Lesson Research Proposal for 2nd Year Higher Level on the topic of 2D transforming to a 3D open rectangular box

For the lesson on: 24-1-18
Location: Cashel Community School, Ms Ann O'Sullivan's class
Instructor: Ann O'Sullivan
Lesson plan developed by: Ann O'Sullivan, Margaret Cully and Donal O'Dwyer

1. Title of the Lesson:

2D or not 2D?

2. Brief description of the lesson

The lesson focuses on students designing an open rectangular box when given the area of its base and total surface area of the box.

Research Theme

(a) Subject Knowledge:
It is expected that all teachers will have a deep understanding of all subject material that they are responsible for teaching where this lacking, it is the teacher's responsibility to acquire the requisite level of understanding through personal research (e.g. Textbooks, websites and research papers) and through collaboration with colleagues in the maths department (e.g through Lesson Study).

(b) **Pedagogical Knowledge:**
As a math's department we recognize that subject knowledge is not sufficient for effective teaching and learning, but that teachers require up to date knowledge of developments in math's pedagogy. To this end, and through participation in lesson study, teachers will work together to research and choose teaching and learning strategies which are most productive.

(c) **Classroom Management:**
We feel the primary goals of mathematical learning are understanding and problem solving. It is agreed that in the coming academic year we will use structured problem solving to introduce new concepts to students. These goals are inextricably related. We feel this can be best supported by problem solving. Problem solving will be central to learning experiences of our students throughout the year. We aim that at least 10% of lessons will clearly incorporate problem solving. We plan to increase this up to 20% in the following two years. The need for teachers to develop these skills is expected to be challenging, hence the gradual introduction of these practices.

3. Background & Rationale

a) Why you chose the topic
This lesson is aimed at Second Year Higher Level Junior Cycle students. The quality of answers to questions in this topic at Junior Cycle level varies greatly between students, once they understand the fundamental concepts of the topic. They struggle to apply their problem solving skills to questions that deviate from standard textbook exams.

b) Your research findings

Whilst teaching both 2d and 3d shapes we have observed the lack of a clear link between 2d and 3d shapes. Whilst students appear to be able to use the given formula, there is no understanding that 3d topics are in essence re-arranged 2d shapes.

Our lesson hopes to develop students understanding of this by highlighting how a 2d sheet can be turned into a 3d rectangular box (without a lid). Students are given guidelines on measurements and size of the given box to enable greater understanding on the ratio of height to total surface area.

4. Relationship of the Unit to the Syllabus

For mathematics, the Primary Curriculum and Post-Primary Syllabus can be accessed at:
Primary: http://www.ncca.ie/uploadedfiles/Curriculum/Maths_Curr.pdf
Junior Certificate: http://www.ncca.ie/en/Curriculum_and_Assessment/Post-Primary_Education/Project_Maths/Syllabuses_and_Assessment/JC_Maths_English_2013.pdf
3 Handbooks.
Leaving Certificate: http://www.ncca.ie/en/Curriculum_and_Assessment/Post-Primary_Education/Project_Maths/Syllabuses_and_Assessment/LC_Maths_English_2013.pdf

