Topic: Patterns
Year group: First Year

Title of the Lesson

"Working with Fred Pattern-son"

Brief description of the lesson

A diagrammatic presentation of patterns using the matchstick character "Fred Pattern-son" culminating in the formation of a linear equation.

Aims of the lesson

a) **Short term aims:** We would like all our students to
   i.   Engage with the task.
   ii. Make sense of the task.
   iii. Analyse the task and break it down into manageable steps.
   iv. Discuss their ideas about the task presented and their possible solution.
   v.   Reflect on their strategies and those of others and adjust their own approaches where necessary.
   vi. Develop their ability to explain and justify their thinking and so gain confidence in their ability to communicate mathematical ideas.
   vii. Make connections within mathematics i.e. between the study of patterns and algebra.
   viii. Develop a deep conceptual knowledge of patterns i.e. to understand the relationship between the diagrammatic presentation of patterns and its transference to the formation of mathematical sentences culminating in the formation of linear equations.
   ix. Build from the concrete (diagrammatic presentation of a pattern) to the abstract (linear equation).
   x.   Develop their understanding of the language and concepts of patterns.

b) **Long term aims:** We would like all our students to
   i.   Build their enthusiasm for the study of mathematics by engaging with stimulating activities
   ii. Make sense of the mathematics they can learn and use when doing a task
   iii. Connect and review the concepts that we have studied already
   iv. Come to the realization that a task may be solved in a variety of ways, each of which is equally valid.

v.   Foster a belief in their ability to become independent learners.
  vi.   Be creative when devising approaches and methods to solve tasks.
  vii.   To develop skills to become independent learners.
  viii. Improve their literacy and numeracy skills.
  ix.   Develop their communication skills.
  x.   Apply the skills learned in their attempt to solve real world problems.

Learning Outcomes

As a result of studying this topic students will be able to:
  a) Gain confidence in their ability to identify what changes and what stays the same
  b) Construct the next term in the patterns by drawing a diagram.
  c) Generalize and explain patterns in words and numbers
  d) Write a linear equation for a particular term in a sequence.

Background and Rationale

Typically in the past the teaching and learning of patterns has been beset with a number of difficulties including:
  a. Students inability to conceptualize the topic and relate it to real world problems
  b. A lack of resources to enable students to engage in hands on activities in the classroom
  c. Timetabling constraints with class duration of 40minutes thus making it difficult for self-discovery learning to take place.

This lesson has been designed in an attempt to overcome such difficulties. Through the use of diagrams to represent a repeating pattern situation, students will be able to generalize and explain patterns and the relationships contained therein in words and numbers. Moreover, students will be able to write arithmetic expressions for particular terms in a sequence by developing a deep conceptual understanding of patters.

Research

In preparation of this lesson plan the following materials have been used:

  a) Junior Certificate Mathematics Guidelines for Teachers (DES 2002)
  b) Junior Cert Mathematics Syllabus (2016)- Appendix: Common Introductory Course for Junior Cycle Mathematics.
  c) Project Maths – Teaching and Learning Plans.
  d) Reflections for Practice Conference and Lesson Plans

About the Unit and the Lesson

According to the Junior Certificate Mathematics Syllabus for examination from 2016 the importance of patterns is highlighted.

> *"Many elements of Science have a quantitative basis and learners are expected to be able to work with data, produces graphs and interpret patterns and trends."*
> (see Syllabus: Related Learning p.7)

The below is an extract from the Junior Certificate Mathematics Syllabus for examination from 2016 that explicitly states the Learning Outcomes related to Patterns:

Topic	Description of topic Students learn about	Learning outcomes Students should be able to
4.1 Generating arithmetic expressions from repeating patterns	Patterns and the rules that govern them; students construct an understanding of a relationship as that which involves a set of inputs, a set of outputs and a correspondence from each input to each output.	– use tables to represent a repeating-pattern situation – generalise and explain patterns and relationships in words and numbers – write arithmetic expressions for particular terms in a sequence
4.2 Representing situations with tables, diagrams and graphs	Relations derived from some kind of context – familiar, everyday situations, imaginary contexts or arrangements of tiles or blocks. Students look at various patterns and make predictions about what comes next.	– use tables, diagrams and graphs as tools for representing and analysing linear, quadratic and exponential patterns and relations (exponential relations limited to doubling and tripling) – develop and use their own generalising strategies and ideas and consider those of others – present and interpret solutions, explaining and justifying methods, inferences and reasoning
4.3 Finding formulae	Ways to express a general relationship arising from a pattern or context.	– find the underlying formula written in words from which the data are derived (linear relations) – **find the underlying formula algebraically from which the data are derived (linear, quadratic relations)**

