

# **Lesson Research Proposal for 3rd Year Mixed Ability (Higher Level) - Trigonometry**

For the lesson on 29 January 2018

At St. Caimin's Community School, Shannon, Tara O'Loughlin's class

Instructor: Tara O'Loughlin

Lesson plan developed by: Parvin Chadda, Tara O'Loughlin, Evelyn Madden & Maggie  
Garrihy

**1. Title of the Lesson:** Angles to find the Treasure

**2. Brief description of the lesson**

The students will be given a map with three different routes to treasure for three pirates. Students must identify the routes and calculate the distances to the treasure using a variety of trigonometric ratios and pythagoras' theorem. Who will win the race to the treasure? The problem will challenge students to use their trigonometry skills in a problem solving context.

**3. Research Theme**

As maths teachers, we want our students to:

- a) Demonstrate a knowledge appropriate to their stage of development and develop an appreciation of others students' views in a group context.
- b) Actively participate and take a sense of ownership in their work, take pride in it and take responsibility for improving their learning.

As maths teachers, we want to:

- a) Collaboratively support our students learning by developing a whole school approach to assessment.
- b) Respond to individual learning needs and differentiates teaching and learning activities, through designing and implementing personalized interventions to help students to overcome challenges.

As a Maths department we will actively support the achievement of these goals in the following ways:

- a) Facilitating an environment where students are presented with problems that are relevant to their experiences and motivate their interest and participation. Students will be provided with the space to arrive at an approach/solution appropriate to their ability and understanding.

- b) To create a problem solving environment where the Math Department develops improves and encourages students problem solving by developing bespoke solutions to non-standardized/ A typical problems.

#### **4. Background & Rationale**

Why we chose this topic:

This lesson is aimed at third year students at lower ability higher level class. Trigonometry is an integral part of the JC and LC course. It is used in many areas to solve real life problems. Students can usually use the ratios when shown, however they struggle to apply them in a real world problem solving context. We would like to improve this situation so students can comprehend and get satisfaction solving real life problems using trigonometry.

Currently the approach is still very much a traditional style of chalk and talk - a show and explain methodology. We are trying to find a different approach to solve the student's difficulties through a problem solving approach.

Our research findings:

Through discussion of members of the research group, we now aim to address this deficit by creating a common agreed approach to the introduction of trigonometry. We identified from our teaching that we predominantly taught division of trigonometry in a purely procedural fashion, neglecting any exploration of visual or real life explanations. We aim to introduce trigonometry in the context of a problem where they can apply their prior knowledge and identify early misconceptions. The primary focus is to develop a deeper understanding of the concept of trigonometry which the students will retain as knowledge as opposed to procedure.