Related prior learning Outcomes	Learning outcomes for this unit	Related later learning outcomes
Primary: Shape & Space **Infant Class:** The child should be enabled to • sort, describe and name 2-D shapes: square, circle, triangle, rectangle collect and sort different shapes e.g. straight, curved, flat, side, corner make shapes with art straws, on geoboard draw shapes found in the environment. **First & Second Class:** 2-D Shapes- The child should be enabled to • sort, describe, compare and name 2-D shapes: square, rectangle, triangle, circle, semicircle describe shapes, referring to size, corners, number and length of sides. • construct and draw 2-D shapes • identify half and quarter of shapes discuss the relationship between halves and quarters. Symmetry: The child should be enabled to • identify line symmetry in shapes and in the environment fold shapes in	**Third Year**: Revision of second year relevant material. Investigating a square. Co-ordinate geometry - revision and extension of second year material. Trigonometry revision and problem solving. Axial Symmetry. Relevant Concepts: Axioms 1. [Two points axiom] There is exactly one line through any two given points. 2. [Ruler axiom] The properties of the distance between points 3. [Protractor Axiom] The properties of the degree measure of an angle 5. [Axiom of Parallels] Given any line l and a point P, there is exactly one line through P that is parallel to l. Theorems: 2. In an isosceles triangle the angles opposite the equal sides are equal. Conversely, if two angles are equal, then the triangle is isosceles. 3. If a transversal makes equal alternate angles	**Fifth & Sixth Year** **2.1 Synthetic geometry** Leaving Certificate students will be expected to understand the meaning of the following terms related to logic and deductive reasoning: Theorem, proof, axiom, corollary, converse, implies, is equivalent to, if and only if, proof by contradiction. A knowledge of the Axioms, concepts, Theorems and Corollaries prescribed for JC-HL will be assumed. Students will study all the theorems and corollaries prescribed for LC-OL, but will not, in general, be asked to reproduce their proofs in examination. However, they may be asked to give proofs of the Theorems 11, 12, 13, concerning ratios, which lay the proper foundation for the proof of Pythagoras studied at JC, and for trigonometry. They will be asked to solve geometrical problems (so-called "cuts") and write reasoned accounts of

half, blob and fold paintings complete shapes or pictures symmetrically collect and sort objects on the basis of symmetry. Angles-The child should be enabled to • explore and recognize angles in the environment; investigate angles as corners; use a right angle made from card to 'measure' corners on 2-D shapes; discuss relate square corner to quarter turn. **Third & Fourth Class:** 2-D Shapes: The child should be enabled to • explore, describe and compare the properties (sides, angles, parallel and non-parallel lines) of 2-D shapes • construct and draw 2-D shapes using a ruler and set square. • solve and complete practical tasks and problems involving 2-D shapes. Symmetry- • use understanding of line symmetry to complete missing half of a shape, picture or pattern in drawings, on geoboard or pegboard where the fold is vertical, horizontal or diagonal. Lines & Angles- The child should be enabled to • identify, describe and classify vertical, horizontal and parallel lines discuss and describe lines in the environment draw and label lines • recognize an angle in terms of a rotation form angles by opening books and doors, by rotating clock hands and geostrip arms, by physically turning (clockwise/anti-clockwise), or on computer • classify angles as greater than, less than or equal to a right angle • solve problems involving lines and angles •	on two lines then the lines are parallel, (and converse). 4. The angles in any triangle add to 180˚. 5. Two lines are parallel if and only if, for any transversal, the corresponding angles are equal. 9. In a parallelogram, opposite sides are equal and opposite angles are equal (and converses). 10. The diagonals of a parallelogram bisect each other. 13. If two triangles are similar, then their sides are proportional, in order (and converse). 14. [Theorem of Pythagoras] In a right-angled triangle the square of the hypotenuse is the sum of the squares of the other two sides. 15. If the square of one side of a triangle is the sum of the squares of the other two sides, then the angle opposite the first side is a right angle. Corollaries: 1. A diagonal divides a parallelogram into 2 congruent triangles. Constructions: 1. Bisector of a given angle, using only compass and straight edge. 5. Line parallel to a given line, through a given point 9. Angle of a given number of degrees with a given ray as one arm. 10. Triangle, given lengths of three sides 11. Triangle, given SAS data 12. Triangle, given ASA data 13. Right-angled triangle, given the length of the hypotenuse and one other side. 14. Right-angled triangle, given one side and one of the acute angles (several cases). 15. Rectangle, given side lengths.	the solutions. These problems will be such that they can be attacked using the given theory. The study of the propositions may be a useful way to prepare for such examination questions. **2.2 Co-ordinate geometry** Students should be able to– use slopes to show that two lines are • parallel • perpendicular – solve problems involving slopes of lines– recognize that $(x-h)^2 + (y-k)^2 = r^2$ represents the relationship between the x and y co-ordinates of points on a circle with center (h, k) and radius r. solve problems involving • the perpendicular distance from a point to a line • the angle between two lines – divide a line segment internally in a given ratio m: n. Students should be able to – use of the theorem of Pythagoras to solve problems (2D only) –solve problems using the sine and cosine rules (2D) – define sin θ and cos θ for all values of θ – define tan θ – work with trigonometric ratios in surd form – use trigonometry to solve problems in 3D– use the radian measure of angles –

