Lesson Research Proposal for Leaving Cert year 1
Geometry, Trigonometry, Area and Volume, Number and Algebra.

For the lesson on 5th Feb 2018
At Pobalscoil Inbhear Scéine, Kenmare
Geraldine Foleys class
Instructor: Geraldine Foley
Lesson plan developed by: Geraldine Foley,
Maureen Foley-Hayes, John O' Connor.

Title of the Lesson: 2D to 3D… That is the Question?

Brief description of the lesson

Students will be presented with a problem in which they need to select and use appropriate knowledge from the different strands in maths, and decipher that which is most useful in solving a problem in the <u>most efficient</u> way possible**.**

Research Theme
For the School Improvement Plan (SIP), 2016 – 2020, Pobalscoil Inbhear Scéine has chosen as the main focus for this year, 2017-18.

- Teachers' plans identify clear, relevant learning intentions that are contextualised to students' learning needs.
- Teachers plan for assessing students' attainment of the learning intentions of the lesson, or series of lessons, using both assessment of learning and assessment for learning.

This lesson study provides an ideal opportunity to focus on these priorities.

Background & Rationale
From discussing the 2015 chief examiners report and meetings of our own Department, the maths teachers feel that:

- Students have '**Difficulty with questions which required them to draw on multiple strands of the syllabus at once'**
- That we, the teachers should **'provide opportunities for students to apply the skills and knowledge from one strand to material from another strand.'**
- While compartmentalising knowledge may help keep it organised, it will restrict the ability to cope with unfamiliar questions, particularly those requiring the synthesis of knowledge and skills from several strands.
- Students should be encouraged to persevere in these types of question – if the initial attempt does not work, they should be prepared to try the question a different way.

Based on our discussions and research findings we have chosen to base our unit and research lesson on creating a more productive approach in our teaching to address the issues outlined above.
We have agreed that:

1. While students can often handle questions well when given at least some dimensions, we recognise that they regularly have difficulties thinking/working with variables particularly in the area of problem solving in geometry.
2. Problem solving tasks that require students to activate mathematical thinking processes as

opposed to imitative thinking processes often cause major difficulties for the majority of students.

3. Students often display difficulties in applying their knowledge and skills to solve problems in unfamiliar contexts.

Relationship of the Unit to the Syllabus

Related prior learning Outcomes	Learning outcomes for this unit	Related later learning outcomes
JCHL Strand 2: Geometry and Trigonometry Students should be able to: 2.1 Synthetic Geometry Recall the axioms as well as apply the results of all theorems, converses and corollaries to solve problems. Complete the constructions specified. 2.3 Trigonometry Apply the theorem of Pythagoras to solve right angled triangle problems. Use trigonometric ratios to solve problems involving surds. JCHL Strand 3: Number 3.1 Number systems Use the binary operations of N, Z, Q and R/Q. Appreciate the Order of Operations. 3.4 Applied Measure Model real world situations and solve a variety of problems involving surface areas, and volumes of cylinders, spheres, cones and prisms. JCHL Strand 4: Algebra 4.6 Expressions Re-arrange formulae.	Students will develop and re-inforce their problem-solving skills. Students should be able to: Apply their knowledge and skills to solve problems in familiar and un familiar contexts. Explain why the procedures they apply are mathematically appropriate. Devise, select and use appropriate mathematical models, formulae or techniques to process information and hence draw relevant conclusions. Analyse a problem and break it down into manageable steps, reflect on their strategies and those of others and adjust their own approaches where necessary.	The general problem solving skills acquired in this unit can be applied across all strands of the syllabus and on to further study of mathematics.

Goals of the Unit

In examining any geometrical problem posed the students will use an integrated approach in extracting and identifying the relevant areas in maths required to solve the problem. Students will have the skills to gather and select confidently, their knowledge from relevant strands/topics to solve problems in unfamiliar contexts.
They will develop a deeper understanding of the connections between topics and strands of the syllabus in order that they can become better problem solvers.

Unit Plan

Lesson	Learning goals and task
1	Revise JC Geometry & Constructions.
3	Investigate constructions: 16, 17, 21 and 22 and theorem 21
4	Trigonometry-Revise JC Trigonometry and Investigate solving non-right angled triangles using the sine rule.
5	Investigating Cosine Rule.
6	Research lesson -Solving a 3D problem
7	Investigate the Surface Area of the 3D Problem in previous lesson.
8	Extension to other 2D and 3D problems.

