Transformation Trickery

For the lesson on 26 January 2017
At Gorey Community School, 3rd Year Higher Level class
Teacher: G. Sunderland
Lesson plan developed by: G. Sunderland, F. Dalton and M. McCarthy

1. Title of the Lesson: Transformation Trickery

2. Brief description of the lesson:

Students are presented with a task. There are several routes to the successful solution. Students are given the equations of two lines (the initial line and the end line) and four translations to apply to the initial line. Drawing on their prior knowledge of reflections and transformations, students are tasked with finding the correct sequence of translations to be applied to the initial line, such that they arrive at the end line.

3. Aims of the Lesson:

Short-term goals:

- I'd like my students to recognise images of points and lines under translation & axial symmetry.

Long-range/thematic goals:

- I'd like my students to appreciate that mathematics can be used to communicate thinking effectively. **(Key Skill: Communicating and Working With Others)**
- I'd like to foster my students to become independent learners. **(Key Skill: Managing Myself)**
- I'd like to emphasise to students that a problem can have several equally valid solutions.
- I'd like to build my students' enthusiasm for the subject by engaging them with stimulating activities. **(Key Skill: Being Numerate)**
- I'd like my students to connect and review the concepts that we have studied already.
- We would like to support our students in developing their literacy and numeracy skills through discussing ideas. [1]

[1] This Lesson Proposal illustrates a number of strategies to support the implementation of Literacy and Numeracy for Learning and Life: the National Strategy to Improve Literacy and Numeracy among Children and Young People 2011-2020 (Department of Education & Skills 2011).

4. Learning Outcomes:

As a result of studying this topic students will be able to:

- Reflect a line in the horizontal & vertical axes.
- Translate a line, given specific instructions.
- To be able to visualize these concepts.
- To be confident with the terminology used.

5. Background and Rationale

Traditionally the teaching of transformation geometry has focused on the translations, symmetries and rotations of points, and arbitrary objects designed to make the study of the topic relate to real life. This approach has value in terms of the initial teaching of the concepts; however, a more detailed exploration of the concepts associated with transformation geometry (particularly relating to linear functions) is required if students' are to be adequately prepared for the transformation geometry specified in the current Leaving Certificate syllabus. In addition, the strengthening of geometric thinking is facilitated by the development of students' power to form and manipulate mental images, and to express what they are imagining in words, diagrams and sometimes objects (Mason *et al* 2005).

6. Research

- *Junior Certificate Guidelines for Teachers* (DES 2002, Government Publications Sales Office).
- First Year Handbook (PMDT).
- Second Year Handbook (PMDT).
- Third Year Handbook (PMDT).
- *Junior Certificate Mathematics Syllabus* (DES 2016, Government Publications Sales Office).
- www.projectmaths.ie
- www.nrich.maths.org
- Chief Examiners Report on Junior Certificate Mathematics 2006 (SEC 2006).
- Chief Examiners Report on Junior Certificate Mathematics 2016 (SEC 2016).
- *Literacy and Numeracy for Learning and Life* (DES 2011).

7. About the Unit and the Lesson

The Junior Certificate Syllabus outlines material that is required to be studied during the three years of junior cycle education. The syllabus outlines the material initially in strands, of which there are five, listed below:

- Statistics and Probability
- Geometry and Trigonometry
- Number
- Algebra
- Functions

Each strand in sub-divided into topics where a description of the topic is given (what the student learns about) and learning outcomes are detailed (what the student should be able to do).

Section 2.4 outlines several learning outcomes that are addressed by this lesson.

2.4 Transformation geometry	Translations, central symmetry, axial symmetry and rotations.	– locate axes of symmetry in simple shapes – recognise images of points and objects under translation, central symmetry, axial symmetry and rotations
Students learn about	**Students should be able to**	
2.5 Synthesis and problem-solving skills	– explore patterns and formulate conjectures – explain findings – justify conclusions – communicate mathematics verbally and in written form – apply their knowledge and skills to solve problems in familiar and unfamiliar contexts – analyse information presented verbally and translate it into mathematical form – devise, select and use appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions.	