use straws to construct oblique and perpendicular lines and diagonals in a square, **Fifth & Sixth Class:** 2-D Shapes: The child should be enabled to • make informal deductions about 2-D shapes and their properties • use angle and line properties to classify and describe triangles and quadrilaterals. • Construct triangles from given sides or angles. • classify 2-D shapes according to their lines of symmetry • plot simple co-ordinates and apply where appropriate use geoboards and squared paper • use 2-D shapes and properties to solve problems. Lines & Angles- The child should be enabled to • recognize, classify and describe angles and relate angles to shape and the environment recognize angles in terms of a rotation examine, measure and record the angles (including the reflex angle) formed by the hands of a clock at a variety of different times extend by using manipulatives, e.g. straws, lollipop sticks, Meccano, string, 360° protractor • estimate, measure and construct angles in degrees measure and record a wide variety of angles using a protractor construct angles of various sizes using a protractor estimate angle sizes and check by measuring with a protractor. • explore the sum of the angles in a triangle. **Post Primary: Strand 2: Geometry & Trigonometry** **First Year:** Measuring angles,		

introduction to the protractor, axial symmetry, co-ordinate geometry and alternate angles. **Second Year:** Introduction to Pythagoras' Theorem Slope of a line. Revision of planes and points and angles, measuring of same and axial symmetry from first year. Properties of parallelograms – squares, rectangles, parallelograms etc. Scaled diagrams of triangles. Trigonometric ratios.		

5. Goals of the Unit

- Students will understand that some types of problems do not have a single solution depending on the dimensions of the base.
- Students may then apply their prior knowledge of 2d and 3d shapes
- Students will engage in a task where the solution is not immediately obvious.
- Students will discuss the problem in pairs and break it down into manageable steps.
- Students will see the direct links between a 2D and a 3D shape which they will construct in the class.

6. Unit Plan

Lesson	**Learning goal(s) and tasks**
1	Review their understanding of prior knowledge: • Dimensions • List 2D and 3D shapes • Area of square and rectangle. • Exploring the link and making connections between the area of a 2d rectangular and the multiplication of two and three digit numbers. • Exploring a link between multiplying algebraic expressions and area.
2	Finding the area of irregular 2d shape Introduction of nets of a rectangular solid.
3 The Research lesson	Introduce surface area in a problem solving context based on an open rectangular box.

7. Goals of the Research Lesson:

The Goals of the lesson should refer to:

a) Mathematical Goals

Describe cognitive or emotional changes within the student. Avoid "students will be able to…" statements; instead say what students need to know or understand and how that will enable to do….

- Understand the concept of 2d and 3d shapes.
- Understanding key words: length, width, height, depth, area, surface-area, net and volume.
- Distinguish between the units used for length, area and volume.

b) Key Skills and Statements of Learning

Refer to how the work carried out during the lesson targets key skills and statements of learning from the Junior Cycle framework.

In the planning and design of this lesson the Junior Cycle Key Skills and Statements of Learning have been considered. This lesson will implement and promote JC Key Skills in the following ways:

1. Being Literate: Students will have the opportunity to express their ideas clearly and accurately.
2. Being Numerate: It will develop a positive disposition towards problem solving.
3. Managing Myself: Student's will have the opportunity to reflect on their own learning.
4. Staying Well: Students' confidence and positive disposition to learning will be promoted.
5. Communicating: Students will present and discuss their mathematical thinking.
6. Being Creative: Students' will explore options and alternatives as they actively participate in the construction of knowledge.
7. Working with Others: Students will learn with and from each other.
8. Managing information and thinking: Students will be encouraged to think creatively and critically.

This lesson is also designed to meet the following JC Statements of Learning in particular:

1. The student communicates effectively using a variety of means in a range of contexts.

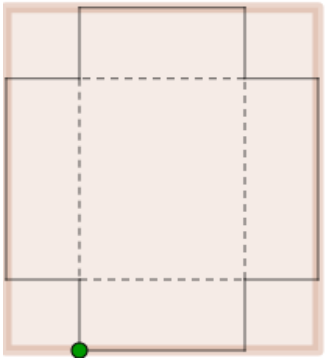
15. The student recognises the potential uses of mathematical knowledge, skills and understanding in all areas of learning.

