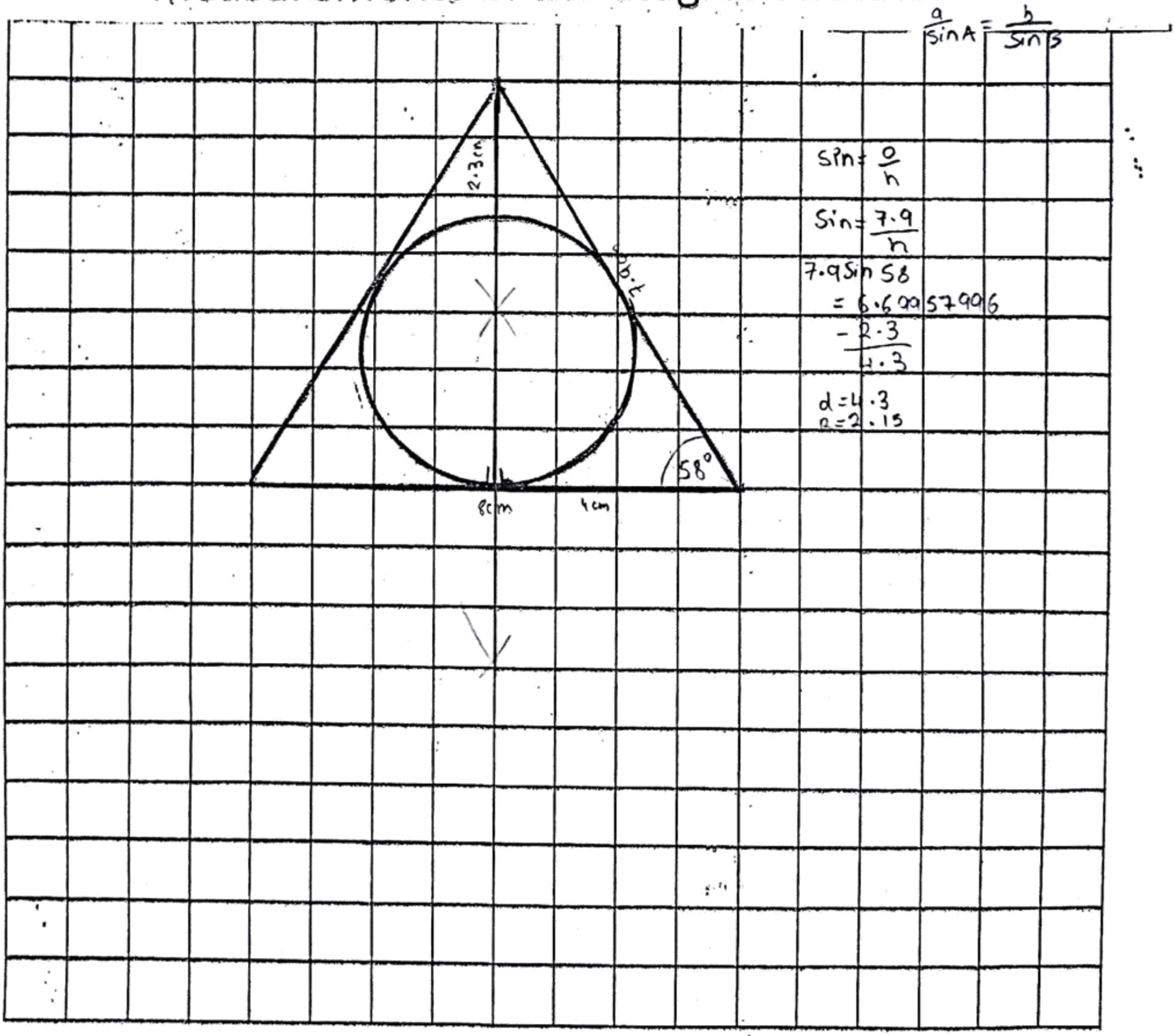
$= 16.619 cm^3$

Question:

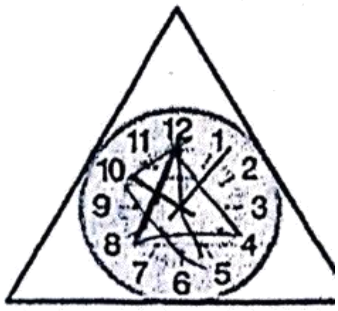
A manufacturer wishes to make



a clock using a triangular sheet of metal. What is the area of the largest circular clock face that she can make, using the measurements in the diagram below?



Tick Tock!



Question:

A manufacturer wishes to make

a clock using a triangular sheet of metal.

What is the area of the largest circular clock face that she can make, using the measurements in the diagram below?

Area of circle

πR^2 $\pi = 3.14$

$\pi (2.25)^2$

15.90 cm^2

Area of Circle

Area of Triangle

$\frac{1}{2}$ Base X $\perp$ h

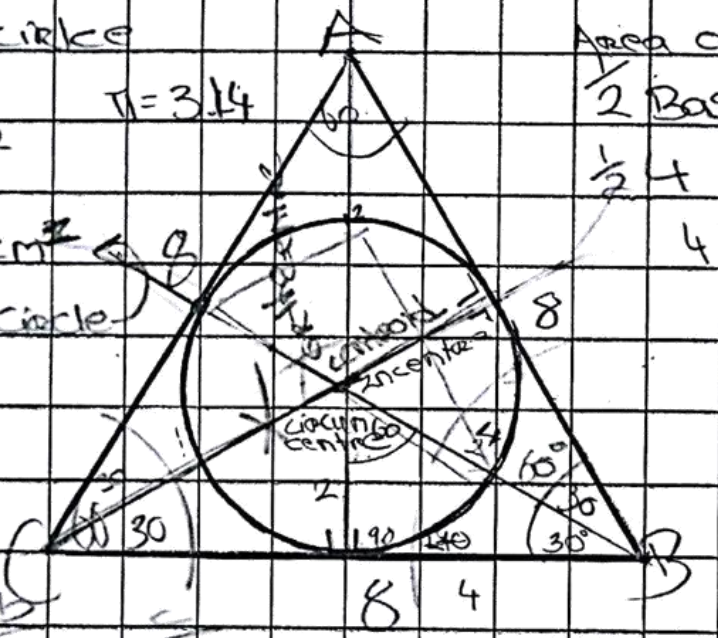
$\frac{1}{2}$ 4 x 2

4 cm units^2

Sin = $\frac{O}{H}$

Cos = $\frac{A}{H}$

Tan = $\frac{O}{A}$



$\frac{a}{\text{Sine A}} = \frac{b}{\text{SineB}} = \frac{c}{\text{SineC}}$

Seating Plan