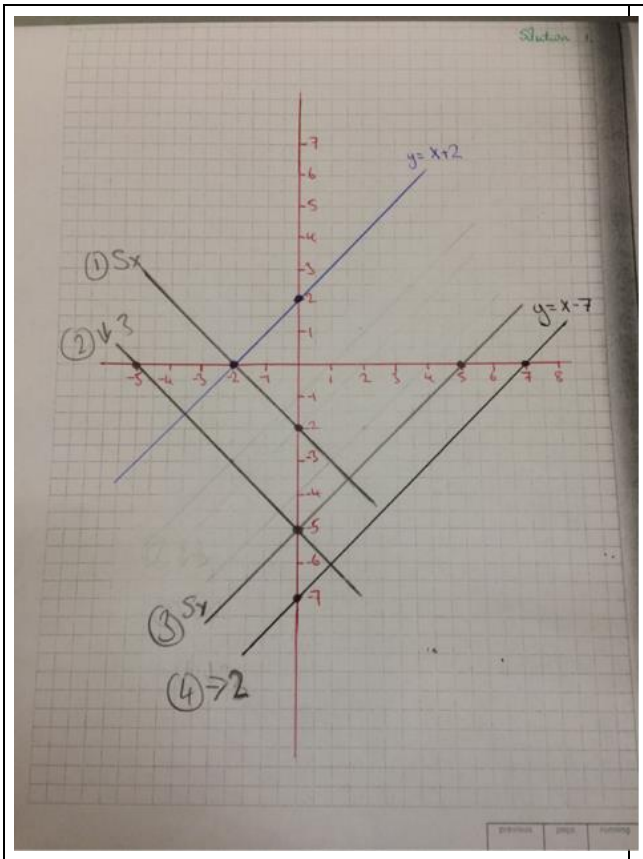
The Junior Cycle syllabus 2016 section 2.5 (Synthesis and problem solving skills) notes that 'most candidates demonstrated good levels of knowledge and comprehension of basic mathematical concepts and relations, which is fundamental to the successful development of mathematical proficiency. Candidates struggled at times when more involved understanding was required, or when the concepts were slightly less standard'. The lesson proposal seeks to develop students' understanding of transformations so that they can tackle questions that are less routine or procedural in nature.

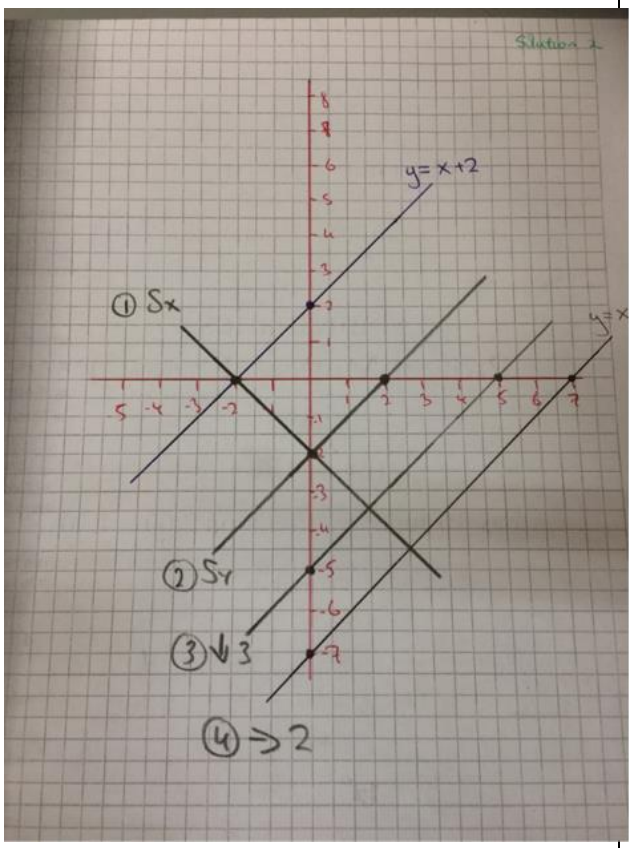
8. Flow of the Unit:

Lesson		# of lesson periods
1	Plotting points on Cartesian plane & equation of a line.	1.5 x 30 min.
2	Graphing lines + Equation of a line (y=mx+c).	4 x 30 min.
3	Transformations –reflection in x and y axes & translations.	1.5 x 30 min.
4	Transformations – Reflections & Translations	1 x 30 min. Research Lesson

9. Flow of the Lesson

Teaching Activity	Points of Consideration
1. Introduction Recap Prior Knowledge (4min) Using the information we have learned to date what can you tell me about each of the following? 1. *Reflections in x and y axes* 2. *Translations*	Teacher draws relevant images on the board to help extract required information.
2. Posing the Task Today's task will involve using your knowledge of reflections & translations to solve the problem on the sheet in front of you. The sheet is divided into 4 sections each on side of the page, all of which have the same problem. The problem posed is as follows; 'Notice that you are given a starting line, coloured red, and the image of that line, coloured blue. You are also given 4 transformations. Can you figure out the order in which you would perform the transformations on the red line in order to end up at the blue line. You are given 10 minutes to solve the given problem in as many ways as you can think of.	Having received the problem, ensure that students are aware that each problem is the same and that there is more than one solution to this problem. Read out the given task and ensure that students are aware what is being required of them. Required materials: pencil and a ruler. During this ten minutes circulate room to prepare and plan for Boardwork and Class Discussion. This in between desk assessment is crucial to the success of the class discussion to follow.
3. Anticipated Student Responses Having walked around the room and observed all of your work I will now ask students to come up and show how they solved the problem.	Instruct students that only those that have the right solutions have to approach the board and so they have nothing to fear. Bring students to the board starting with the most common approach to the most sophisticated. Summarise each solution after students have presented and explain how they got their answer. Guide students through any solutions that they have not found/developed.