## 5. Relationship of the Unit to the Syllabus

Related prior learning Outcomes	Learning outcomes for this unit	Related later learning outcomes
<p><b>Primary School</b></p> <p><b><u>Syllabus states</u></b></p> <p>From fifth to sixth class the child should be enabled to...</p> <p><b><u>Strand Unit Fractions</u></b></p> <ul style="list-style-type: none"> <li>understand and use simple ratios</li> </ul> <p><b><u>Strand Unit Decimals</u></b></p> <ul style="list-style-type: none"> <li>solve problems involving operations with whole numbers, fractions, decimals and ..... use diagrams; estimate and compute answers with a calculator.</li> </ul> <p><b><u>Strand Unit Equations</u></b></p> <ul style="list-style-type: none"> <li>... translate word problems with a variable into number sentences</li> </ul> <p><b><u>Strand Unit 2D Shapes</u></b></p> <ul style="list-style-type: none"> <li>make informal deductions about 2-D shapes and their properties</li> <li>use angle and line properties to classify and describe triangles</li> <li>use 2-D shapes and properties to solve problems....</li> </ul> <p><b><u>Strand Unit Lines and Angles</u></b></p> <ul style="list-style-type: none"> <li>recognise angles in terms of a rotation</li> <li>explore the sum of the angles in a triangle cut off</li> </ul>	<p><b>Junior Certificate</b></p> <p><b><u>Syllabus states</u></b></p> <p>Right-angled triangles. Trigonometric ratios. Working with trigonometric ratios in surd form for angles of <math>30^\circ</math>, <math>45^\circ</math> and <math>60^\circ</math> Right-angled triangles. Decimal and DMS values of angles.</p> <p>Geometry and Trigonometry Strand Students should be able to:</p> <ul style="list-style-type: none"> <li>calculate, interpret, and apply units of measure and time</li> <li>investigate 2D shapes .... so that they can find the perimeter and area of plane figures made from combinations of .... triangles, .....</li> <li>apply the theorem of Pythagoras to solve right-angled triangle problems of a simple nature involving heights and distances</li> <li>evaluate and use trigonometric ratios (sin, cos, and tan, defined in terms of</li> </ul>	<p><b>Leaving Certificate</b></p> <p><b><u>Syllabus states</u></b></p> <p><b><u>2.3 Trigonometry Right-angled triangles.</u></b> <b><u>Trigonometric ratios</u></b></p> <ul style="list-style-type: none"> <li>apply the result of the theorem of Pythagoras to solve right-angled triangle problems of a simple nature involving heights and distances</li> <li>– use trigonometric ratios to solve real world problems involving angles</li> <li>use of the theorem of Pythagoras to solve problems (2D only)</li> <li>– define <math>\sin \theta</math> and <math>\cos \theta</math> for all values of <math>\theta</math> – define <math>\tan \theta</math></li> <li>– work with trigonometric ratios in surd form</li> </ul> <p><b><u>3.3 Arithmetic</u></b></p> <ul style="list-style-type: none"> <li>– make estimates of measures in the physical world around them..</li> </ul> <p><b><u>3.4 Length, area and volume</u></b></p> <ul style="list-style-type: none"> <li>– solve problems involving the length of the perimeter and the area of plane figures: ..., triangle,.....and figures made from combinations of these</li> </ul> <p><b><u>4.1 Expressions</u></b></p> <ul style="list-style-type: none"> <li>– evaluate expressions given the value of the</li> </ul>

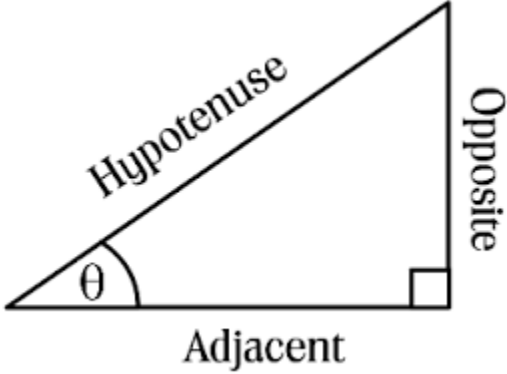
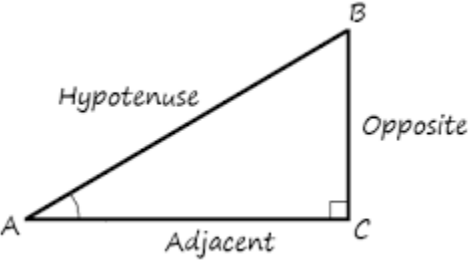
the three corners of a paper triangle and put them together to make $180^\circ$	right-angled triangles) and their inverses, involving angles between 0 and 90 in decimal form	variables
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## 6. Goals of the Unit

- a) Students apply their mathematical knowledge of Pythagoras and trigonometric ratios in a wide variety of problems across different topics, including interpreting and converting a narrative into a diagrammatic representation.
- b) Students develop a positive disposition when using mathematics to solve and model real-world situations. Following, students understand the value of trigonometric skills relating to everyday scenarios.
- c) Students have an opportunity to engage their creativity in exploring many different methods of solving problems.
- d) Students will engage with digital technology to conceptualise 2D and 3D problems.

## 7. Unit Plan

Lesson	Learning goal(s) and tasks
1	Revision of formula relating to shape and space.
2	Revision of Pythagoras, observing patterns in side measure i.e. Pythagorean triples. Introduction to naming sides in relation to specific angle in right angled triangle.