16. The students describes, illustrates, interprets, predicts and explains patterns and relationships.

17. The students devises and evaluates strategies for investigating and solving problems using mathematical knowledge, reasoning and skills.

21. applies practical skills as she/he develop models and products using a variety of materials and technologies.

8. Flow of the Research Lesson:

Steps, Learning Activities Teacher's Questions and Expected Student Reactions	Teacher Support	Assessment
Introduction (5 mins) At the beginning of class, students are asked to list the factors of 48. Think Pair share the factors. Each pair receives a sheet of A3 graph paper with the problem posed. Students also receive rulers, scissors, pencils, markers, selotape, bluetac and A4 show me boards to help with rough-work process. Today we are going to use our mathematical knowledge to solve a problem. We're going to try to solve the problem in pairs and then we're going to come together as a class and use all your knowledge to learn something new…	Factors of 48: 1x48 2x24 3x36 4x12 6x8	Students begin to review what are the factors of 48. This is essential to solutions in problem posed.
Posing the Task (5 +10 minutes) John wants to design an open rectangular box. He was given a number of criteria by the packaging company: • The base of this box must be 48 units squared. • The dimensions of the base must be whole numbers • The total surface area of the open rectangular box must equal 160 units squared. **<u>In pairs,</u>** • Decide on the possible dimensions of the box that will meet the above criteria. • Draw a net of the open rectangular box based on the criteria given. **Clarifying the problem:** Geogebra could be used to help clarify the problem with a 2D <u>square</u> base transforming into a 3d open box. Students must then come up with a suitable <u>rectangular based</u> box to meet the above criteria.	Present on the board an illustration to make the meaning of the problem easy to understand. An open box already made could be used as a real life visual aid.  https://www.geogebra.org/m/sWvJYnkd	Do students understand the task? (if they don't, it's probably not a good idea to move on) Some pairs may need guidance and further explanation. Are students eager to solve the problem? Some students may approach this as a challenge and others may see it as confusing and uncertain. Students will be informed they will be given 10 minutes to complete the task.