y= x+2
y= x-7
① Sx
② ↓3
③ Sy
④ ⇒2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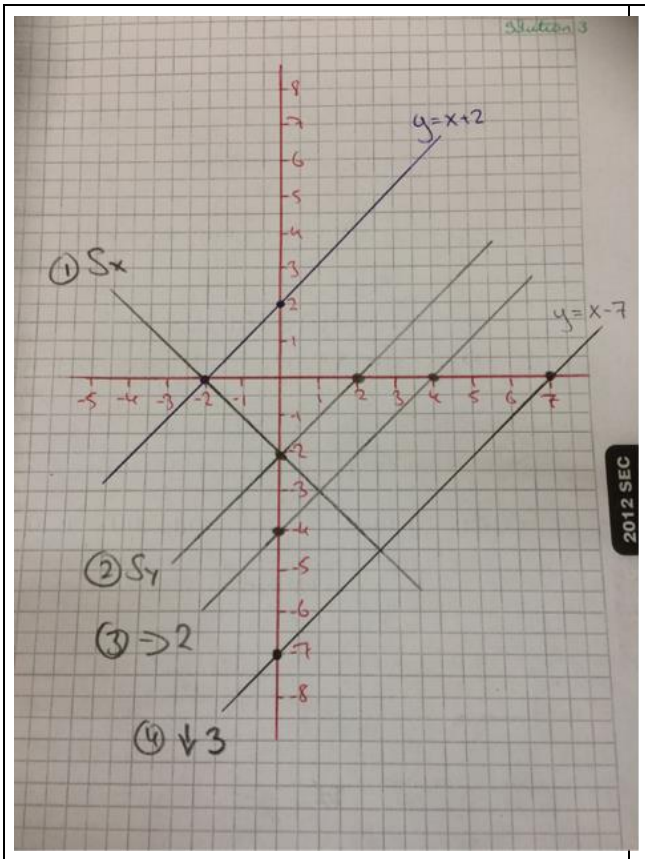
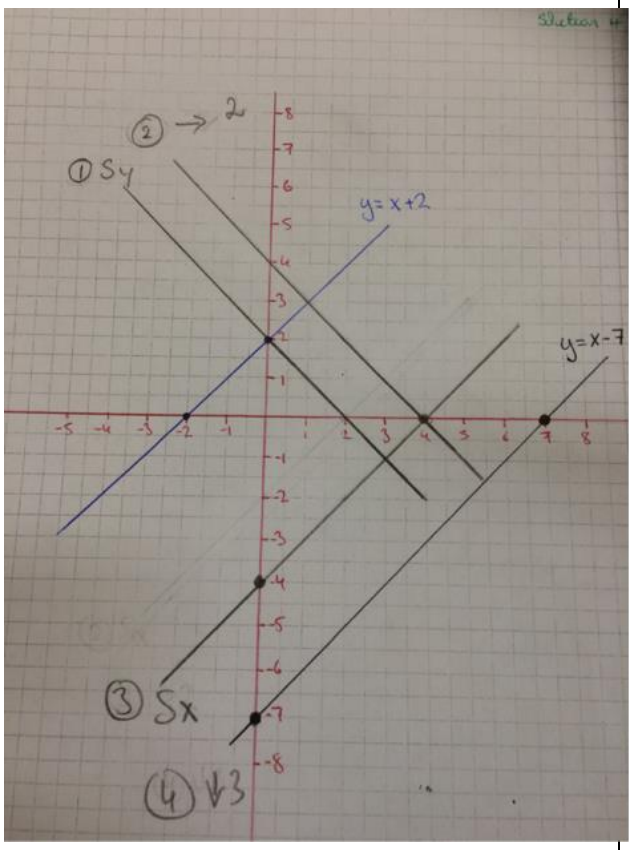
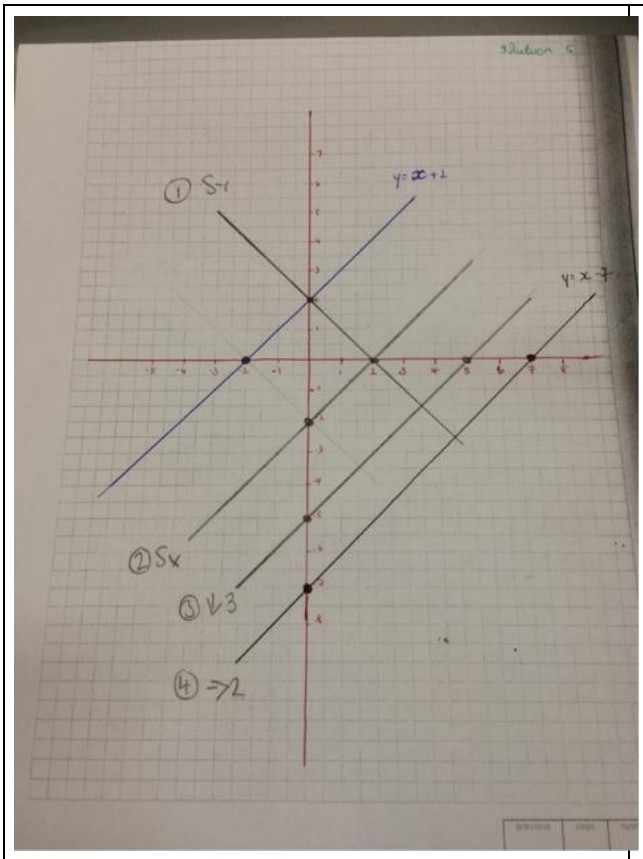
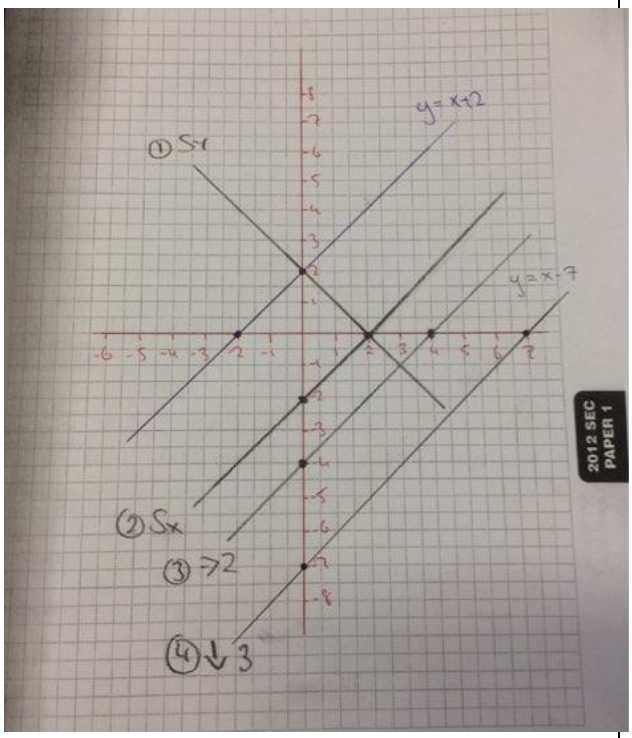
① Sx
② Sy
③ ↓3
④ ⇒2