Each strand also states that students should learn synthesis and problem-solving skills. The first skill listed in this section, in each strand is *"..explore patterns..."*. The importance is reiterated given that this is one of the key areas that students work with at the start of the secondary school career. Below is an extract from what is covered as part of the CIC.

All Strands Synthesis and problem-solving skills	– explore patterns and formulate conjectures – explain findings – justify conclusions – communicate mathematics verbally and in written form – apply their knowledge and skills to solve problems in familiar and unfamiliar contexts – analyse information presented verbally and translate it into mathematical form – devise, select and use appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions.

Flow of the Unit

Lesson	Details		# of lesson periods
1	Examine concepts of patterns, numbers, colours and shapes. Investigate if all lists form patterns. Students form their own patterns. Using LEGO to build a three colour repeating pattern allow students to see		1 period 40 mins
2	Identify characteristics of patterns, start term, common difference and ability to predict the next term. Use Activity sheets A&B from Project Maths Teaching and Learning Book.		1 period 40 mins
3	Introduce FRED PATTERN-SON. Students to draw FRED at year 3 and year 4 Identify what changed and what stayed the same. Using matchsticks make FRED at year 8 & 10. Turning a pictorial presentation into numerical pattern. Work towards formulating a general term Tn		1 period 40 mins
4	General term T_n formation for FRED'S development and referring back to previous covered patterns. $T_n = dn + c$		1 period 40 mins
5	Graphing Fred's development as a linear pattern. Recap plotting points. Discussion around reading future values from graph.		1 period 40 mins
6	Examine non-linear patterns. Identify second difference. Draw a graph to see curve forming. Class assessment in a week.		1 period 40 mins

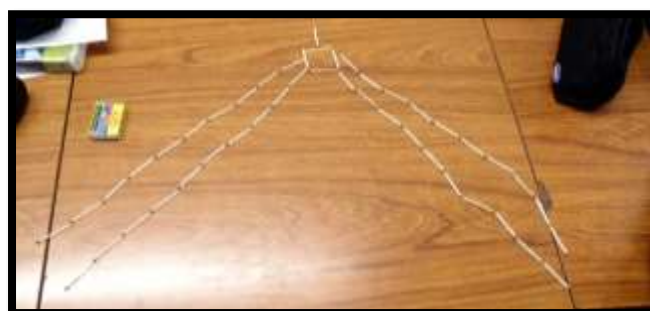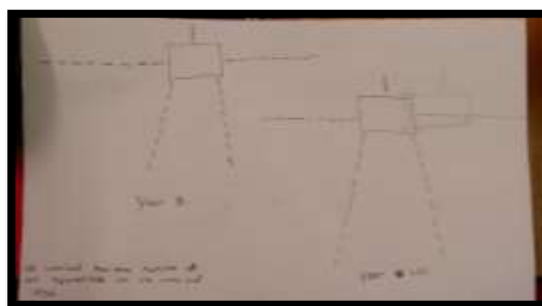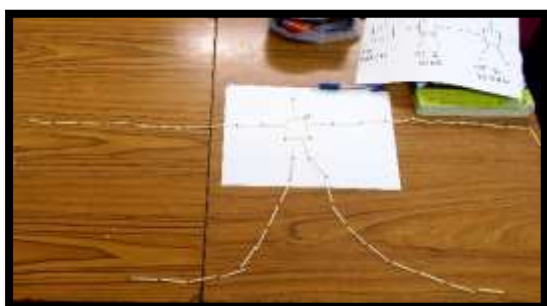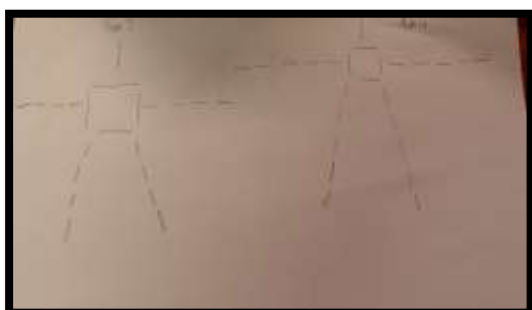| Teaching Activities and Students' Anticipated Responses	Points of Consideration
1. Introduction (5 minutes) **Prior Knowledge:** ● Prior knowledge will be tested and revised with the use of a simple pattern e.g. 1, 3, 5, 7 ● Students had come up with concepts such as ● Difference being the same ● Predicting the next term ● Numbers Positions i.e. term1 and term2 Use this time to re-enforce key words and to ensure that each student understands these terms	Teacher puts relevant material on the board if needed to extract the required information from students.
2. Posing the Task (5 minutes) Introducing "Fred Pattern-son" Three pictures placed on the whiteboard showing: ● Fred at Birth ● Fred at Year 1 ● Fred at Year 2 Students were given Fred on a Handout and are asked to use their pencil and paper and how they see Fred developing into Year 3 and Year 4 	The teacher will read out the given task in order to make sure that all students are aware of what is being required of them. Materials required: Paper and Pencil The teacher will circulate the room and observe students work. At this point the teacher will decide on students to bring to the board.
3. Students individual work (5 minutes) Students will be given 5 minutes to work on the problem on their own.	Teacher circulates the room assessing students' work.
4. Anticipated Student Responses: **Student 1 – Asked to draw Year Three** Arms getting longer and Legs getting longer. Trunk staying the same	Students' names will be written beside their work to give them ownership of their work.