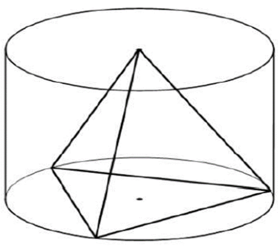
Goals of the Research Lesson:
a) Mathematical Goals

1. Students will use the theorem of Pythagoras, trigonometric ratios, sine rule, and cosine rule appropriately in context.
2. Students will recall and apply suitable axioms, theorems, corollaries, and constructions
3. Students will manipulate formulae accurately to write one variable in terms of another.
4. Students will know how to apply surds in context.
5. Students will link variables to efficiently find solutions.

b) Key Skills
This lesson will implement and promote the Senior Cycle key skills in the following ways:

1. Information processing: Students will be presented with tasks that will require them to make sense of, or interpret, the information to which they are exposed.
2. Critical and creative thinking: Students will engage in an investigative process where they will critically evaluate information and think creatively about it. They will be required to solve problems in a variety of ways, exploring options and alternatives, and making judgements about their choices.
3. Communicating: During the lesson students will present and discuss approaches and solutions to the problem.
4. Being personally effective: Students will engage in a problem solving task in an environment where all student’s solutions and contributions are valued, thus providing opportunities for students to develop their self-confidence and personal effectiveness.

Flow of the Research Lesson:

Learning Activities Teacher's Questions and Expected Student Reactions	Teacher Support	Assessment
Introduction 5 mins **Learning Intention:** Select and use appropriate knowledge from any number of areas/topics in maths and decipher that which is useful in solving a problem posed in the <u>most efficient</u> way possible.		
Posing the Task The teacher introduces the problem placing it on the board, and also distributing it to the students. The teacher tells students to 'read and examine the problem carefully.'	The teacher hands out a copy of the problem to the students.	
Problem  A wooden puzzle consists of several pieces that can be assembled to make a regular tetrahedron. A regular tetrahedron has four faces each of which is an equilateral triangle. The manufacturer wants to package the assembled tetrahedron in a clear cylindrical container, with one face flat against the bottom and the apex just touching the top. The length of one edge of the tetrahedron is $2a$. Find the volume of the container in terms of a and π.		
Clarify the problem and initiate prior learning 5 mins Teacher states that this question explicitly asks for volume of the container in terms of a and π, and asks: 'What are we looking for and what form do you think the answer would take?' Students suggest that we will use the formula for volume of a cylinder and that ' we will not have h or r in the answer, it is in terms of a and π only.' The teacher writes on the board:		Are students clear about what is being asked in the question? Are they engaged?

<table>
<tr>
<td>

$$V = \pi r^2 h$$

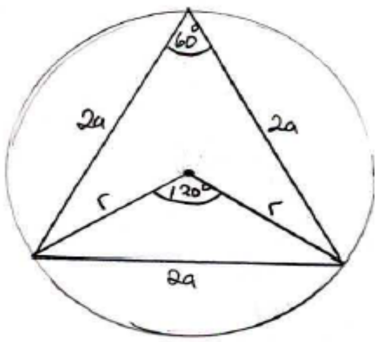
and $$V = \ldots\ldots \pi a \text{ units}^3$$

The teacher asks ‘How many flat surfaces can you see, and is there any one in particular that might be best to use as a starting point?’
After some suggestions, students realise that to start at the base of the container and firstly to find r in terms of a is the best option.

Task 1
Teacher instructs students to:
‘Use worksheet 1 to find the radius r in terms of a in as many ways you can. Work individually or in pairs.’
Students are also instructed to leave their answers in surd form and quickly elicit from students the reason for this.

Student individual work. 10 mins

Task 1
Response 1
Cosine Rule



$(2a)^2 = r^2 + r^2 - 2(r)(r)\cos 120^\circ$
$4a^2 = 2r^2 - 2r^2(-\frac{1}{2})$
$4a^2 = 2r^2 + r^2$
$4a^2 = 3r^2$
$\sqrt{\frac{4a^2}{3}} = r^2$
$\frac{2a}{\sqrt{3}} = r$

</td>
<td>

Task 1

The teacher tells students that they may be called on to present their solutions on the board and that they may be asked to offer their suggestions or explain another’s solution at any time during the class.

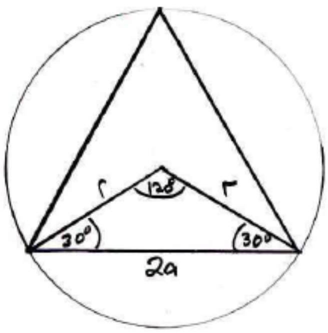
The teacher moves around the classroom observing students work.
Observe if students can carry out procedures accurately (sine rule, cosine rule, trigonometric ratios, manipulating formulae simplifying fractions involving surds. Some <u>limited</u> help may be given here but any misconceptions will be dealt with during ceardaíocht.