y=x+2
y=x-7
① Sx
② Sy
③ →2
④ ↓3
2012 SEC


② → 2
① Sy
y=x+2
y=x-7
③ Sx
④ ↓3


Solution 6
① S-1
y=x+2
y=x-7
②Sx
③↓3
④→2


y=x+2
① S-1
y=x-7
②Sx
③→2
④↓3
2012 SEC PAPER 1

Having observed all of the students solutions, probe the class to see if anyone has thought of any other way since finishing the task.	
4. Summing up What did you learn today? Which solution did you find to be the best? How will what you have learned today help you in the future?	Reinforce with the class that this one problem had many solutions and this can be the case with so many problems in mathematics. For homework they are to try and get another solution if not all of the six solutions are achieved in class. Homework / Extension: Is the following statement always true, sometimes true or never true? A reflection in an axis followed by a reflection in a different axis, can be replaced by one reflection in a single line.

Start	**1st move**		**2nd Move**		**3rd Move**		**4th Move**	**Finish**
y=x+2	S_x	y=-x-2	S_y	y=x-2	Down 3	y=x-5	Right 2	y=x-7
y=x+2	S_x	y=-x-2	S_y	y=x-2	Right 2	y=x-5	Down 3	y=x-7
y=x+2	S_y	y=x+2	S_x	y=x-2	Down 3	y=x-5	Right 2	y=x-7
y=x+2	S_y	y=-x+2	S_x	y=x-2	Right 2	y=x-4	Down 3	y=x-7
y=x+2	S_x	y=-x-2	Down 3	y=-x-5	S_y	y=x-5	Right 2	y=x-7
y=x+2	S_y	y=-x+2	Right 2	y=-x+4	S_x	y=x-4	Down 3	y=x-7

