Topic: Parallelograms
Year Group: 5^{th} Year OL

For the lesson on: Parallelograms Jan 2017
At: Larkin Community College,
Class & Level: LC Mixed OL Class
Teacher: Kathy Barrett
Lesson plan developed by: Kevin Golden, Kathy Barrett & Anne-Marie Hynes.

1. **Title of the Lesson:** "What makes a parallelogram?"

2. **Brief description of the lesson:** Students will identify a number of ways to find the area of parallelograms and hence, identify the properties of parallelograms.

3. **Aims of the Lesson:**

   The lesson should ensure that:

- Students apply trigonometry to solve problems.
- Students will develop the skill of listening to each other by identifying the differences and similarities in their methods.
- Students learn there are a variety of ways to solve problems.
- Literacy and numeracy skills are developed through the use of effective mathematical language and terminology.
- Students will be able to identify, connect and review the concepts that we have studied and become independent learners.
- Students will develop the confidence to apply a variety of methods to solve more complex and unseen problems in the future.

4. Examples of short-term goals

Students will deduce the properties of parallelograms through problem solving using multiple approaches from Trigonometry and Geometry.

5. Learning Outcomes:

As a result of studying this topic students will be able to:

- Apply trigonometric methods along with using geometry and previous knowledge to find the areas of parallelograms in various ways (Theorem 18) and make connections between parallelograms, diagonals and their sides (Theorem 17).

- Identify unique attributes of parallelograms through analysis and problem solving and reinforce through homework questions.

6. Background and Rationale

According to the curriculum, learners should be able to explore patterns, explain findings, justify conclusions, communicate mathematics verbally and in written form, apply their knowledge and skills to solve problems in familiar and unfamiliar contexts, analyse information presented verbally and translate it into mathematical form, select and devise appropriate mathematical models and use appropriate formulas or techniques to process information and to draw relevant conclusions (NCCA, 2013).

Students tend to have difficulty with the topics of geometry and trigonometry. Common misconceptions around parallel lines, use of protractors and alternate and corresponding angles occur in the classroom. This comes from students understanding of key properties and theorems. Students are often informed of these without any reasoning or investigation.
Through this lesson, it is intended that students investigate these topics thoroughly and understand them through investigation and practice. They will learn from each other's different methods and deduce properties of a parallelogram through the use of trigonometry.

7. Research

- Project Maths Teaching & Learning Handbooks: Geometry & Trigonometry (from The Maths Development Team Website: www.projectmaths.ie.)
- Texts & Tests 3 Leaving Certificate Ordinary Level Maths Book.
- Junior Certificate Mathematics Guidelines for Teachers (DES 2002).
- First and second year teacher's handbooks (from the Math's Development Team's website: www.projectmaths.ie).

8. About the Unit and the Lesson

This lesson is designed to get students to draw on prior knowledge from Junior Certificate Level geometry and trigonometry. This will enable them to answer more tedious and challenging questions in the future. Students will have participated in previous lesson in relation to:

- straight lines
- parallel lines
- intersecting lines
- diagonals
- opposite and alternate angles
- Cosine and Sine Rules
- The area of right angled and non-right angled triangles
- The Theorem of Pythagoras

We hope this will guide them to complete the problems at hand in the lesson.

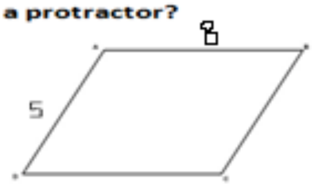
This follows the Leaving Certificate Mathematics Syllabus 2016, strand 2, section 2.3, trigonometry, page 25 and Section B, section 6.7, quadrilaterals and parallelograms, page 68-70.

9. Flow of the Unit:

Lesson		# of lesson periods
1	• Geometry: quadrilateral shapes and properties of certain types of angles (corresponding and alternate angles…). Including basic constructions i.e. perpendicular lines.	4 x 60 min.
2	• Revision of trigonometry concepts and problem solving right angles triangles and non-right angled triangles.	4 x 60 min.
3	• Using trigonometry, Geometry and Constructions to deduce the properties of parallelograms. • Theorem 9 (Opposite sides and opposite angles are equal) will be the only parallelogram theorem used in prior knowledge.	1 x 60 min. **(research lesson)**
4	• Deducing the diagonals of a parallelogram bisect each other (Theorem 10). Using geometry and trigonometry to problem solve.	1 x 60 min.
5	• Exploring exam questions requiring a prior knowledge of geometry and trigonometry.	1 x 60 min.