	 <p style="text-align: center;">Adjacent</p>
3	<p>Revision of fraction/ratio and intro to trigonometric ratios.</p> <div style="display: flex; align-items: center;">  <div style="margin-left: 20px;"> <math display="block">\sin(A) = \frac{\text{opposite}}{\text{hypotenuse}}</math> <math display="block">\cos(A) = \frac{\text{adjacent}}{\text{hypotenuse}}</math> <math display="block">\tan(A) = \frac{\text{opposite}}{\text{adjacent}}</math> </div> </div> <p>Oh Hell Another hour Of Algebra</p>
4	<p>Use trigonometric ratios to solve simple right angle triangle problems. Practice using clinometer in class to find angle of elevation in preparation for use in next class to measure various heights of structures the school.</p>
5	<p>Discovery learning lesson  Organise students in differentiated groups, allocate roles and instruments i.e. measuring tapes, clinometers and worksheets.  Set the task to measure various structures and ask students which trigonometric ratio they will use to find the heights given the measurements they can gather.</p>
6	<p>Groups reflect on their findings from previous lesson and allocate a spokesperson to present summary to class.  After each presentation students will have time to discuss reasons for alternative findings and the value of accurate readings, example. Making sure measuring tape is taut and clinometer is used correctly.</p>
7	<p>Research Lesson</p>

## 8. Goals of the Research Lesson:

### a) Mathematical Goals

Students will:

- Have enriching experience of the application of trigonometry
- Understand the characteristics of trigonometry and use trigonometry to solve right angled related problems
- Galvanise their appreciation of trigonometry in the real world
- Understand the relationship between angles and distances and use their previous knowledge to understand how they are related
- Achieve an understanding of how trigonometry can be used in practical real world problems understand how to solve problems that involve finding heights and distances from right-angled triangles (2D only)
- know how to use the theorem of Pythagoras to solve problems (2D only)
- solve problems that involve calculating the cosine, sine and tangent of angles between  $0^\circ$  and  $90^\circ$
- Apply the use of trigonometry to solve real world word problems

### b) Key Skills and Statements of Learning

In the planning and design of this lesson the Junior Cycle Key Skills and Statements of learning have been considered. This lesson will implement and promote JC Key Skills in the following ways:

1. Being Literate: Students will have the opportunity to express their ideas clearly and accurately.
2. Being numerate: It will develop a positive disposition towards problem solving
3. Managing myself: Student's will have the opportunity to reflect on their own learning
4. Staying well: Students confidence and positive disposition to learning will be promoted
5. Communicating: Students will present and discuss their mathematical thinking
6. Being Creative: Students' will explore options and alternatives as they actively participate in the construction of knowledge
7. Working with others: Students will learn from each other
8. Managing information and thinking: Students will be encouraged to think creatively and critically.




This lesson is also designed to meet the following JC Statements of learning in particular:

1. The student communicates effectively using a variety of means in a range of contexts
15. The student recognises the potential uses of mathematical knowledge, skills and understanding in all areas of learning
16. The student describes, illustrates, interprets, predicts and explains patterns and relationships



17 The students devise and evaluate strategies for investigating and solving problems using mathematical knowledge, reasoning and skills.


## 9. Flow of the Research Lesson:


The problem

Black Beard, Blue Beard and Orange Beard are anchored  at Clare Island and need to find the quickest route to the treasure.   They can't go directly due east because of weather conditions.

They head off at the same time.