10. Evaluation

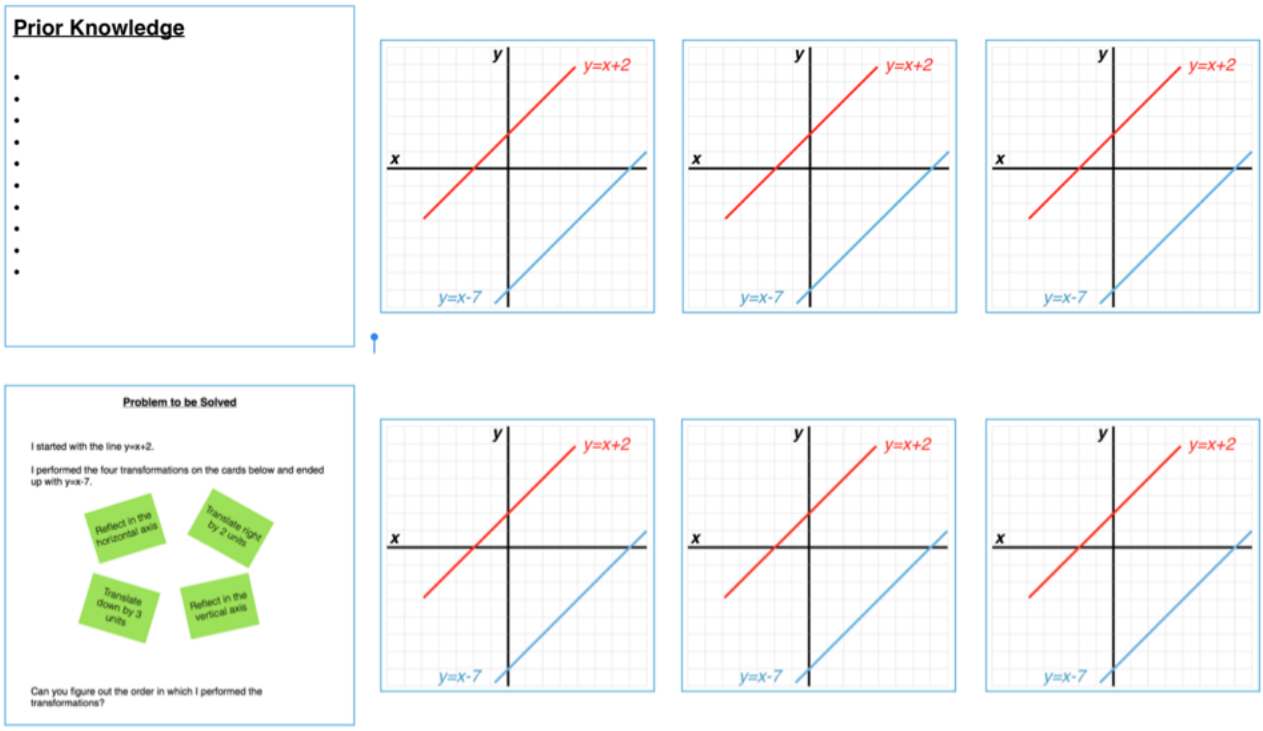
There will be three observing teachers in the room. The room is divided into zones and each observing teacher has been assigned a zone. Samples of student work will be recorded using iPads. One observing teacher will use LessonNote to record interactions during the lesson.

Observing teachers will pay particular attention to the following:

- The distribution of trial and error versus logical procedure as a strategy to solve the task.
- Whether students transform a point on the line or transform the entire line when attempting the task.
- Any misconceptions in student knowledge.

A formative assessment component has been built into the lesson to allow for student feedback.

11. Board Plan



12. Post-lesson reflection

- What are the major patterns and tendencies in the evidence?
  - There were six correct solutions to the task. Several students found multiple solutions. No individual student found all six. Students displayed a high level of understanding and mathematical fluency in the area of co-ordinate geometry. In particular, co-ordinates were plotted and annoted correctly.
  - Students have excellent line drawing skills. There is clear evidence of a high level of ability and accuracy in terms of students' use of mathematical instruments. Again this facilitated a high level of student engagement with the task.
  - All students, with one exception, used the points on the axes to transform a given line. Lessons following the Research Lesson could focus on transforming the line using points other than the intercepts.
  - Students approached the task mainly using the method of structured trial and error. Approximately half of the students deleted (rubbed out) workings which did not lead to a correct solution. It is evident that students' learned from their incorrect work and that in many instances it helped the student progress towards a correct solution. However, students must be reminded of the value or recording all workings.
  - Several students began with the translations of the lines rather than reflecting in the x or y axis. This surprised observers as it was expected that all students would begin by reflecting the lines in one of the axes.

- In what ways did students achieve or not achieve the learning goals?
  - The task gave students the opportunity to embed their prior knowledge of transformations and apply it in a problem solving context. The design of the task also allows for a discussion of parallel and perpendicular lines. Lessons following the Research Lesson could be used to explore the relationship between the start line and the fact that non parallel lines are perpendicular.
  - Students developed their presentation and communication skills. They were comfortable coming to the board to explain their workings/approaches and were pleased to share their thought processes.