10. Flow of the Lesson

Teaching Activity	Points of Consideration
1. Introduction7mins Recap the prior knowledge of right angled and non-right angled triangles. What approaches do we have to solve both sets of triangles? <u>Right Angled TrianglesNon-Right Angled Triangles</u> SOHCAHTOA The Sine Rule Pythagoras' Theorem The Cosine Rule ? ⇐ The area of a triangle formula What is a Quadrilateral? Types of quadrilaterals? What is a parallelogram? Theorem 9: Opposite sides and opposite angles are equal.	Students will have been asked to bring a full maths set to class. The title of the lesson will not be given as leading questions of prior learning should recap key problem solving tools for the lesson.
2. Posing the Task 3mins	Problem 1 has 6 diagrams of the same parallelogram so this will be emphasised to students as well as the emphasis of trying to find multiple solutions to this same problem. Students will need prior knowledge particularly constructing perpendicular lines to complete the task. Students may need to be guided towards constructing perpendicular lines accurately.

Problem 1: **How many ways can you find the area of the parallelograms without using a protractor?**  Key Point: A ruler can only be used for constructions and not to measure.	
3(i) Anticipated Student Responses **See Appendix 1** **Individual student work 8mins**	Teacher will circulate the room to plan and prepare board work and class discussion. How the board will link prior knowledge and key words, using magnets and posters for students to demonstrate their work and a worksheet to practice questions. Use of a ruler may need to be encouraged here to act a catalyst for individual problem solving. **R1:** Count the boxes **R2:** Construct a diagonal to form two triangles (2 ways) **R3:** Construct perpendicular lines to form two triangles and a rectangle (3-4 ways)
4(i) Comparing and Discussing 12mins	Use of A4 sheets with the problem enlarged which studetns will use to demonstrate their work. A student with Solution 1 will be called up first followed by alternative Solution 2 & 3's. Therefore calling students up starting with the most common approach to more sophisticated approaches. Students present and explain how they got their answers followed by a teacher summary as each solution is presented. The discussion will compare differences in approaches and identify similar approaches with 3 approaches dominating the 6-7 suggested solutions. *Constructing perpendicular lines accurately will be a requirement.
3(ii) Individual student work 8mins **Problem 2:** **Using strategies from part one or otherwise, how many ways can you find the area of these parallelograms by only using trigonometry and a protractor?**  Key Point: A ruler can only be used for constructions and not to measure.	Students use protractors to work out angles and trigonometry tools to problem-solve a new parallelogram given 4 times so different ways of choice. **R1:** Anticipated responses will have two solutions with diagonals with 1/2absinC as the method. **R2:** However to complete two more solutions will require use of SOHCAHTOA and/or Pythagoras' Theorem.
4(ii) Comparing and Discussing 12mins	Students will be asked to come up to the board with solutions requiring diagonals. Followed by solutions with right angled triangles will be presented by students

	explaining their use of their protractor, SOHCAHTOA, Pythagoras' theorem. Students will be told that only students with the right answers will be asked to approach the board to put students at their ease during the lesson. Guide students through solutions they did not get. Board work will be now collated to align similar approached from Problem 1 & Problem 2 and deduce Theorem 17 area of a parallelogram formula & 18 diagonal bisecting the area of a parallelogram.
5. Summing up 10mins	Students write down: 3 things they have learned today? And 1 thing they would like to find more about. A worksheet will be given out with 6 different parallelograms given one side in each one to find the area using a protractor and trigonometry.

