1. **Title of the Lesson:** ‘Tri to be shady’ Shaded area in a 2-D shape.

2. **Brief description of the lesson:** Calculate the shaded area in a given 2-D shape. Discover multiple solutions to a problem.

3. **Aims of the Lesson:**
   a) **Short term aim:**
      - Review area of a square, a rectangle and a triangle
      - Discover multiple solutions for solving shaded area problems
   b) **Long term aims:**
      We would like our students:
      - to appreciate that mathematics can be used to solve real world problems
      - to become independent learners. (Key Skills: Managing Information & Thinking: Thinking creatively and critically)
      - We would like to support our students in developing their literacy and numeracy skills through discussing ideas.

4. **Learning Outcomes:**

   As a result of studying this topic, students will be able to:
   - Use knowledge of 2-D shapes to solve surface and volume of 3-D shapes
   - Use knowledge of 2-D shapes to introduce nets
   - Develop observational skills so as to recognise nets of 3-D shapes
   - To develop problem skills that can be applied to real life scenarios
   - Present logical ideas to their peers. (Key Skill: Communicating)

5. **Background and Rationale**

   (a) What the students need to learn according to the syllabus;

   3.4 Applied Measure – 2-D shapes, Select and use suitable strategies to solve area of a triangle and square.

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1 This Lesson Proposal illustrates a number of strategies to support the implementation of Literacy and Numeracy for Learning and Life: the National Strategy to Improve Literacy and Numeracy among Children and Young People 2011-2020 (Department of Education & Skills 2011).
(b) Difficulties students have had in the past with the subject matter

- Difficulties with spatial awareness
- Mixing up what perimeter and area are
- Recognising the relationships between different shapes
- Problems with shaded areas

(c) The thematic focus of this lesson study, i.e. larger (see above in number 3 for ideas) goals the team will try to address, and why.

- More than one way to solve a problem
- Foster independent learning
- Practical use of maths

6. Research

b) First and second year Teachers Handbooks (from the Maths Development Team’s website [www.projectmaths.ie](http://www.projectmaths.ie))
c) Consulted many Junior Certificate Mathematics textbooks

7. About the Unit and the Lesson

This lesson is designed to engage the students and their prior knowledge of 2-D shapes. This lesson will be a continuation from their basic understanding of the area of a square and triangle. This lesson is designed to introduce the link between different shapes and how you may use this to solve more complicated questions. We hope to show the students that there are many ways to discover the solution and help develop their problem solving skills. Through active learning we hope to develop independent learning and spatial awareness.

8. Flow of the Unit:

<table>
<thead>
<tr>
<th>Lesson</th>
<th>Teaching Activity</th>
<th># of lesson periods</th>
</tr>
</thead>
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| 1      | • Revision of area of square, rectangle and triangle.  
         • Introduction of key words.  
         • Problem solving based on 2-D shapes. | 1 x 40 min. |
| 2      | • Calculate areas in a given 2-D shape.  
         • Discover multiple solutions to a problem. | 1 x 40 min.  
         (research lesson) |
| 3      | • Volume of a cube, cuboid and a cylinder.  
         • Problem solving with 3D shapes. | 2 x 40 min. |
| 4      | • Surface area of 3D shapes and their nets. | 3 x 40 min. |
| 5      | • Finding missing dimensions of 2D and 3D shapes.  
         • Problem solving with scale diagrams. | 1 x 40 min. |

9. Flow of the Lesson

<table>
<thead>
<tr>
<th>Teaching Activity</th>
<th>Points of Consideration</th>
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| **1. Introduction (5 mins)**  
Students recount prior knowledge which will be made visible throughout the lesson on the board. | Students prompted to recall formulae encountered in the previous lesson. Board work prepared on board for students to engage with during the introductory task. |
| **2. Posing the Task (5 mins)**  
Each student receives an A3 sheet with 6 copies of the same problem. The question will then be put to the students and they will be encouraged to find as many solutions as they can. | Students asked if they have any problems comprehending the task at hand. Task is re-read until all students have verbally confirmed they are comfortable with attempting the task. Students will be informed at this point that they have a designated 10 minutes on to complete the task. |
| **The task:**  
Calculate the shaded area in this 2-D shape and find multiple solutions to the problem. | |
### 3. Anticipated Student Responses

1. Students find the area of the square and use fractions/decimals to find the shaded area i.e. divide by 4, multiply by $\frac{1}{4}$ or multiply 0.25. Alternatively students can find 25% of the given area.
2. Students find the area of the square and divide by 2 twice to get the area of the triangle.
3. Students find the area of the square and divide by 2 twice to get the area of the triangle.

4. Use the formula for the area of a triangle to find the area of the shaded portion.
5. Use the formula for the area of a triangle to find the area of the largest triangle and divide by two.
6. Use the formula for the area of a triangle to find the area of the unshaded part of the square and take it away from the overall area of the square.