- Student feedback indicates student feedback indicates that the short and long term goals of the lesson were achieved.

  "It is possible for there to be more than one solution"

  "I learned how to move lines correctly"

  "I learned how to solve a problem with translations in it"

  "I learned that when a point is on the x/y axis and when you reflect in horizontal or vertical axis, it doesn't move"

  Student feedback also provided information for the teacher about what needed to be addressed at the beginning of the next lesson.

  "I found axial symmetry hard."

  "Reflections between y and x were confusing."

  Student feedback also provides evidence that problem solving type questions and teaching through problem solving adds significant value to students' mathematical skill set.

  "I found it difficult doing questions before I saw some examples."

- Based on your analysis, how would you change or revise the lesson?

  - The duration of the Research Lesson (40 minutes) could be extended to allow students time to experience the 'struggle' of being stuck when problem solving. It would also allow them to develop strategies for getting unstuck, and develop the resilience required for problem solving in mathematics.
  - The discussion and analysis section of the lesson could be facilitated by the use of a colour coded card display for correct solutions to the task.
  - Depending on the ability range of the class, students could work individually for five minutes and in pairs for the remainder of the problem solving session.

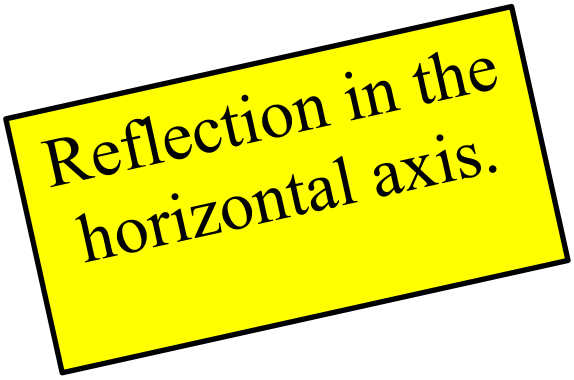
Transformation Trickery

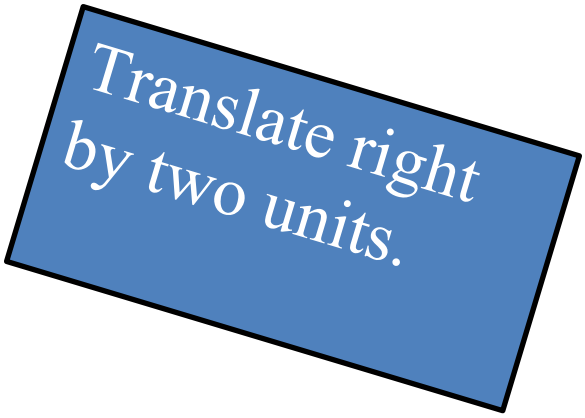
I started with the line y = x + 2

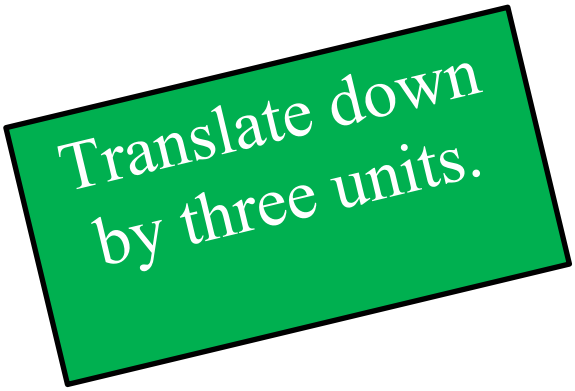
I performed four transformations.

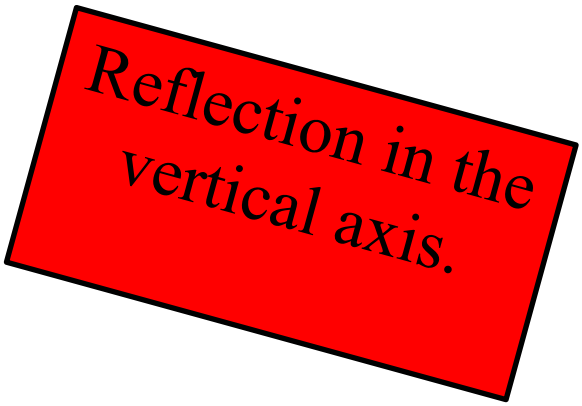
They are listed on the cards below.