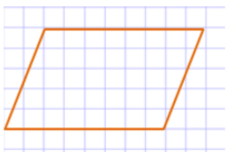
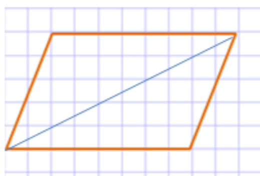
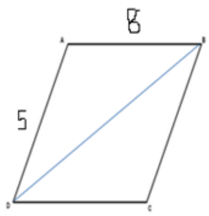
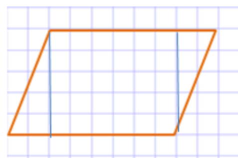
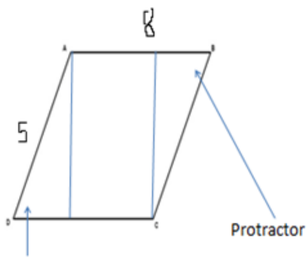
11. Evaluation

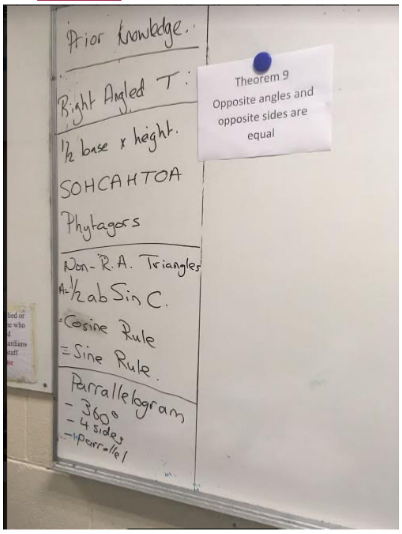
- A seating plan has been designed by the teacher
- Two observers with 8 students per observer, and an Observation Form was created and agreed so Pen and Paper will be used as opposed to LessonNote.
- Additional kinds of evidence to be collected (exemplars of students' work, photographs, end of lesson reflection)
- Types of student thinking and behaviour observers will focus on:

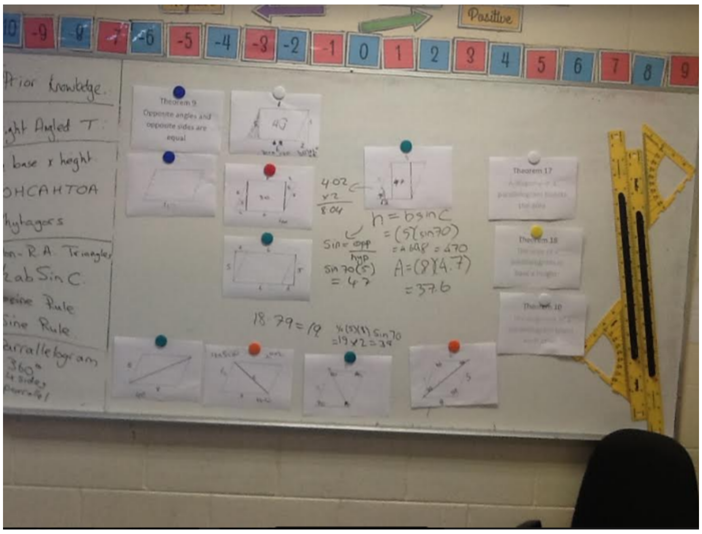
Introduction, posing the task	Can students recall the prior knowledge of basic trigonometry? Was wording of the task clear? Questions asked by students What grey areas did the problem pose for some students? Will this aid in rewording the question in the future?
Individual work	Can students construct accurate perpendicular lines? Were prompts required to facilitate multiple problem solving approaches? What anticipated responses did students actually achieve? Did any unanticipated solutions crop up? What strategies/responses does each student use/employ? How many can solve the problem? How long do students spend on the task? What kind of questions do students ask? Do they persist with the task?
Discussion	Are students attentive to what is happening on the board?

	Are clarifications needed to presenters' board work? Did the discussion promote student learning?

12. Board Plan

Prior Knowledge		Non-right angled triangles	Right angled triangles
What are Parallelograms? Theorem 9 Constructing lines to make compound shapes. Different ways to find the area of triangles.	*Student 1*  COUNT THE BOXES	*Student 2*  ½ base x perpendicular height  ½ abSinC	*Student 3*  Two triangles & a rectangle  Protractor & SOCAHTOA





13. Post-lesson reflection

What are the major patterns and tendencies in the evidence?