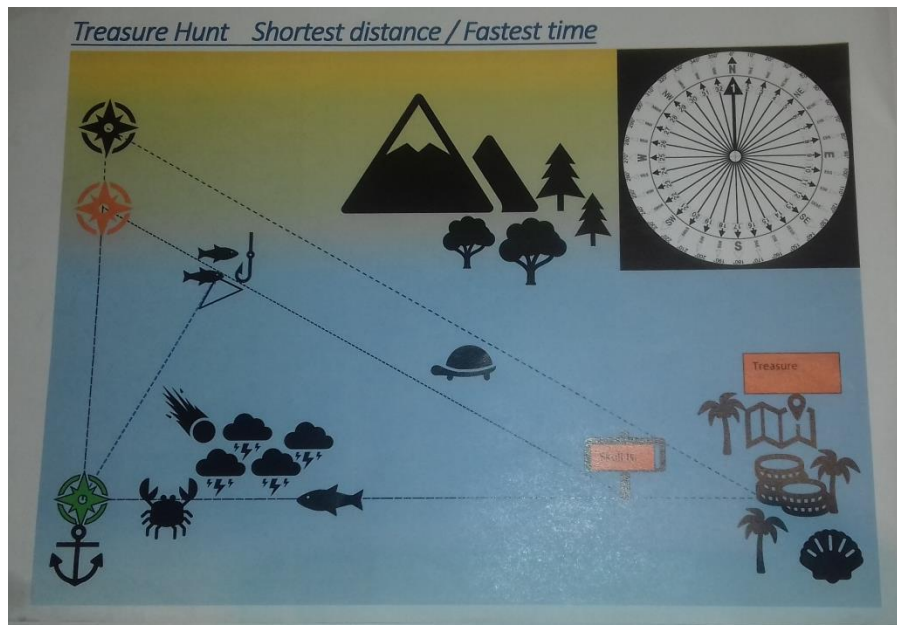
Bluebeard thinks the shortest route is to take a  $N30^\circ E$  travelling for 80km to the fishing grounds   where he will make a  $90^\circ$  turn and head for Skull Island where he will then turn due East to reach the treasure.

Orange Beard travels to  then turns and heads to Skull Island and then he takes a due east bearing to grab the treasure.

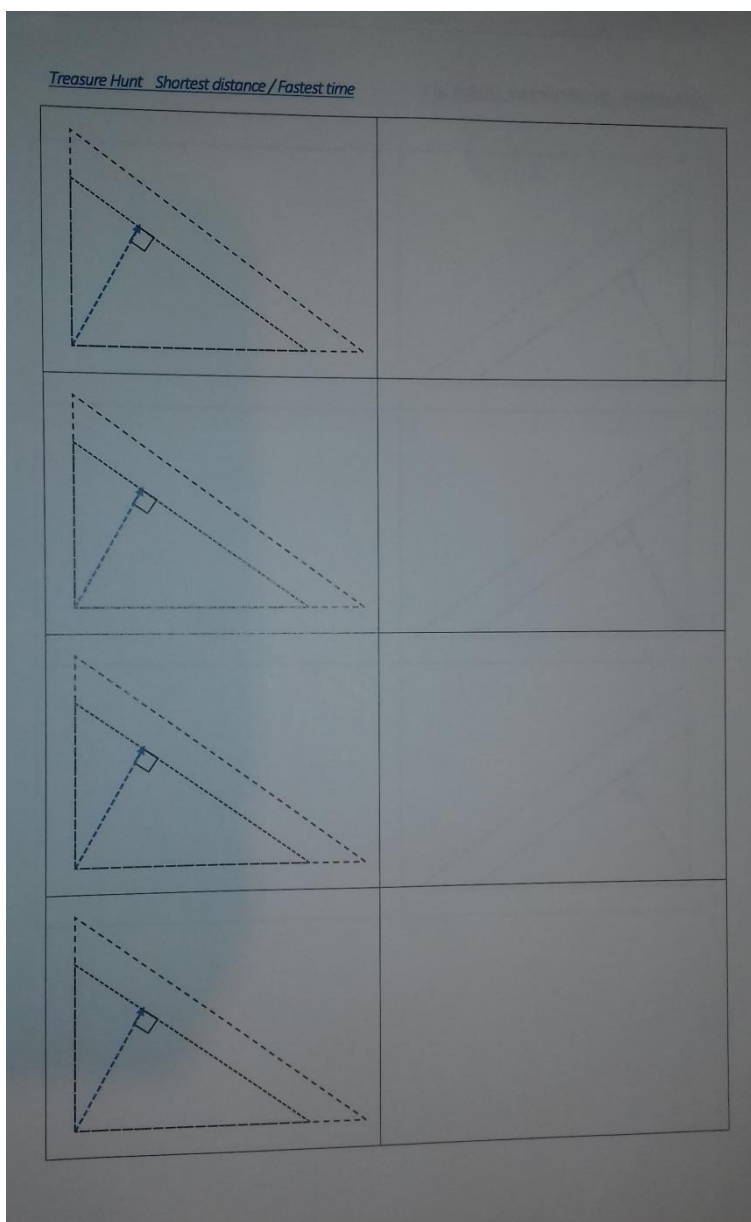
Black Beard also heads due north but travels an additional 20km further than Orange Beard before turning at  and taking a parallel route to Orange Beard.