</td>
<td>

It is expected that students will vary in their ability in finding different solutions to the problem posed but it would be appropriate to continue if it is observed that the majority of students can achieve a reasonable level of success.

</td>
</tr>
</table>

Response 2
Sine Rule



$$\frac{r}{\sin 30^\circ} = \frac{2a}{\sin 120^\circ}$$

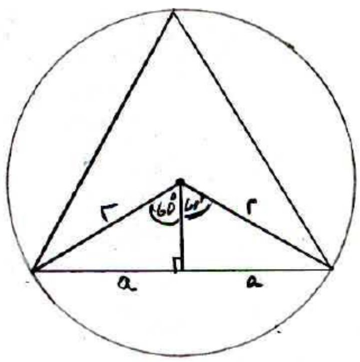
$$r = \frac{2a \sin 30^\circ}{\sin 120^\circ}$$

$$r = \frac{2a\left(\frac{1}{2}\right)}{\frac{\sqrt{3}}{2}}$$

$$r = \frac{a}{\frac{\sqrt{3}}{2}} \cdot \frac{\frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}}}$$

$$r = \frac{2a}{\sqrt{3}}$$

Response 3
Trigonometric ratios.



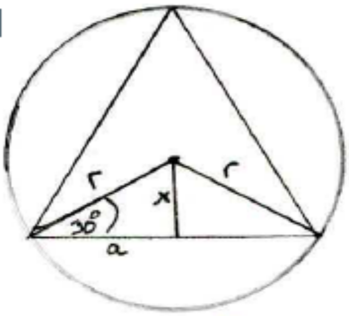
$$\sin 60^\circ = \frac{a}{r}$$

$$r \cdot \frac{\sqrt{3}}{2} = a$$

$$r = \frac{a}{\frac{\sqrt{3}}{2}}$$

$$r = \frac{2a}{\sqrt{3}}$$

Response 4
Trigonometric ratios.



$$\cos 30^\circ = \frac{a}{r}$$

$$r\cos 30^\circ = a$$

$$r = \frac{a}{\cos 30^\circ}$$

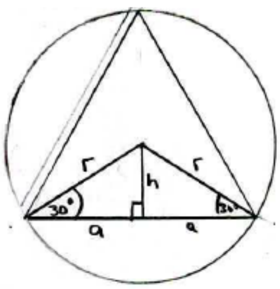
$$r = \frac{a}{\frac{\sqrt{3}}{2}}$$

$$r = \frac{2a}{\sqrt{3}}$$

$$r = \frac{2a}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$r = \frac{2\sqrt{3}a}{3}$$

Response 5
Trigonometric Ratios done twice with a third variable *h* introduced and equating to get *r*.



$$\sin 30^\circ = \frac{h}{r}$$

$$r\sin 30^\circ = h$$

$$r = \frac{h}{\sin 30}$$

$$r = \frac{h}{\frac{1}{2}}$$

$$r = 2h$$

$$\frac{r}{2} = h$$

$$\tan 30^\circ = \frac{h}{a}$$

$$a\tan 30^\circ = h$$

$$a = \frac{h}{\tan 30^\circ}$$

$$a = \frac{h}{\frac{1}{\sqrt{3}}}$$

$$a = \sqrt{3}h$$

$$\frac{a}{\sqrt{3}} = h$$

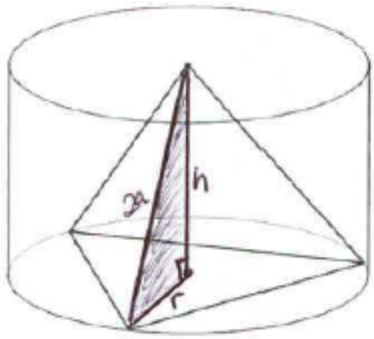
$$\Rightarrow \quad \frac{r}{2} = \frac{a}{\sqrt{3}}$$