Student 2 – Asked to draw Year Four Arms getting longer and Legs getting longer. Trunk staying the same	When referring to the work, the students' names will be used.
5. Comparing and Discussing (5 minutes) Two students will be selected to draw Stage 3 & 4 on the whiteboard Discussion to follow. From this, the students will hopefully be able to come up with the idea of ● What happened to Fred? ● What changed? ● What stayed the same? If the students refer to the word 'difference' it should be reiterated. .	The teacher will lead a discussion in describing student drawings.
6. Task 2: Paired Work (5 minutes) Students are asked how many matches it would take to make Fred at Year 8 and at year 10. Students given boxes of matches and sheets of paper. Working in pairs, some students are to draw Fred at Year 8 and Year 10, while others will make him with matchsticks.	Teacher circulates the room assessing students' work and giving guidance where it is required.
7. Comparing and Discussing for Task 2 (5 minutes) ● How many matches did it take to make Fred at Year 8 and Year 10? ● In what way did Fred change? ● By how many matchsticks did each limb increase? ● Did you have to do Year 9 to find Year 10? ● How many extra matchsticks were needed to do Year 10 following Year 8? ● Was it possible to predict how many matchsticks would be needed for Year 10? *Numerical Structure:* Identifying the constant i.e His Trunk =5 Identifying the difference i.e =4 Yr 1: 5+4= 5+4(1) Yr 2: 5+8= 5+4(2) Reinforcing idea of multiples as mentioned by Yr 3: 5+12 = 5+4(3) students in previous class.	Through group discussion, work towards putting a numerical structure on each stage.
8. Summing up (5 minutes) Recap of key concepts. Homework. Homework is a follow on question that will lead into identifying that the difference is the coefficient of '*n*' in the general formula '*Tn*' in the next lesson.	Reinforce the findings in today's lesson through use of correct terminology associated with the lesson and also that there were multiple avenues to the same result.