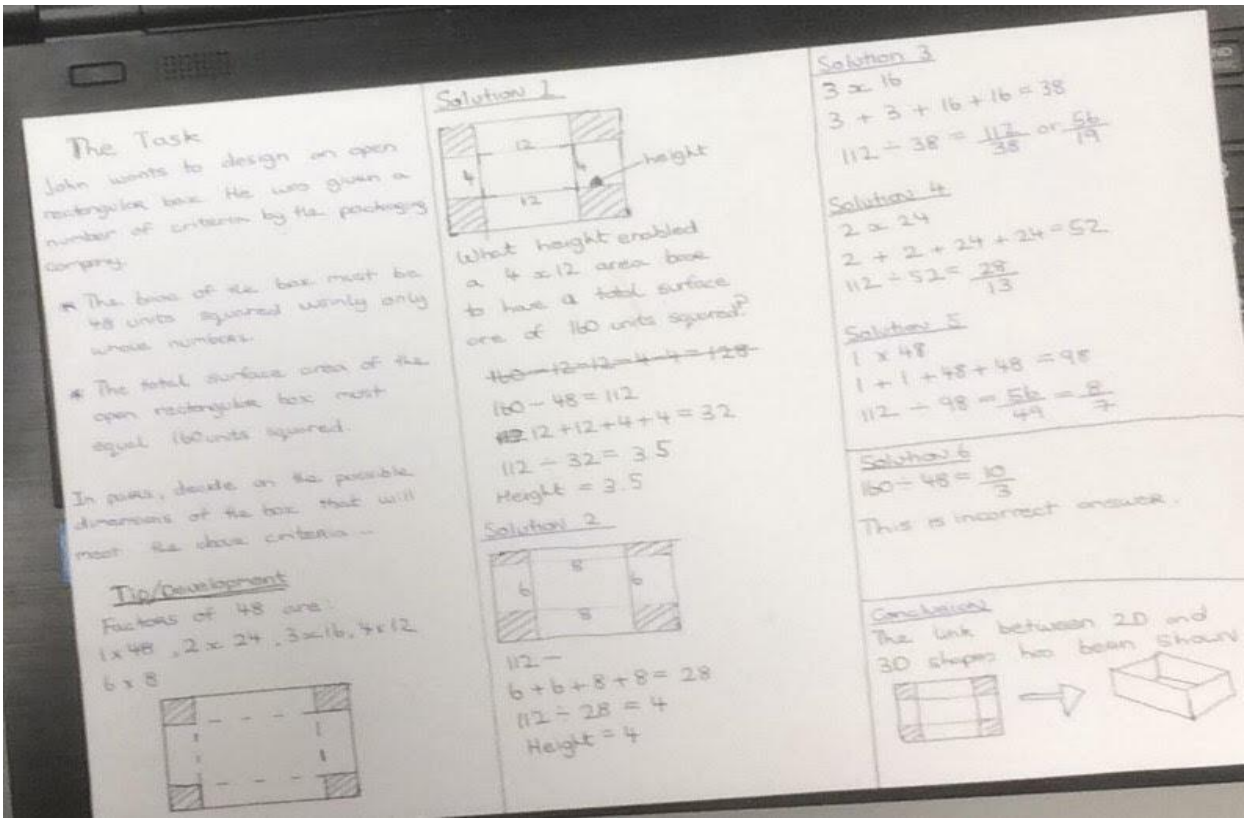
		Do students understand that they should use the factors of 48 to get the possible dimensions of the base. Do students understand there is more than one solution to the problem. Do they understand that using different dimensions for the base will give different heights.
Student Individual Work **Comparing and discussing (15 minutes)** Using dimensions for the base, the most likely correct solutions are as follows: R1:Base 6x8. Height of 4. R2: Base 4x12. Height of 3.5. R3: Base of 1x48. Height of 8/7. R4: Base of 3x16. Height of 56/19 R5: Base of 2x24. Height of 28/13 Possible incorrect: **R6:** Students confusing surface area with volume, by dividing the surface area by the base area. 160 divided by 48= 3.33333333333	**R1:** Please raise your hands if you used 6x8/etc/etc as the dimensions for the base? Why did you use these dimensions… Please raise your hand if you found the corresponding height for this base? What height did you get for this base? **R2:** Please raise your hands if you used 4x12 as the dimensions for the base? Why did you use these dimensions… Please raise your hand if you found the corresponding height for this base? What height did you get for this base? **R3:** Please raise your hands if you used 48x1 as the dimensions for the base?	Are students able to tackle the problem? Are students listening to each other's ideas? Are students responding to each other's ideas? Are students working together in pairs? Is their any dominant figures among the pairs? Is having a strong and weaker students being paired together beneficial?

	Why did you use these dimensions… Please raise your hand if you found the corresponding height for this base? What height did you get for this base? **R4:** Please raise your hands if you used 3x16 as the dimensions for the base? Why did you use these dimensions… Please raise your hand if you found the corresponding height for this base? What height did you get for this base? **R5:** Please raise your hands if you used 4x12 as the dimensions for the base? Why did you use these dimensions… Please raise your hand if you found the corresponding height for this base? What height did you get for this base? **R6:** Did anyone get different dimensions to the above problem than the previous 5 responses? What answers did you get and how did you get them? Can anybody suggest why this solution will not work?	

2D to 3D Using the net, construct a box that John could have used.		Are all the boxes the same? Look at the boxes that have been constructed. Which box do you think would store the most? Do you think it would cost the company the same amount of money to make all the boxes?
Ceardaíocht /Comparing and Discussing Teacher goes through each of the five solutions one by one by asking students which ones answered which. Teacher also goes through incorrect solutions such as the dividing the total surface area by the base area…	• "Why do you think ____ was the most popular solution"? (ask another student(s) other than the presenter) • "Which one is nearest to a box"? (Looking for evidence). • "Did anyone else solve it the same way? Can you explain this method" • "Could you make a cube if you were allowed use non-whole numbers??" • "Did any group use 2 boxes per unit rather than 1? Did you do this correctly ie 2x2 instead of incorrect 2x1"	Are students defending their ideas? Are they responding to each othe r's ideas?"