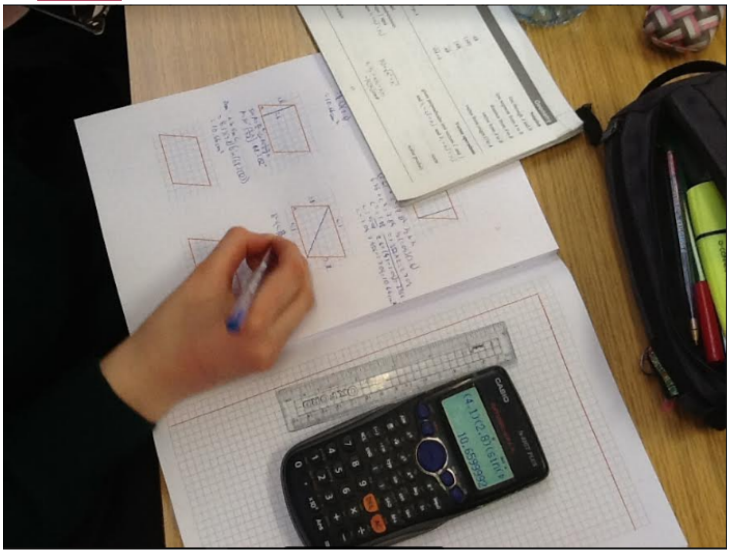
It is vital that students are aware of what they are trying to solve i.e. initial question must be very clear. Students must have clear understanding of what they are asked to do.

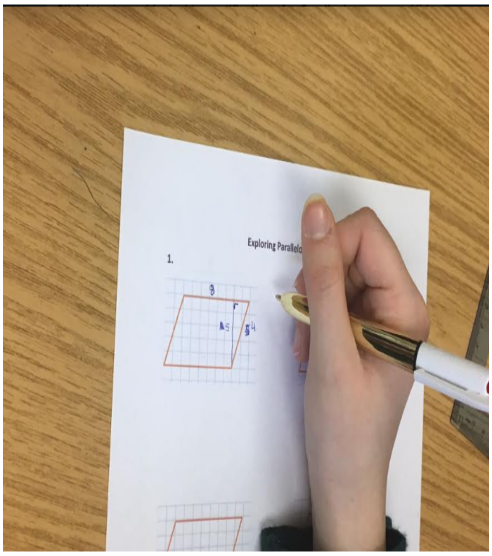
It was also observed that many students used the ruler to measure the parallelograms instead of counting boxes. The instruction to only use the ruler for constructions as opposed to measurements is important for this activity.

We found that the students constructed triangles and a rectangle at first as opposed to counting the boxes which we found surprising. They then continued on to draw a diagonal and then they began to realise the easier approaches. When they got this initial success it developed their confidence to look at other approaches.

What are the key observations or representative examples of student learning and thinking?

Students started off by over complicating their initial approach by constructing triangles and a rectangle. They were cautious at first and once they constructed the diagonal they then began to use the simpler approaches.





When it came to the strategy of counting boxes some students posed the question "Do I count the bits of boxes as a full box?" This brings us back to basic estimation and approximation skills. They were all very clear on what they had to do and once they had found their own approach they began to look at what the person beside them had and some discussed it with them. Although the individual problem solving phase is a key part of the lesson, we felt for some students communicating their solution to a friend instilled the confidence to pursue further problem solving individually.

In the second task the students did apply the method of constructing a diagonal and problem solving successfully but many struggled to problem solve right angled triangles using trigonometry.

It was a mixed ability group and as we are DEIS school we found some students needed the support of a peer to reassure that they were on the correct path. The majority of the class worked individually but some students were more suited to paired work and discussing approaches as previously mentioned. It is envisaged the more we use lesson templates like this it will accommodate more individualised work from students.

Students were reluctant coming up to the board at first but when the first student peer taught to his classmates, reinforced by the teacher, all students in the class were then very clear on what was required and soon began to see other approaches themselves.   Some approaches namely splitting the parallelogram into four triangles surprised us.

As the lesson progressed the students learning and thinking became more open with comments such as "but I did that a different way" and they began debating amongst themselves about who had the best approach. They also began questioning the student who was presenting about the validity of their figures.