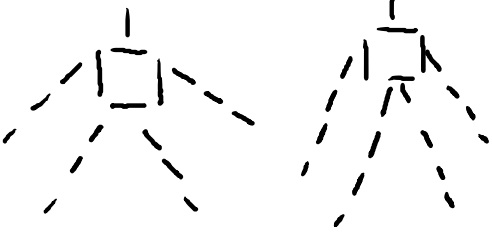## Evaluation: Plan for observing students

- A seating plan provided by the teacher
- Three observers, 6 students per observer, two observers using pen and paper with observation sheet (Appendix 2) and one observer using lessonnote.
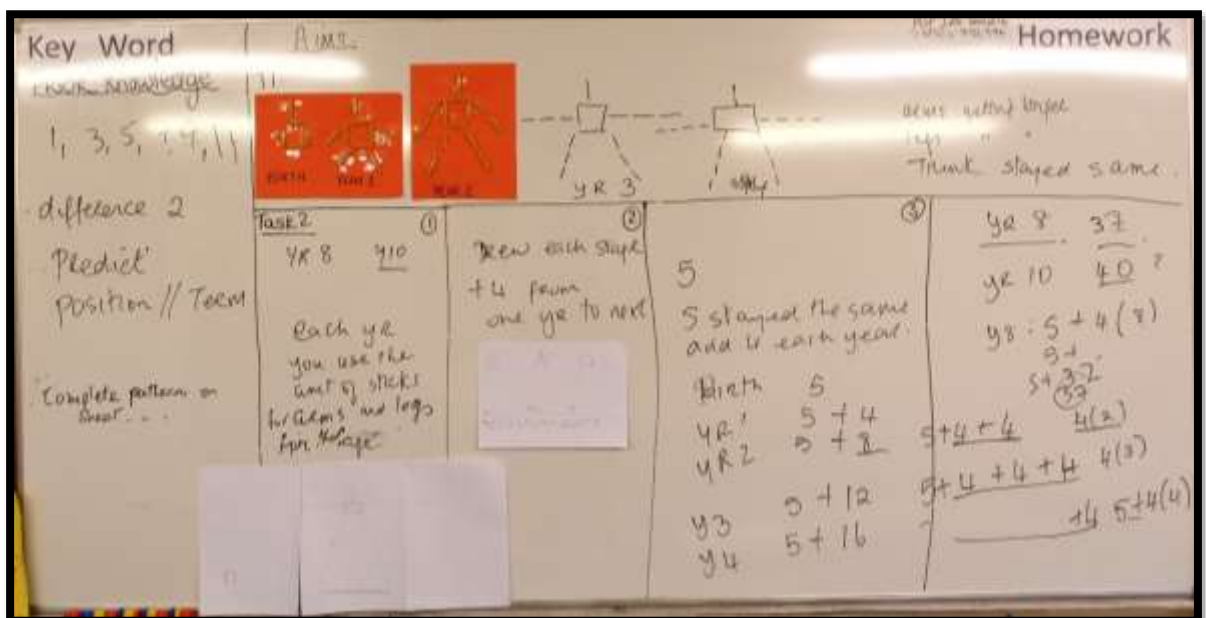- Types of student thinking and behaviour observers will focus on:

Introduction, posing the tasks	Can students recall the answer to all questions asked during the prior knowledge phase?
	Was wording of the task clear?
	Questions asked by students
Individual work/Pair work	What methods are used?
	Are prompts required?
	How long do students spend on the task?
	What kind of questions do students ask?
	Do they persist with the task?
Discussion	Are students attentive to what is happening on the board?
	Are clarifications needed to presenters' board work?
	Did the discussion promote student learning?

- Additional kinds of evidence collected (exemplars of students' work, photographs, end of lesson reflection)
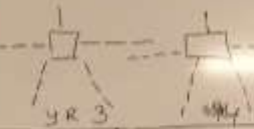
Board Plan

Prior Knowledge	Task 1:			Follow on
Revising Patterns: - Number patters (Linear) -Shape Patterns - Colour Patterns **Keywords:** "Difference" "Position" "Term"	Year 3 (Student 1) Year 4 (Student 2)			What stayed the same? What changed? Explain in words… Can we make this into a sum? … working to $T_n = dn + c$

	Task 2: How many matchsticks would it take to make Fred at Year 8 and Year 10?			
	Student 3	Student 4	Student 5	
	Either using drawing or Matchsticks to create Fred at these years and count the number of Matchsticks needed.	Using the fact that the limbs are increasing each year so starting at a known year (year 4) and adding on 4 each year to the number of matchsticks needed.	The body doesn't change – 5 Matchsticks The limbs increase by 4 each year. So 4 X Years + Body can be used to work out the number of matchsticks needed.	