Summing up & Reflection Students are now expected to build a 3D shape from the net they drew. Teacher sets homework based on class activity. "Today I noticed how a 2d object can transform into a 3d shape" **Summing up (5-10 minutes)** What did we learn: • How to get factors of a number. • How to get the different base sized rectangular boxes. • How the base size effects the height of the overall box if a given criteria is given. • There is a limited range of solutions, if whole numbers are not used this can be unlimited. • Discussion about which box would be most desirable. Ask students to write a reflection. Appropriate homework.	The teacher may use the layout of the boardwork to help provide students with a summary of the progression in their learning.	Do the students' reflections represent the teacher's view of the lesson?

9. Board Plan



10. Evaluation

Were the goals and research theme of the lesson met?
Yes: students broke down the problem and discussed very well each step as they went – this goal was met. Students eagerly engaged in the task at hand, unaware of the solution at first glance. Students' sketches of the problem showed use of prior knowledge of 2D and 3D shapes, by drawing the net. Students recognise this is a 'box', which they constructed at the end of the class. After the teacher re-instructed the class, students realised that there were multiple solutions to this question and set to task again.

What methods did the students use to solve the problem?
All 10 groups apart from 3 chose the factor for the base as 6 and 8. 2nd response was 4 x 12, with a height of 3.5 (2 methods used and explained – one using algebra and one using trial and error). 3rd response was 3 x 16, with a height of56/19. 4th response was 1x 48 with height of 8/7. Teacher highlighted our expected R6 where the surface area was divided by the base and explained to the class how this is incorrect.

What comments or questions did the students have?
Students almost sought confirmation and reassurance from the teacher for calculating factors. One said 'Does the surface area include the inside and the outside of the box?', perhaps relating back to the students' prior learning. Some asked 'will we draw the net?'.

What were the common misconceptions and misunderstandings?
Some students were initially hesitant to work as a pair. As predicted, a pair of students skipped to a volume-related question rather than focusing on the area of the 5 sides of the box. Students were fearful to use fractions/decimals for the sides of the box so the teacher reassured the class that the base of the box must be whole numbers, not the sides.

How and when did students' understanding change?
Teacher stepped in to reassure students that the dimensions of the base must be whole numbers. This helped many students' understanding of the problem. As students worked together, this clearly spurred on their understanding of the problem. Students realised there were multiple answers to this problem once the teacher said the sides of the box do not have to be whole numbers necessarily. During the Ceardíocht some students realised there were many ways to find the solution, not just the one method. Some students used algebra to calculate the side, more used trial and error.

Did the students' presentation and discussion promote their thinking and learning?
Students' discussion was essential to this problem. They all effectively tried using factors in pairs. During the Ceardaíocht, students were eager to see all methods used and realized that during the time allowed to work, they had been on the right method but gave up because the decimals looked incorrect.

Was the flow of the lesson coherent?
Teacher effectively moves from calculation of factors to the problem at hand and asks the class 'Does everyone understand the question you have to do?' This added to the smooth flow of class. Each time the teacher spoke, the class pushed on to tackle the question from another angle, which showed the teacher only spoke when absolutely necessary to get the most out of her class. The construction of the box brought the lesson very effectively from a problem about 2D shapes to the introduction of 3D shapes.

Did the students display a positive disposition?

Students are thoroughly engaged with teacher instruction, explanation and interaction. When told to work on the actual problem, all students got busy immediately with the problem. All students worked very hard for their time allowed and were genuinely interested in trying out different factors to complete the box. Students were eagerly engaged with all aspects of this lesson and especially at the end with the construction of the box; it is clear they thoroughly enjoyed the lesson and the learning experience.