$$r = \frac{2a}{\sqrt{3}}$$

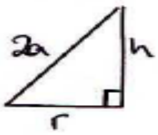
Ceardaíocht/Comparing and Discussing **Task 1** 10 mins Pre-selected students come to the board and present their solutions to the class in the following order according to the method they used to solve the problem: Response 1 Using the cosine rule. Large triangle equilateral so all angles measure 60° and all sides measure 2a. Theorem-The angle at the centre of a circle standing on a given arc is twice the angle at any point of the circle standing on the same arc. Equilateral triangle so angle at centre is twice 60°. Response 2 Using the sine rule. A circle can be used to construct any polygon, the angle at centre = *360°/ n* where *n*= number of sides of the polygon so here polygon is a triangle - 3 sides→angle = *360°/3 =120°*. Isosceles triangle: two radii as arms so angles opposite *r = 30°* Response 3 or 4 Using trigonometric ratios. Construction of circumcircle- circumcentre is intersection of the perpendicular bisector of sides. Links variables *a* and *r*. During ceardaíocht the teacher places post-it notes on the board alongside the students work to highlight the theorem, construction, algebra used in the solution. The results $r = \frac{2a}{\sqrt{3}}$ or $r = \frac{2\sqrt{3}a}{3}$ or both are posted on each solution also. **Task 2** The teacher refers to the board work from Task 1 and asks students to: 'Use worksheet 2 to find h in as many ways as possible.'	The teacher encourages students to present their solutions clearly and, that all other students are engaged fully in the class. The teacher will ask questions to check students understanding of the presenter's work. An example of questions that could be asked are: 'Has anyone else used this method to write r in terms of a?' 'Can you explain how you decided that the angle must be……..?' 'Can you prove to me that….?' 'Does it matter which triangle……?' Is it always true that….?' 'How did know that …….?'	Are students defending their ideas? Are they responding to each other's ideas? Can they explain their ideas well? Are they using mathematical procedures accurately? Can they justify their methods?

Student Individual work 10 mins

Response 1
3D diagram and one edge and radius. (1 step)



Pythagoras theorem



$$(2a)^2 = h^2 + r^2 \qquad r = \frac{2a}{\sqrt{3}}$$

$$4a^2 = h^2 + \frac{4a^2}{3}$$

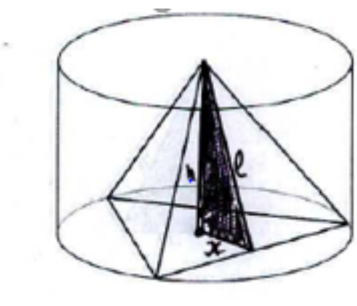
$$4a^2 - \frac{4a^2}{3} = h^2$$

$$\frac{8a^2}{3} = h^2$$

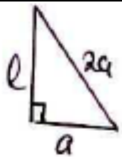
$$\sqrt{\frac{8}{3}}\,a = h$$

$$h = \frac{2\sqrt{6}a}{3}$$

Response 2
3D diagram and slant height of face. (3 Steps)



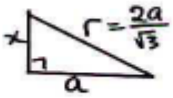
l = slant height



$$(2a)^2 = l^2 + a^2$$
$$3a^2 = l^2$$
$$l = \sqrt{3}a$$

Find *x*, the perpendicular from the base edge to the centre of the circle

$$x = ?$$

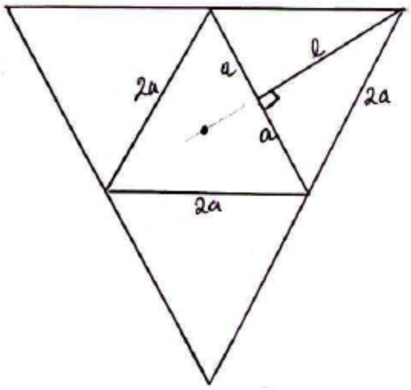


$$r^2 = x^2 + a^2$$
$$\left(\frac{2a}{\sqrt{3}}\right)^2 - a^2 = x^2$$
$$\frac{4a^2}{3} - a^2 = x^2$$
$$\frac{a^2}{3} = x^2$$
$$x = \frac{a}{\sqrt{3}}$$

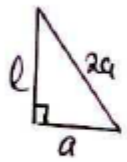
Find *h* the perpendicular height

$$h^2 = l^2 - x^2$$
$$h^2 = (\sqrt{3}a)^2 - \left(\frac{\sqrt{3}a}{3}\right)^2$$
$$h^2 = 3a^2 - \frac{a^2}{3}$$
$$h^2 = \frac{9a^2 - a^2}{3}$$
$$h^2 = \frac{8a^2}{3}$$
$$h = \sqrt{\frac{8}{3}}a$$
$$h = \frac{2\sqrt{6}a}{3}$$

Response 3
Using the net of the tetrahedron. (3 steps)



Finding the slant height *l*



l = slant height

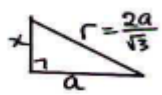
$(2a)^2 = l^2 + a^2$

$3a^2 = l^2$

$l = \sqrt{3}a$

Find *x*, the perpendicular from the base edge to the centre of the circle.