They also showed a good understanding of prior knowledge as they were overheard saying to each other "opposite sides and angles are equal", which are the principles of theorem 9.

What does the evidence suggest about student thinking such as their misconceptions, difficulties, confusion, insights, surprising ideas, etc?

Students were clear about their understanding of basic trigonometry. They understood what parallelograms were and theorem 9. By the end of the lesson all students had a good understanding of theorems 17 and 18 as well as the various methods used to find the area of a parallelogram.

We found that the students constructed triangles and a rectangle at first as opposed to counting the boxes which we found surprising. We also noted that no student used proper constructions when drawing perpendicular lines even though they had the equipment to do so.

There were three students who showed a complete lack of effort to engage with the lesson; this is consistent with previous lessons with this group. The lack of effort wasn't directly associated with the approach of the lesson but more with the student's openness and willingness to learn.

Student's confidence in using right angled triangles to solve problem proved problematic for some. Students understood the area of a non right angled triangle however when it came to applying more comprehensive trigonometry they struggled.

In what ways did students achieve or not achieve the learning goals?

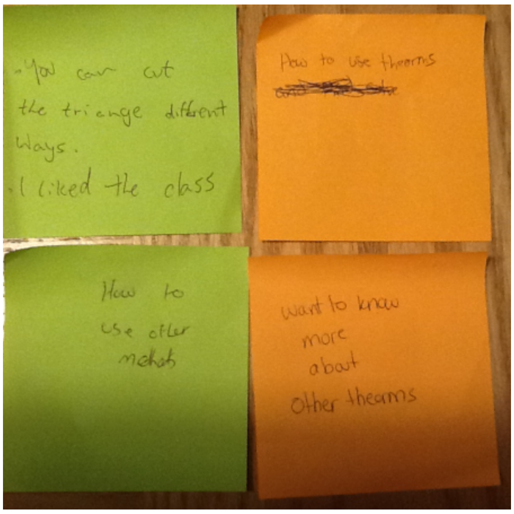
Learning Outcome 1: Apply trigonometric methods along with using geometry and previous knowledge to find the areas of parallelograms in various ways (Theorem 18) and make connections between parallelograms, diagonals and their sides (Theorem 17).

Learning Outcome 2: Identify unique attributes of parallelograms through analysis and problem solving and reinforce through homework questions.

Students achieved the learning goals as all students were able to discover two methods to find the area of a parallelogram. With regard to Problem 1 approximately half of the students in the class arrived at three anticipated solutions. Problem 2 proved more challenging especially the last solution with right angled triangles as it challenged students to have a comprehensive understanding of the whole topic of trigonometry.

The students were able to articulate their methods clearly at the board to their peers. The main concepts were reinforced and summarised by both the students and the teacher after each student presented their work. The pre-prepared posters worked very well in getting all students to follow the flow of the lesson. The final board plan worked very well as a final revision of the class. Overall, it was clear from questioning and overall student engagement in this class that the students understood the concepts.

This linked with the feedback students gave us at the end of the lesson which was all very positive. They were asked to write on a post it note three things they learned and one thing they want to know more about. Samples are below that elude to goals and aims of the lesson being achieved.



Based on your analysis, how would you change or revise the lesson?

- We would recommend a huge amount of work prior to the lesson on problem solving compound shapes using trigonometric methods so that students feel competent finishing the second task, particularly problem solving right angled triangles.

  Recommendation: Prior lesson(s) on problem solving compound shapes.

- Asking the students to justify their methods is important when doing the board work. It is important that they are allowed the time to fully explain their solution before being questioned by the rest of the class.

  Recommendation: This expectation needs to be explained to the class at the start of the lesson.

- No student drew out a proper construction of a perpendicular line.

  Recommendation: The Maths teacher should always ensure that they are utilising a full board Maths set to lead by example in all constructions when teaching geometry.

- At the start of the lesson the teacher needs to emphasise that the ruler cannot be used to measure the parallelograms in problem 1 as this was the first thing a number of students did.

  Recommendation: Emphasise to students not to use the ruler for measurement

- We felt the timings in the lesson plan were accurate. However this lesson was done in a DEIS school in a mixed ability class so alterations could be made in a more mainstream school.