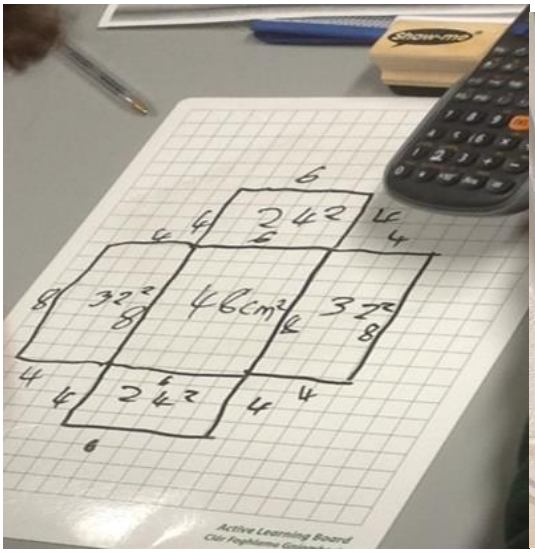
Did the activities support the goals?

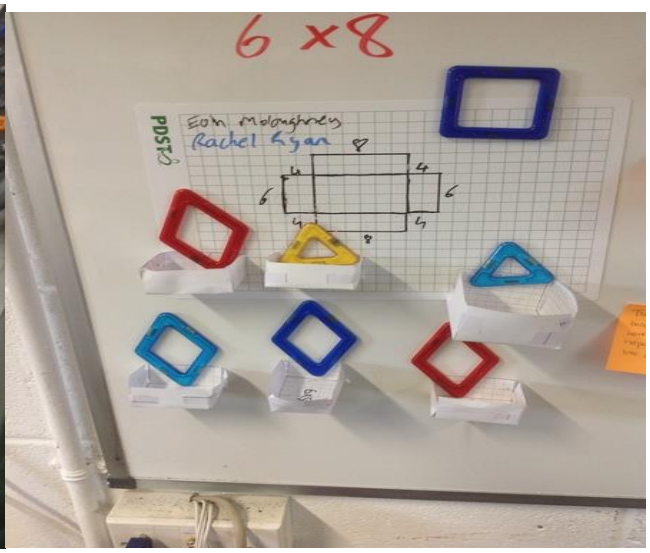
Listing the factors of 48 proved very useful in the latter section of the lesson. Explanation of 'dimensions of a box' by the student was probed very well by teacher. The activity suited the goals very well as is clear from the productive pair work and productive discussions between students. The Ceardaíocht proved to be extremely useful and effective with all pairs as all groups were keen to see their own attempt again and change numbers to suit the dimensions. The construction of the box at the end rounded off the learning experience for bringing 2D shapes to 3D shapes excellently.

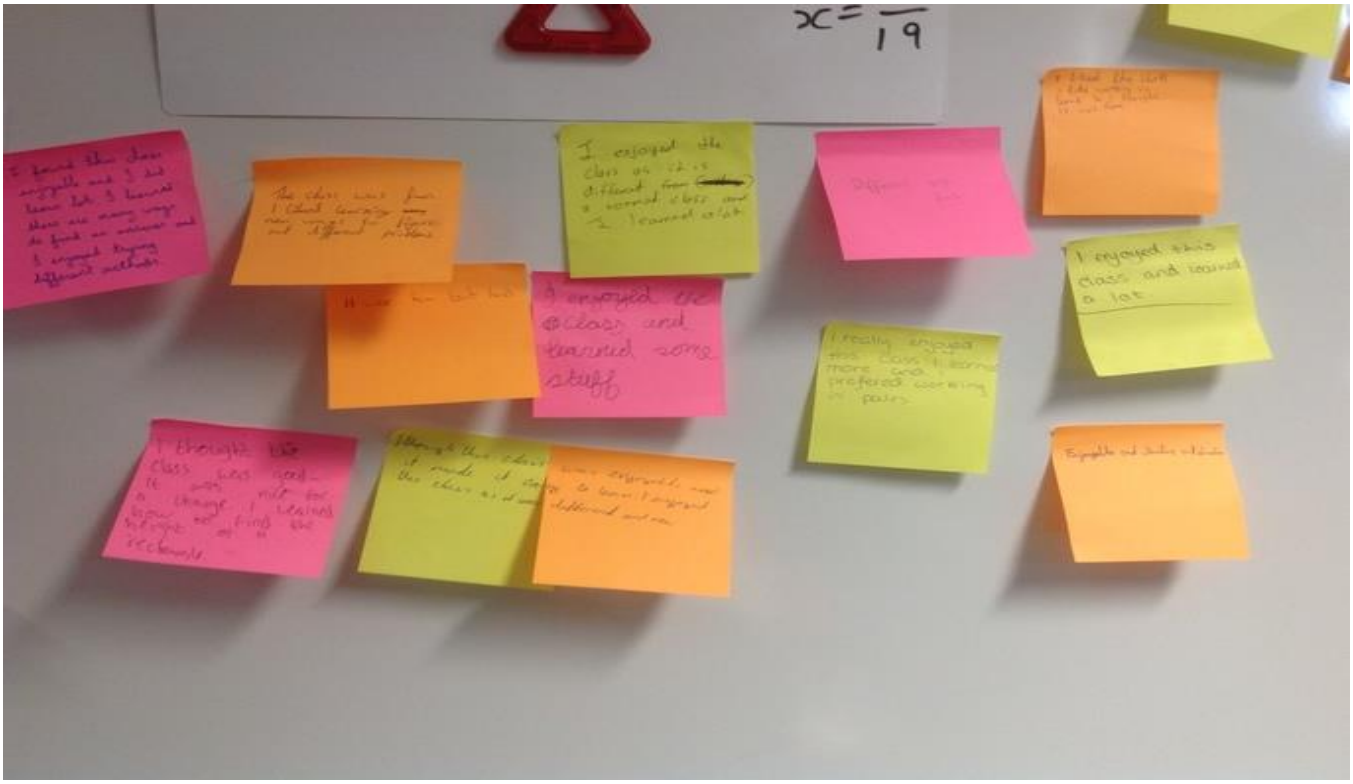
Were Key Skills met?