$x = ?$



$r^2 = x^2 + a^2$

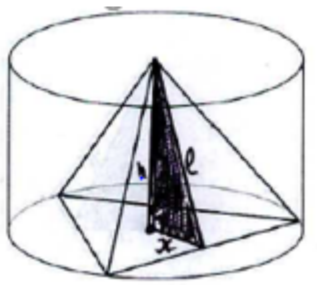
$\left(\frac{2a}{\sqrt{3}}\right)^2 - a^2 = x^2$

$\frac{4a^2}{3} - a^2 = x^2$

$\frac{a^2}{3} = x^2$

$x = \frac{a}{\sqrt{3}}$

Find *h* the perpendicular height.



$$h^2 = l^2 - x^2$$

$$h^2 = (\sqrt{3}a)^2 - \left(\frac{\sqrt{3}a}{3}\right)^2$$

$$h^2 = 3a^2 - \frac{a^2}{3}$$

$$h^2 = \frac{9a^2 - a^2}{3}$$

$$h^2 = \frac{8a^2}{3}$$

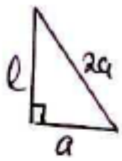
$$h = \sqrt{\frac{8}{3}}a$$

$$h = \frac{2\sqrt{6}a}{3}$$

Response 4 (3 Steps)

Using any one face of the tetrahedron to find the slant height *l*.

l = slant height

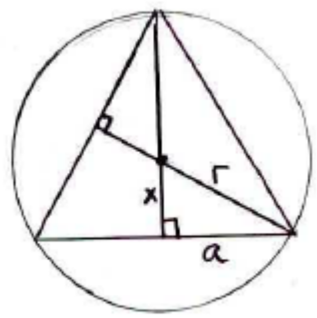


$$(2a)^2 = l^2 + a^2$$

$$3a^2 = l^2$$

$$l = \sqrt{3}a$$

Find *x* on using the circumscribed base of the tetrahedron.