Who arrives at the treasure first assuming they are all travelling at the same speed?

The Map



The answer sheet



<b>Steps, Learning Activities</b> <b>Teacher's Questions and Expected Student Reactions</b>	<b>Teacher Support</b>	<b>Assessment</b>
<b>Introduction 5 min</b> Distribute documents and explain the first part of the task. Ask students to find the various routes to the treasure.	Give the problem, the map and the answer sheet to the students.	.Are students motivated and interested?

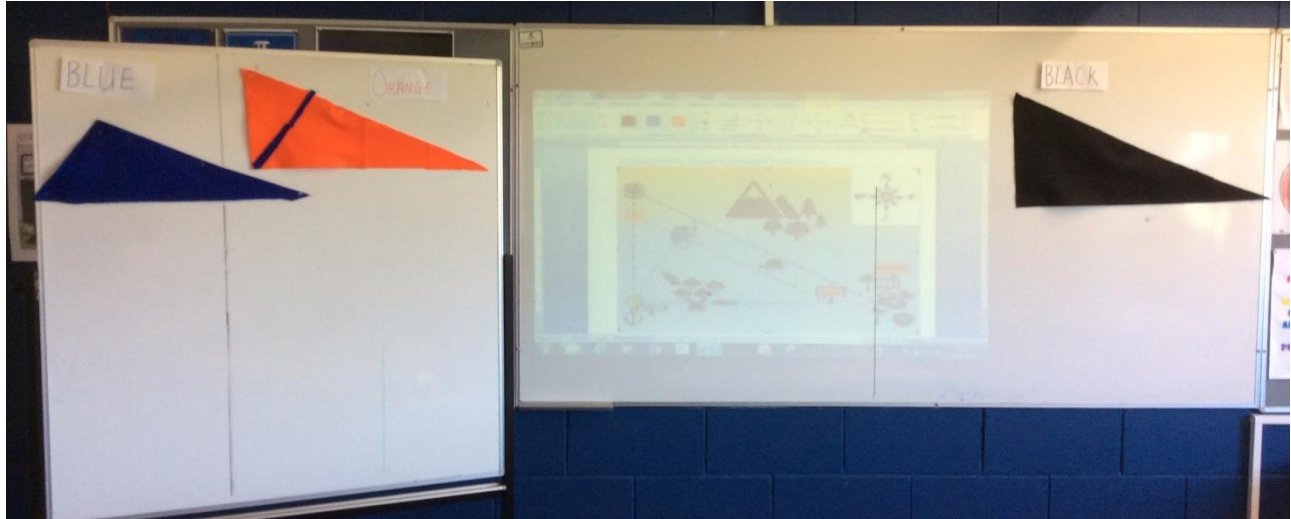


<p><b>Problem Solving 1 5 min</b> Students find the various routes to the treasure</p>	<p>Teacher observes</p>	
<p><b>Ceardaíocht 1 5 min</b> 3 students illustrate the routes on the board clarifying the next part of the task Any misconceptions are clarified.</p>	<p>Teacher seeks clarity</p>	
<p><b>Posing the Task Problem Solving</b> Who arrives at the treasure first assuming they are all travelling at the same speed ? <b>Clarifying the problem</b></p> <ul style="list-style-type: none"> <li>• What is the shortest route to the treasure</li> <li>• Students are asked to write down their estimate for the shortest route</li> </ul>	<p>Have a diagram illustrating each of the routes.</p>	<p>Do the students understand the task and can they explain to teacher?</p>
<p><b>Student Individual Work (15 min) And Ceardaíocht (20 min)</b></p> <p><b>Student Response 1</b> (this may be split between a 3 students - one for each route)</p> <ul style="list-style-type: none"> <li>• Fill in all possible angles and measurements stated in the question.</li> <li>• Work out the other angles based on previous task.</li> <li>• Blue Beard 's journey is represented on the board</li> </ul> <p><b>Student Response 2</b></p> <ul style="list-style-type: none"> <li>• Blue Beard</li> <li>• Using Tan 30 find distance from the fishing ground to Skull Island.</li> </ul> <p><b>Student Response 3</b></p> <ul style="list-style-type: none"> <li>• Do we need to calculate Orange Beard 's total distance</li> </ul> <p><b>Students Response 4</b></p> <ul style="list-style-type: none"> <li>• To calculate Black Beard's distance from Clare Island to his turning point for the treasure we need to first look at Orange Beard's journey</li> <li>• Orange Triangle used on board</li> </ul>	<p>Invitation to board to fill in information given and calculated to one student</p> <p>Invitation to board to show method and calculations</p> <p>Ask Student to explain his / her answer.