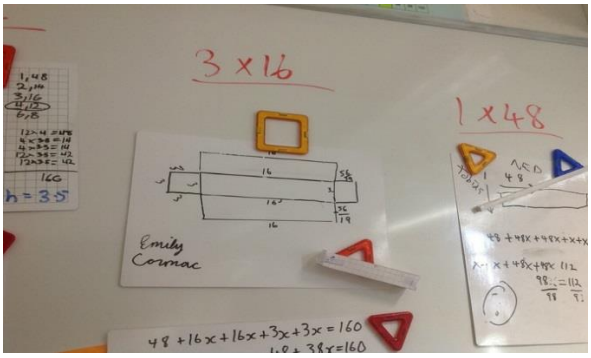
Being Literate: Clearly understood instructions from teacher
Being Numerate: Trialling all possible factors
Managing Myself: Students were able
Staying Well: Working and discussing in pairs
Communicating: Students talked very well with each other and learned both from each other and the teacher.
Being Creative: Construction of box
Working with Others: Students worked effectively with each other during the discussion and work time and also during the construction at the end.
Managing information and thinking: Students managed a lot of information especially during the Ceardaíocht.

Actual Board Plan


Active Learning Board


6 x 8
Eoin Mclaughlin
Rachel Ryan


I enjoyed the class and learned a lot


3 x 16
1 x 48
Emily
Cormac
48 + 16x + 16x + 3x + 3x = 160

11. Reflection

- The prior knowledge supported the success of the first task which was getting the dimensions of the base. All students quickly got the pairs of factors of 48 and decided on the dimensions of their base.
- Task 2 proved more challenging for students, it required more investigation and discussion which was certainly evident. This required students to get the height of their box. Most students now appreciate that there are different ways to solve a problem. Some students used trial and error while others set up an algebraic equation to find the height of the box.
- All groups produced at least two boxes based on the specifications given in the task.
- At the end of the lesson, the feedback from the students was very positive. All teachers agreed that it would be very beneficial to introduce more problem solving lessons to encourage independent learning.
- Students enjoyed taking ownership of their own work by bringing their work to the board and sharing the methodology used. Teachers agreed it would be useful to incorporate this into future lessons to encourage peer learning.
- In the follow-up lesson, the class were asked why a manufacturers would produce one box rather than another. Some groups concentrated on the cost of producing the boxes while other groups looked at the capacity of the boxes. This led on nicely to the concept of volume.