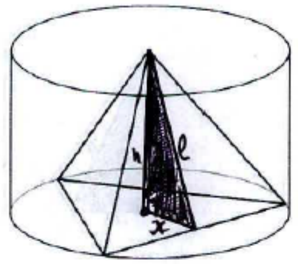
All Equilateral Δ's $\Rightarrow$

Circumcentre = centroid

$\div$'s median in ratio 2:1

$\Rightarrow \quad x = \frac{1}{3}l$

$x = \frac{\sqrt{3}a}{3}$



$h^2 = l^2 - x^2$

$h^2 = (\sqrt{3}a)^2 - \left(\frac{\sqrt{3}a}{3}\right)^2$

$h^2 = 3a^2 - \frac{a^2}{3}$

$h^2 = \frac{9a^2 - a^2}{3}$

$h^2 = \frac{8a^2}{3}$

$h = \sqrt{\frac{8}{3}}a$

$h = \frac{2\sqrt{6}a}{3}$

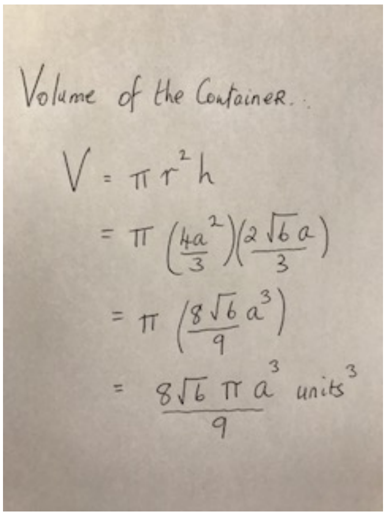
Ceardaíocht;Comparing and Discussing

10 mins

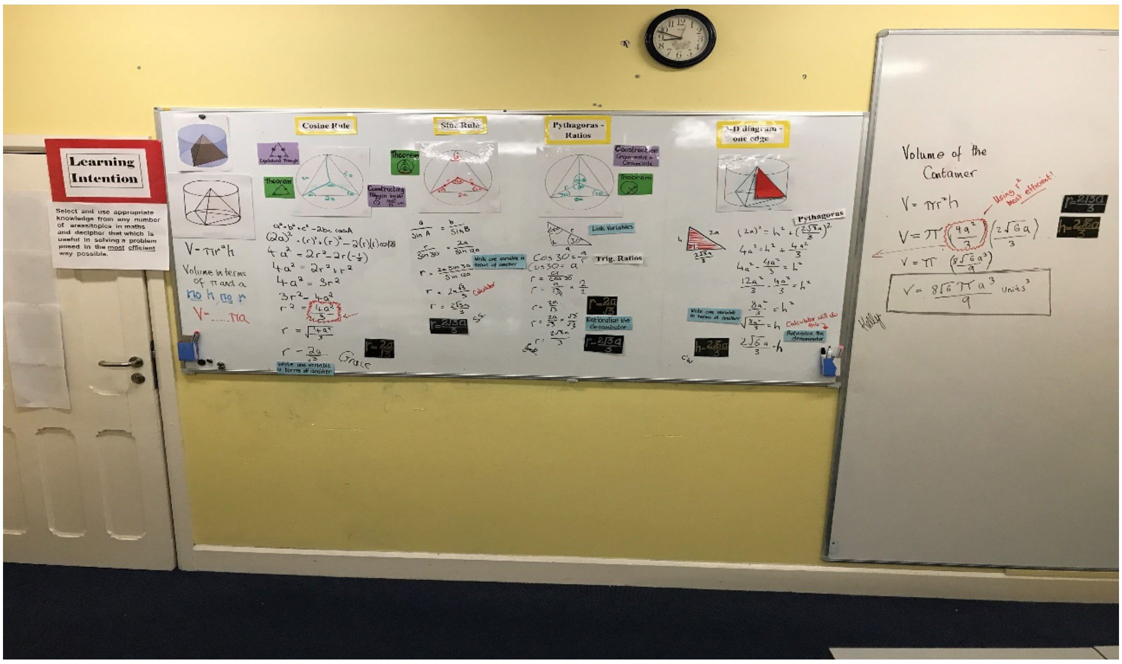
Task 2

Students present, on the board, their solutions to the class in the following order according to the method used to solve the problem:

Again teacher encourages students to present their solutions clearly and, that all other students are engaged fully in the class noting any misconceptions they may have had.

Response 1. The 3- D diagram, radius and any one edge is used. Pythagoras theorem. Response 2. The 3-D diagram and the slant height of one face and the perpendicular from the centre to the base edge. Response 3 The net and slant height of a face and the perpendicular from the centre to the base edge. **Task 3** **5 mins** The teacher instructs the students that having found the height and radius in terms of a to now go to worksheet 3 and find the volume of the cylinder in terms of a and π. Response 1 	Teacher emphasises the importance of being able to rationalise the denominator themselves as well as using the calculator to write $\sqrt{\frac{8}{3}}$ in its simplest form. Teacher elicits from the students the efficiency of response 1 in comparison to all the extra steps required in the other solutions. The last student will be asked to present his/her work on the final part of the task: finding the volume of the cylinder. Emphasis is put on using $units^3$ in the answer. Substituting for r^2 directly from **Task 1** would be nice and efficient here.	
Summing up & Reflection **5 mins** Students then are asked to fill the reflection sheet to reflect on their learning. Some students will share their reflections.	The board work will be used to recap on the work done. Students will suggest which methods they feel was most efficient in solving the problem posed. As an extension, students will be asked to find the surface area of the tetrahedron as a lead on to the following class where the nets of polygons will be developed.	

Board Plan



Evaluation

Did the lesson go as planned?

The beginning of the lesson was designed to guide (without leading too much) the students towards an efficient starting point. We anticipated that students would find this starting point more quickly and the discussion took 15 minutes and not 5-6 as planned. The remainder did go to plan. It succeeded in terms of the broad theme of using a defined Lesson Intention before, during and after the lesson to give focus to assessment of and assessment for learning and was also successful in linking various strands of the syllabus.

Did the Activities support the intention of the lesson?

Pair work promoted good discussion and learning from each other. The worksheets included a diagram to give focus and did keep this part of lesson on target. The pre and post reflections did highlight the broad range of mathematics involved in solving 3D problems.

Was the flow of the lesson coherent?

It took longer to get into it but the rest did flow. From the moment that the students realised that they were looking for *r* and *h*, they knew exactly what they had to achieve and worked very well. The student Board work went to plan and there was good comparison and discussion of their various methods in Task 1 of problem. This helped the students to solve Task 2 efficiently albeit, using only one method.

How were students solving the problem?

Individually first and then in pairs, they came up with 3 -4 different solutions. Most came up with just two methods, mainly the Sine Rule and Cosine Rule. Trigonometric ratios were used to a lesser extent.