</p> <p>Invitation to board to demonstrate calculations.</p>	<p>Observe and assess the students work during the students individual work.</p> <p>Are students able to work out missing angles based on information given</p> <p>Do students realise that other calculations are required first before Blue Beard's total distance can be found</p> <p>Can students see that Blue Beard's journey is shorter</p> <p>Can students assess outstanding</p>

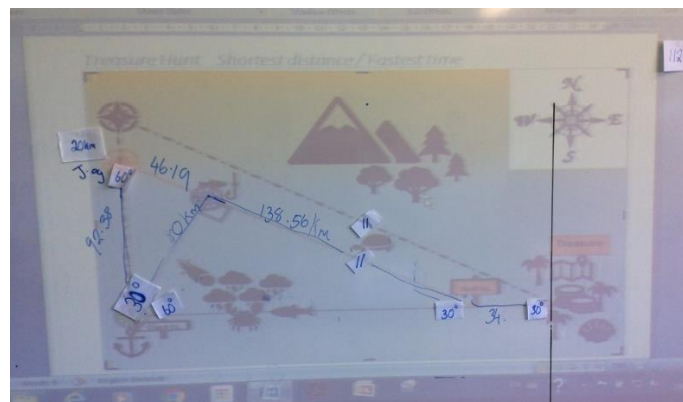
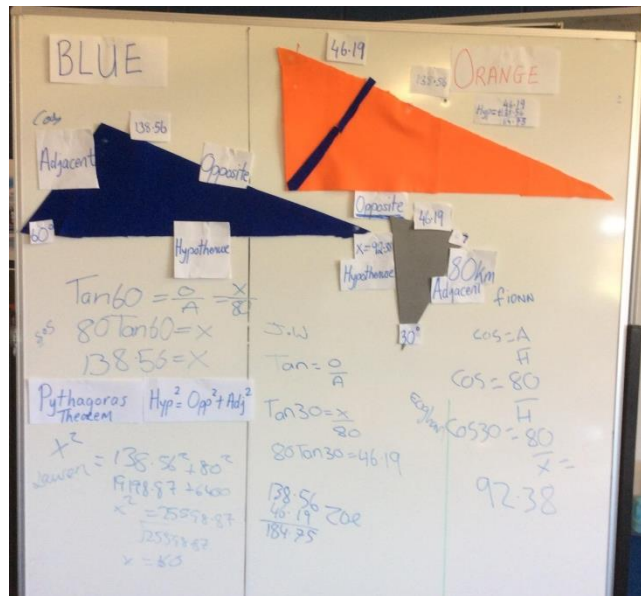
<p>separately to enhance understanding</p> <ul style="list-style-type: none"> <li>• <math>\tan 60 = x/80</math></li> </ul> <p><b>Students Response 5</b></p> <ul style="list-style-type: none"> <li>• Complete the distance of Black Beard's journey to turning point</li> </ul> <p><b>Students Response 6</b></p> <ul style="list-style-type: none"> <li>• Find the distance from when Black Beard turned to the treasure</li> <li>• <math>\sin 30 = 112.37 / X = 224.74\text{km}</math></li> </ul> <p><b>Students Response 7</b></p> <ul style="list-style-type: none"> <li>• Calculate the total distance for Black Beard</li> </ul> <p><b>Student Response 8</b></p> <ul style="list-style-type: none"> <li>• Finish Blue Beard's journey</li> <li>• Small Triangle Blue Beard - pythagoras to find from Clare Island direct to Skull Island</li> </ul> <p><b>Student Response 9</b></p> <ul style="list-style-type: none"> <li>• Large triangle - use pythagoras theorem to find distance from Clare Island to the Treasure</li> </ul> <p><b>Student Response 10</b></p> <ul style="list-style-type: none"> <li>• Subtract both to find distance from Skull Island to Treasure - this means Blue Beards journey is complete</li> <li>• Add all relevant measurements to get total distance <b>for</b> Blue Beard.</li> </ul>	<p>Student will simply add this <math>92.37 + 20</math></p> <p>You might ask the class who is using <math>\sin 30</math> or <math>\cos 60</math></p> <p>Invitation Board to add both distances to get Black Beard's total journey</p> <p>Investigate if using pythagoras theorem is the only way to find out these measurements</p>	<p>information?</p> <p>Have students remembered details given in question</p> <p>Do students realise <math>\sin 30</math> or <math>\cos 60</math> would give same results?</p>
<p><b>Summing up &amp; Reflection (5 min)</b></p> <p>.Which was the shortest distance.</p> <p>Was your estimate correct?</p> <p>Reflection sheet</p>	<p>Give the students a reflection sheet to complete.</p>	