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Teacher prompts without indicating whether the answer is correct or not.

Differentiated through extension work

Students make conjecture based on work they have carried out already
7. Flip over the top half of the shaded triangle to form a square and use square formula to find the area.
8. Find the area of the smallest shaded triangle and double it.

9. Place the problem on graph paper and count the boxes to find the area.
10. Place the problem on a coordinate plane and use coordinate geometry to find the area.
4. Comparing and Discussing (15 mins)
Teacher provides a pre made enlarged version of the problem for students to work on during the discussion.

Students asked to come to the board in a pre-determined order to discuss their solutions.

Teacher asked students to make comparisons between particular solutions.

Students encouraged to use appropriate language when discussing solutions.

Teacher highlights that there are multiple solutions to the problem all of which are equally valid.

5. Summing up (5 mins)
Teacher prompts students to reflect on their learning by writing a minimum of 2 points on the work completed.

Teacher sets homework task based on class activity.

10. Evaluation

What is your plan for observing students?

Tables in the classroom are set up as groups of four. Tables are back to back. One group has 6 chairs to accommodate extra students in a different class group. One teacher will take responsibility for 3 groups of 4 students on the left hand side of the room. One teacher will take responsibility for 3 groups of 4 students on the right hand side of the room. Finally one teacher will monitor the group of six students in the middle of the class at the larger table.
Discuss logistical issues such as who will observe, what will be observed, how to record data, etc.

Data will be recorded using lesson note. All three teachers will make note of successful outcomes by students to ensure an even spread of results from the different groups. Pre-made solutions will be used by all 3 teachers as a guide to getting a variety of results.

**What observational strategies will you use (e.g., notes related to lesson plan, questions they ask)?**

Students will not be told whether their solutions are correct or not. Students will be encouraged to keep attempting the task and will be encouraged to work independently.

**What types of student thinking and behaviour will observers focus on?**

Students will be encouraged to engage with prior knowledge as a means to help with the problem solving task. Teachers will encourage strategic and systematic approaches to problem solving. Students will be praised for demonstrating resilience towards the task. Students will be encouraged to make practical attempts using all mathematical equipment to complete the task.

**What additional kinds of evidence will be collected (e.g., student work and performance related to the learning goal)?**

Students will be monitored to see if they are aware of getting different answers resulting from different methods and did students give up on the task or stick to one method i.e. counting the boxes.

**11. Board Plan**
12. Post-lesson reflection

What are the major patterns and tendencies in the evidence?
Students tended to use fractions and in particular working off quarters. Very few students counted the boxes. There was a tangible unfamiliarity with the area of a triangle formula.

What are the key observations or representative examples of student learning and thinking?
Most students now appreciate that there are a number of different ways to solve a single problem (This was evident through feed-back comments). The orientation of the triangle was off-putting for students. As mentioned previously students tended not to use the area of a triangle formula.
Students spotted familiarities between different solutions.
Further of division of square into eights highlights axis of symmetry.

What does the evidence suggest about student thinking such as their misconceptions, difficulties, confusion, insights, surprising ideas, etc.?
Students who counted the squares in the picture often forgot or did not notice that 4 squares were needed to make 1 cm squared.
One student used prior knowledge of circle geometry to solve the problem (full square = 360 degree’s then 90 degree’s represented the triangle area.)
Students measured the dimensions of the triangle but stopped at that point.
The area of square was the last formula to be recounted during the prior knowledge introduction.

In what ways did students achieve or not achieve the learning goals?
All students had more than 2 methods for solving the problem. Students all accessed prior knowledge for the problem solving elements of the lesson.
Students in general did not use the area of triangle formula.

Based on your analysis, how would you change or revise the lesson?
We would promote pair work at the early stages of the lesson as a small number of students struggled to get started.
We would move the position of the co-ordinate grid to the top left hand side of the work sheet to encourage students to start with the easiest solution.
We would ensure adequate time at the end of the lesson for feedback and to introduce the homework task.

What are the implications for teaching in your field? All teachers agreed that it would very beneficial to introduce more problem solving elements to lessons to promote independent learning. This teaching methodology allowed for easy differentiation throughout the lesson and this is something that we all agreed would be useful to incorporate into future lessons. Another aspect that we all agreed would an important part of all future lessons was having the students come up to the board and to share their results and ideas as it encouraged and easily facilitated peer learning.
Appendix 1: Worksheet

Problem: How many different ways can you find the area of the shaded section of the square tile above?
Extension: Make statements underneath each solution exploring your method or system.

Appendix 2. Homework

*Problem Solving Homework Task*

If each tile costs £2.40 and the shaded area of the tile makes up £1.80 of their cost, how much would it cost to put the tile design sketched to the left on a wall in your kitchen?

Extension: Design your own tile pattern to save to another group's kitchen.