Manipulating surds and algebraic equations was well handled by most. The volume of maths and linking to various topics and strands was impressive.

Overall, they engaged with the problem and with each other really well and learned a lot from each other.

Did student's presentation and discussion promote their thinking and learning?

It took a long time to write out the solutions on the Board and a lack of practice at explaining their work was evident by some. The students clearly understood their chosen method well. The class did engage with the questions posed by the teacher.

What did the students learn?

They learned that there is more than one way to solve a problem and the importance of being be able to pull from different strands in problem solving. The importance of algebra and number skills (calculator use in rationalising the denominator versus using the traditional method) was discussed during the lesson.

Was the time used effectively?
Students could have done with more time to come up with more solutions.

What will the students need tomorrow?

They will be asked to find the surface area of the tetrahedron and will need to find the slant height.

Reflection

- We learned that the lesson plan was over ambitious in terms of content for time allotted (1 hour). 2 lesson would be more suitable. A natural ending point to the first lesson would be when finding r in terms of a and π. With this extra time, further alternative solutions and "non-solutions/dead ends" could have been explored.

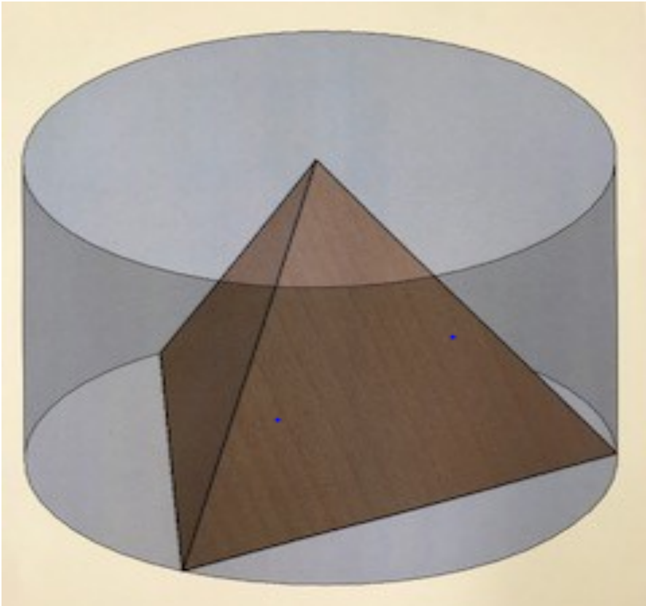
- On reflection, if doing the lesson again, it might have been more effective to get the students to write out their solutions on a large A3/A2 page with marker and stick it to the board rather than write them out. This would have saved some time and would have allowed the students to concentrate on presenting rather than writing. Another solution would have been to send photos of the work to a projector etc.

- The students 'basic maths' were very good and they used proper language throughout i.e. not cancelling but multiplying across.

- There was very good use of the board to link the learning.
- A large volume of maths were seen in the lesson which would also make it an effective revision lesson for 6th years.
- Alternative and less efficient attempts/solutions should be given time for consideration.
- The end of lesson summing up and reflection for the students could be simplified to 2 Stars and a Wish or an exit ticket as it would be more efficient.

Appendix1: Student Work Sheets

The Problem



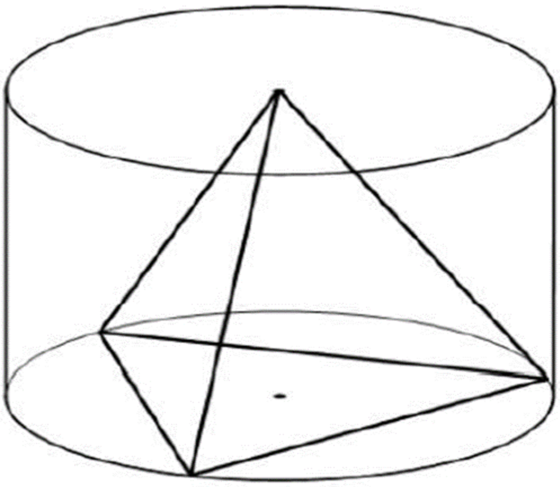
A wooden puzzle consists of several pieces that can be assembled to make a regular tetrahedron.A regular tetrahedron has four faces each of which is an equilateral triangle.

The manufacturer wants to package the assembled tetrahedron in a clear cylindrical container, with one face flat against the bottom and the apex just touching the top.