  Recommendation: For example Problem 1 may only take six minutes to facilitate Problem 2 taking ten minutes. A stopwatch displayed on the board would also further motivate and engage students while also keeping the teacher on plan.

What are the implications for teaching in your field?

During the process of planning the lesson we found it difficult to find a problem with multiple solutions that could be done during a single class period. However the experience and process of trying to find a suitable problem allowed us to explore concepts in Maths thoroughly and to really understand and anticipate student misconceptions and understandings.

We understand that problem solving involves engaging in a task for which the solution method is not known beforehand. We want students to experience the frustration of problem solving but also the satisfaction of success. We want to develop student's confidence in the subject so that they know that it is ok to make mistakes and that often in Maths there is more than one way of solving a problem.

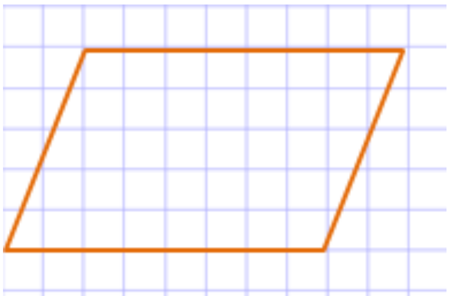
We sometimes don't allow our students enough time to solve a problem and end up showing them too quickly. We found in executing our lesson that the students were more engaged because they were given more time to work individually on the problem.

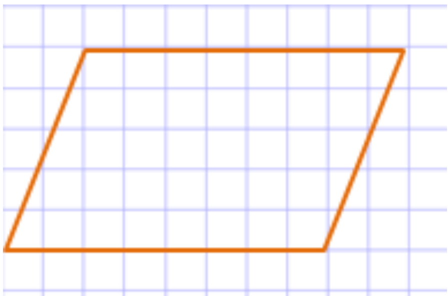
We felt this increased student's engagement in class as well as deepening their understanding of trigonometry. We also feel it helped give them more confidence in their own mathematical ability.
We feel it would be worthwhile planning some more lessons in a similar fashion.

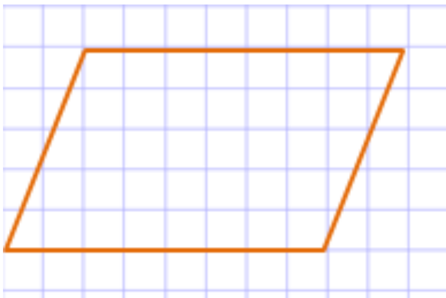
Appendix 1. Worksheet

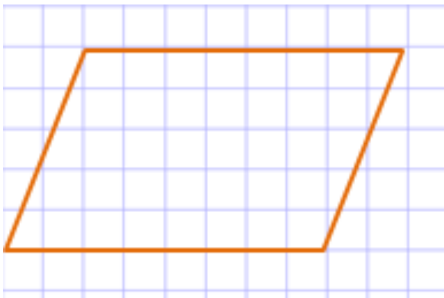
Exploring Parallelograms

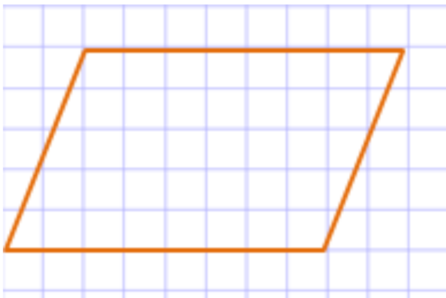
1.