What are the major patterns and tendencies in the evidence?

It is vital that students are aware of what they are trying to solve i.e. initial question must be very clear. Students must have clear understanding of what they are asked to do.

We found that all students got engaged in the problem solving and seemed to enjoy working out the patterns. We noticed that even the weaker students got very involved and enjoyed taking charge of their own learning with a few prompts from the teacher. It was also observed that many students took it upon themselves to solve the problem and the teacher was present only as a guide.

What are the key observations or representative examples of student learning and thinking?

Students were able to make Fred at his various birthdays. Students were able to ask for the correct amount of matchsticks needed to make Fred. Most students were able to make a link between Fred's body and his birthday.

As the lesson was being planned, those involved were split with their opinions of how much the students would be do with the problem. Some members felt that getting to the sum (4 times years + 5) would never happen with a first year class, while others felt that some students could get to this. Everyone was very pleasantly surprised when many students came up with this idea. This is a great example of how sometimes, we can, as teachers, underestimate what students can do.

What does the evidence suggest about student thinking such as their misconceptions, difficulties, confusion, insights, surprising ideas, etc?

Students were all able to make Fred. All of them used method 2 (Which was to work from the years we knew and add 4 on for each extra year) to come up with the number of matchsticks to make Fred. Some students lacked the vocabulary to describe Fred accurately at each birthday.

In what ways did students achieve or not achieve the learning goals?

Students achieved their learning goals as all of them got very involved in the problem solving. It made students look practically at patterns. Students took control of their own learning.

Based on your analysis, how would you change or revise the lesson?

1. Issue/Element that you would change: The size of the desks as they were too small to make Fred.

Recommendation: Students could be asked in advance to clear their desks of everything else. Since students were working in pairs they could also be warned that Fred could get very big and to work across both desks. Double desks would be ideal for this exercise.

2. Issue/Element that you would change: Students should have been brought up to the board to display their answers.

Recommendation: During the flow of the lesson and the layout of the class students were not brought up to the bored, but this meant the lesson still felt teacher led. The students should be brought up to the board to show their work and to talk about how they came up with this solution. Even though the students could not bring any matchstick creations to the board, an ipad could be used to take a picture and display it using the projector.

3. Issue/Element that you would change: More focus could have been placed on getting students to think about the number of matchsticks needed. During the lesson, the students were simply asked to draw/create Fred rather than "How many matchsticks are needed to build Fred…".

Recommendation: Lesson plan should be followed. The task planned was added in at the end, but students did not have enough time to work through it. It would be useful for the teacher teaching the lesson to have a summary of the flow somewhere visible to avoid this.

4. Issue/Element that you would change: It was hard for the teachers to hand over control to the students, yet it was very successful.

Recommendation: More time with the students beforehand on that topic. The more lessons that are carried out giving the students control would make this easier. The teachers involved should be aware of the extra effort that is required to ensure you don't inadvertently take back the leading of the lesson beyond what was planned.

What are the implications for teaching in your field?

The lesson allowed differentiated teaching to happen by choice. Mixed ability group worked better using this method. Both weaker and stronger students were able to fully engage with the lesson without need for a different task, or without some students being bored or left behind with the work.