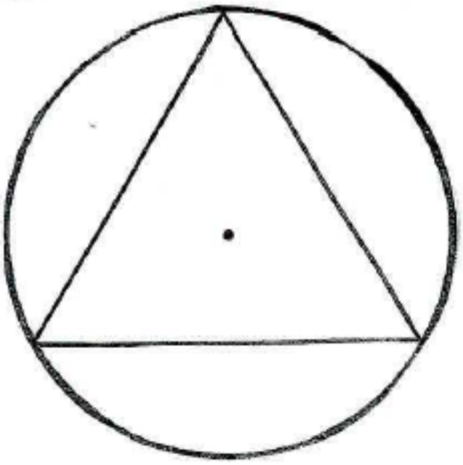
The length of one edge of the tetrahedron is $2a$.

Find the volume of the container in terms of a and π.

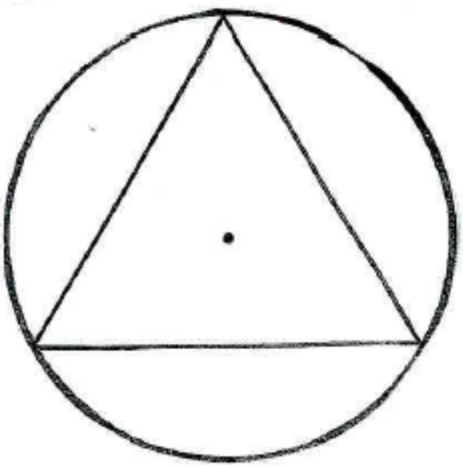
You may find the diagram below helpful to solve the problem.



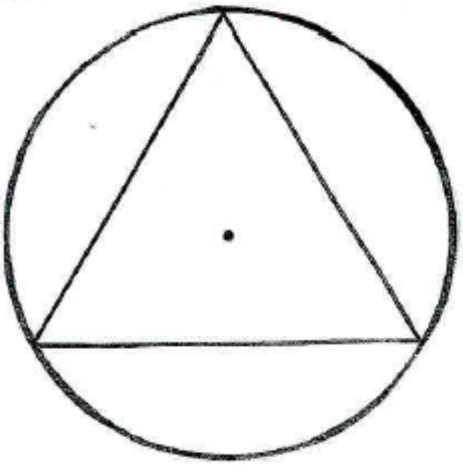
Solution 1



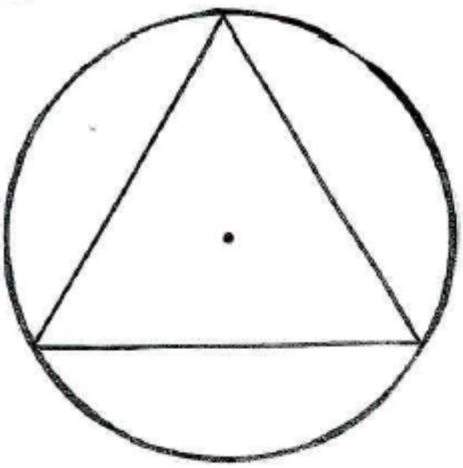
Solution 2



Solution 3



Solution 4



Reflection Sheet: ______________________________

Methods for 2 D part: (r in terms of a)	Maths knowledge used in this method
1: Cosine Rule	• Theorem 19: Angle at centre is twice angle at circle standing on same arc **or** • Construction: Polygons and Circles (360 /n sides) • Trigonometry: Cosine Rule • Algebra/Number: Manipulating Formula i.e. writing one variable in terms of another Rationalising the denominator i.e. no Surd in the denominator
2: Sine Rule	• Theorem: Isosceles triangle… • Constructions: __ • Trigonometry: Sine Rule • Rationalising the denominator i.e. no Surd in the denominator
3: Pythagoras/ Trig. Ratios	• Theorem: Perpendicular from centre bisects the chord • Constructions: Circumcentre and Circumcircle • Trigonometry: Sine Rule • Algebra/Number: Rationalising the denominator i.e. no Surd in the denominator
Method(s) for 3D part (h in terms of a) A. 3-D Diagram-one edge B. Net-slant height of face, l.	• Theorem of Pythagoras to find perpendicular height using one edge and radius: • Theorem of Pythagoras to find: 1) slant height, **l** using edge and half side of any face 2) **l** to find x (perp distance from centre to one edge on the base) 3) Use results 1, 2 to find h in terms of a

A. Circle which method <u>you</u> would use to most efficiently solve this problem?
B. Highlight 2 areas you need to work on, if any.

C. Any other interesting methods will be looked at later.