I ended up with the line y = x – 7





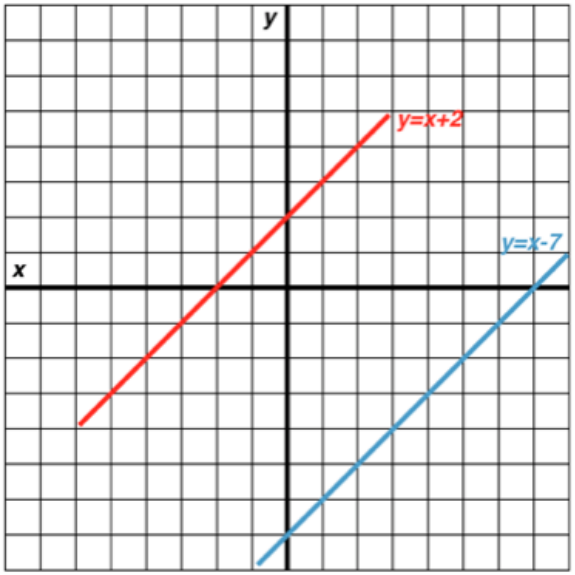




Can you figure out the order in which I performed the transformations?

Appendix 1

Student Response Sheet

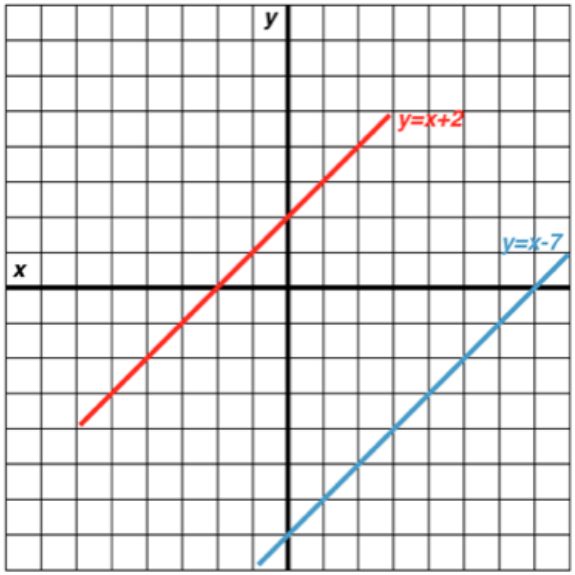


Step 1: _____

Step 2: _____

Step 3: _____

Step 4: _____

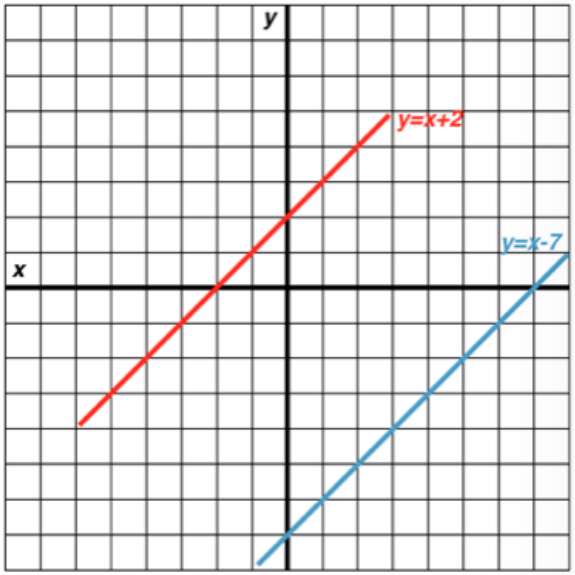


Step 1: _____

Step 2: _____

Step 3: _____

Step 4: _____



Step 1: _____

Step 2: _____

Step 3: _____

Step 4: _____