Appendix 1. Worksheet for Task 1

At Birth

At Year 1

At Year 2
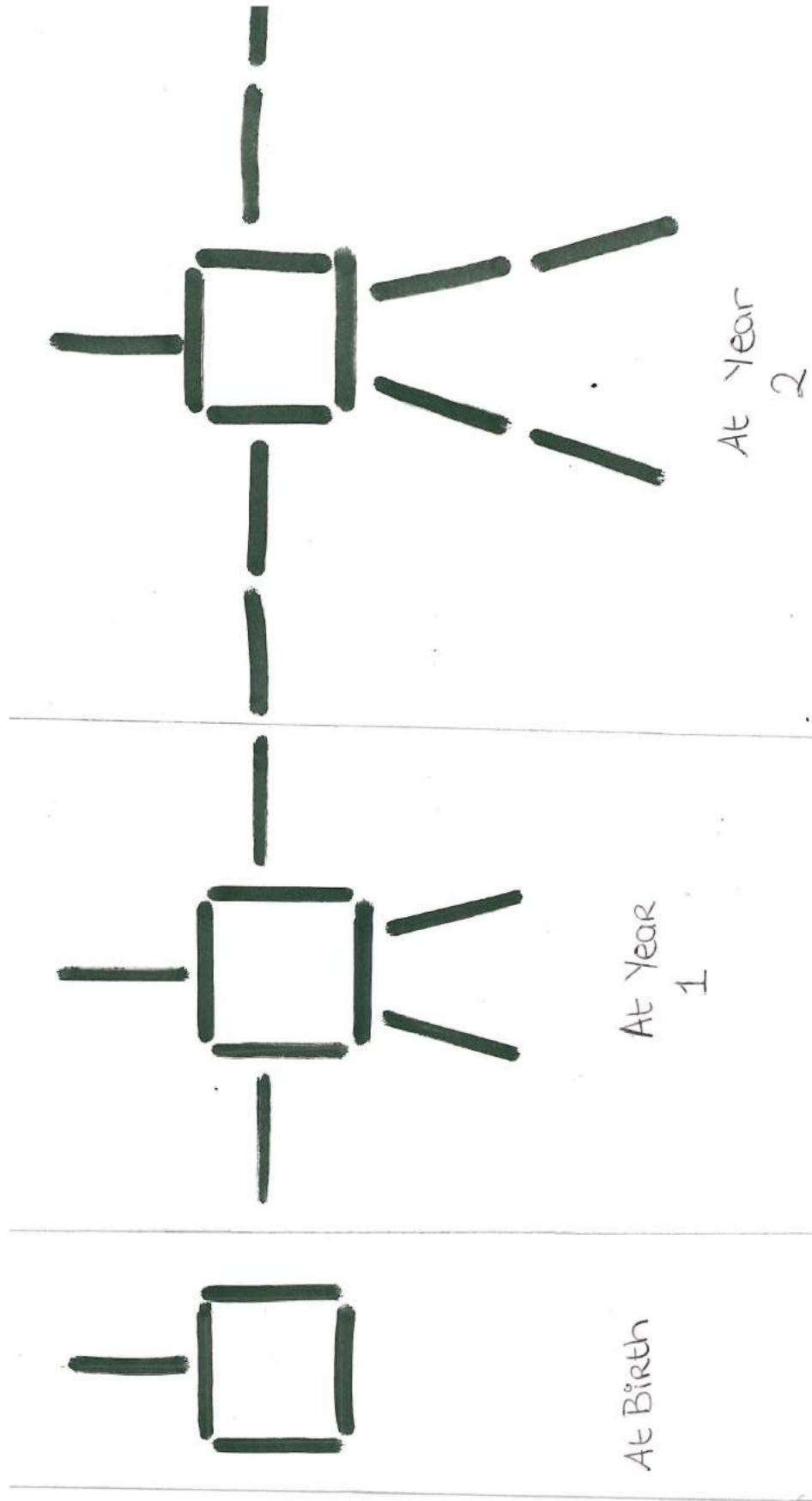
Appendix 2. Observation Sheet

<u>Lesson Observation</u>

Observation of student responses	Student 1	Student 2	Student 3	Student 4	Student 5	Student 6	Student 7
Fred age 3							
Fred age 4							
Task 2							
Method 1 (drawing all stages – counts matchsticks)							
Method 2 (Just one stage – counts matchsticks)							
Method 3 (Using 4X8 + 5)							
Other Methods							
Other observations							

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	Student 1	Student 2	Student 3	Student 4	Student 5	Student 6
Were the students engaged throughout the lesson? 1-Very little engagement 2-Some engagement 3-Engaged Comment on any other observations						

Questions asked by students:

What changes do you think need to be made to the lesson plan?

Did you notice any problems that could possibly arise again in future classes that may limit understanding of the topic?