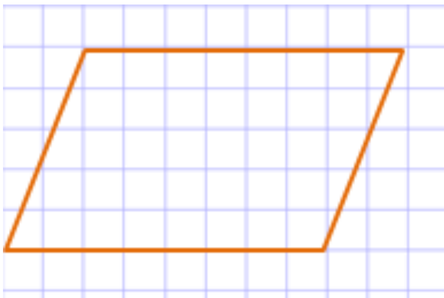












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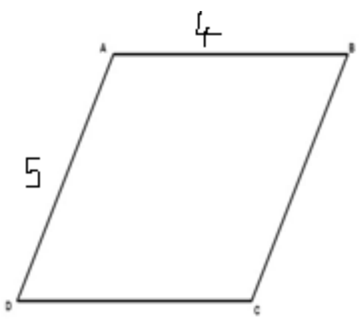
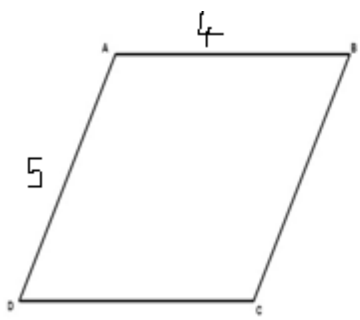
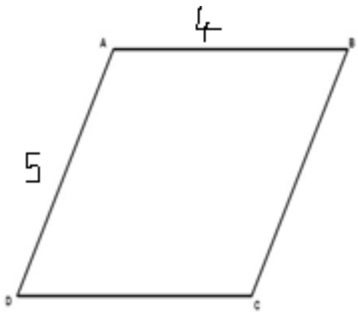
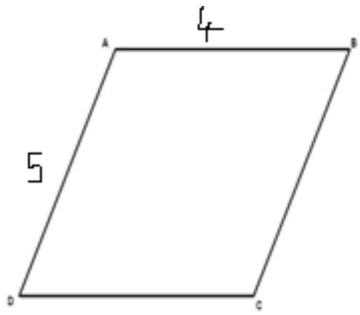


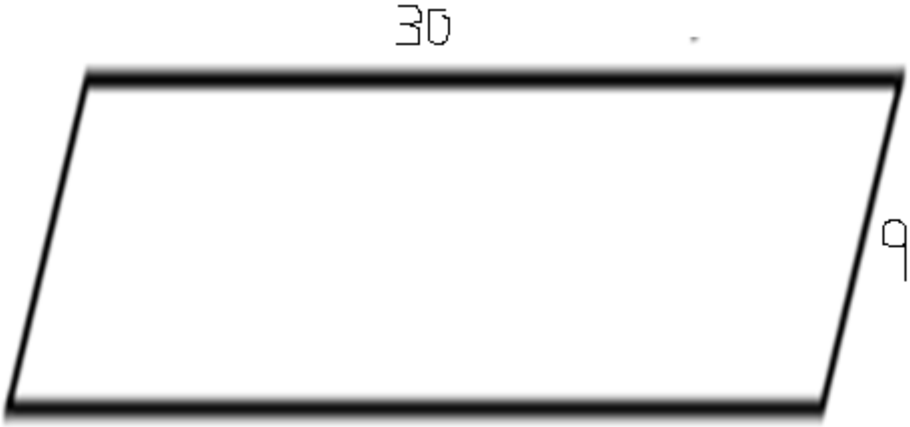






Appendix 2.Homework

Find the area of the following parallelograms by only using trigonometry, constructions and a protractor, and then check your answers using the area of a parallelogram formula:



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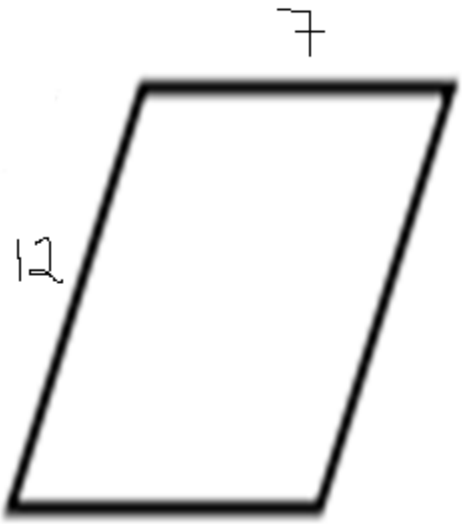

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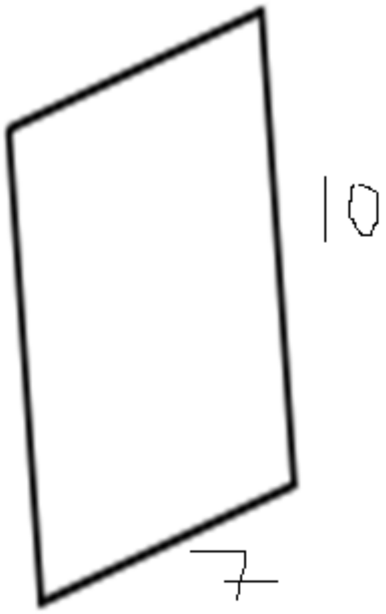

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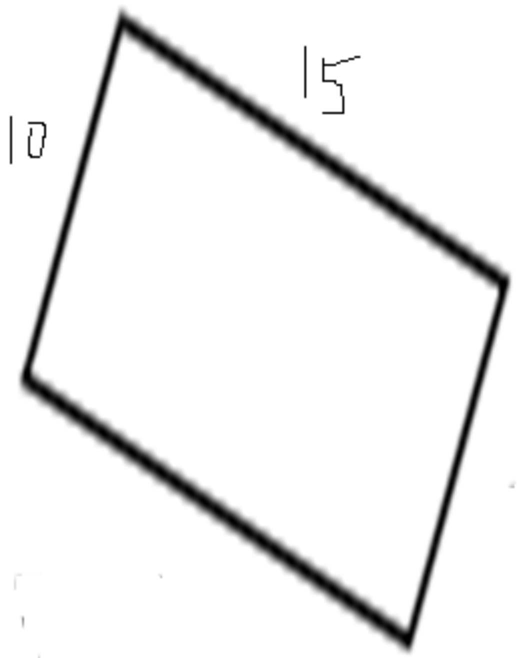


4.

Appendix 3. The next lesson...

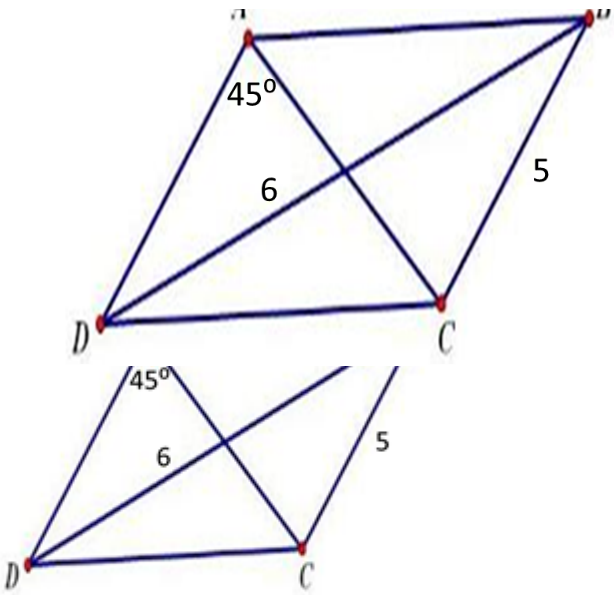
The example below will facilitate students to also utilise further trigonometry and geometry principles to deduce Theorem 10: The diagonals of a parallelogram bisect each other.

Instruction: How many values you would need to find all the angles and sides in this parallelogram:

Use the below diagram by PowerPoint. Have no values on the shape and then fly in one value at a time and discuss how many sides and angles we may find as a result.

The exercise should plateau at 3 values as below and students must problem solve all the remaining sides and angles informed by prior whole class discussions.

Key Point: The task needs to be done without a ruler or protractor.



Appendix 4. Teacher Checklist

Observation of student responses	Student 1	Student 2	Student 3	Student 4	Student 5	Student 6	Student 7	Student 8
Did the student understand what was being asked of him/her?								
Question 1 The student:								
counted the boxes								
constructed a diagonal to form two triangles (1- 2 ways)								
constructed perpendicular lines to form two triangles and a rectangle (1-4 ways)								
Other observations								
Question 2 The student:								
constructed diagonals, used protractor and used $\frac{1}{2}$ abSinC (1 - 2ways)								
constructed perpendiculars and used SOHCAHTOA or Pythgoras' Theorem (1 - 4 ways)								

2-Some engagement 3-Engaged Comment on any other observations								

		What changes do you think need to be made to the lesson plan?
		Did you notice any problems that could possibly arise again in future classes that may limit understanding of the topic?