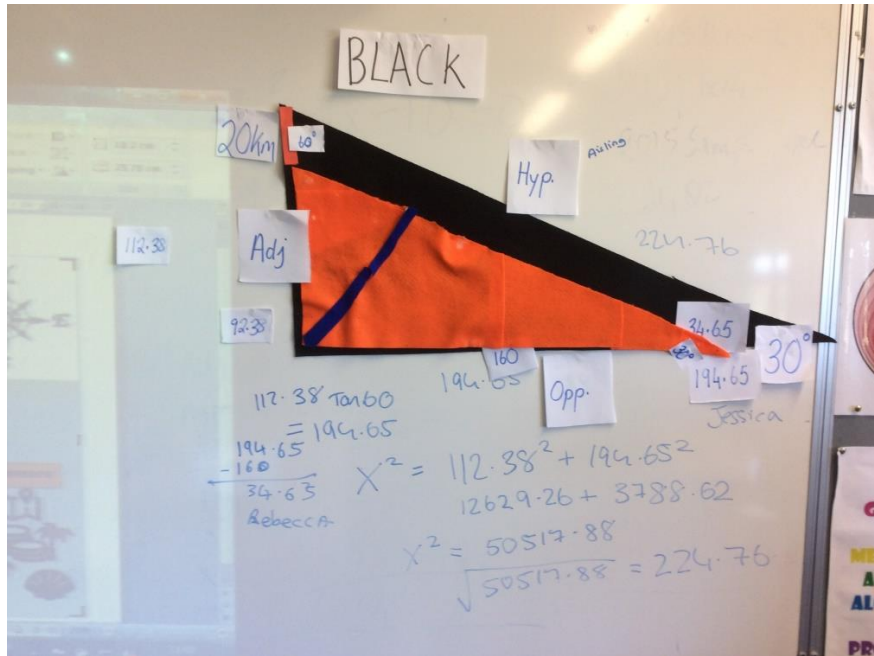
# 10. Board Plan

Before

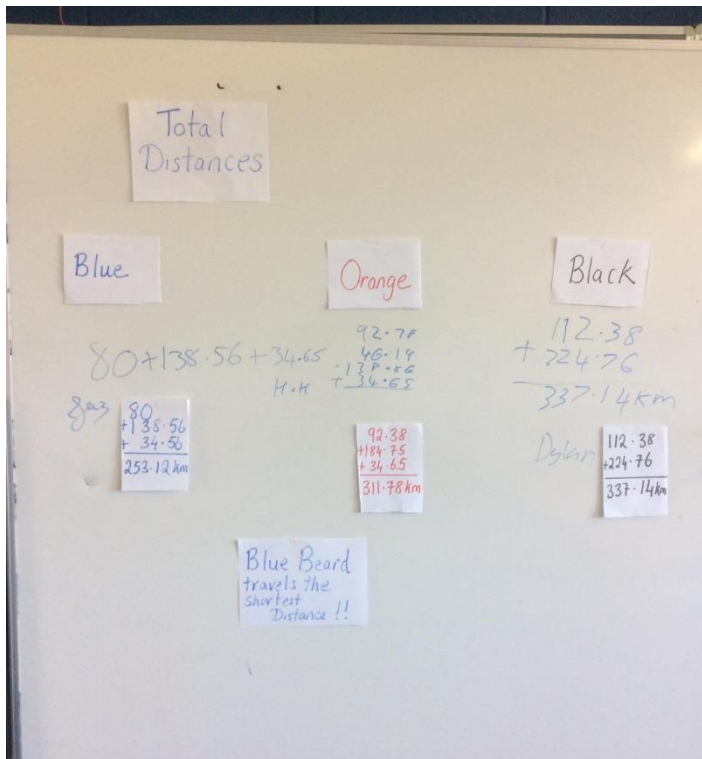


After





A summary of distances to be travelled were prepared for completion by students on the rear of the portable board.



## 11. Evaluation

The consensus was that the lesson was very successful, and the goals stated by the group at the outset had been met. It was clear to the team that the students were engaged in the task from the beginning. The students responded well to the prior knowledge and once the task was set they engaged immediately in trying to find the routes and to calculate distances. All students were successful in finding routes to the treasure. The majority of students calculated the distance to the treasure of one of the pirates and all students attempted to do this work. One student was observed struggling significantly with the problem and disengaging as the class progressed.

Once the Ceardaíocht began, and the students saw their classmates work, all students self-corrected and there were a few moments where students said “Yes, we were right”. This self-correction based on students contributions to the boardwork and the expert handling of the teacher was a notable positive feature of the class. A number of students calculated  $30\tan 80$  instead of  $80\tan 30$  but they self-corrected during the ceardaíocht. Overall, all students learned during the class and were challenged by the problem which indicates that the problem is an excellent, engaging and ideal problem for a mixed ability higher level 3rd year class.

## 12. Reflection

The objective was that students engaged mathematically with a stimulating trigonometric problem. It was expected that students would use Pythagoras’ theorem along with the Sin, Cos and Tan ratios to solve the problem. The ceardaíocht would then allow students to analyse and correct their misconceptions.

The class worked very well and students were engaged. They spent significant time (up to 4 minutes) reading the problem. Two students progressed very quickly and completed all the calculations but the majority of students continued to work for the full 15 minute period. The students only began discussing the problem amongst themselves towards the end of their allotted period. Fifteen students contributed to the board work with contributions at various levels of sophistication. Students were observed smiling after they made their contribution. They clearly enjoyed having an impact in the class’s learning.

The students reflections stated that the lesson improved their understanding of trigonometric ratios. They became more aware of the application of Pythagoras’ theorem. They enjoyed the pirate’s dilemma and found the visual aids beneficial.

There were a few standout moments where students delight at understanding the problem and solving the problem was observed. After Bluebeard’s distance had been calculated, a student was observed saying “I know how to do it, It is so annoying”. This student then raced to do the next part of the problem before it had been completed in the ceardaíocht. This self-correction was observed on a number of occasions during the research lesson.

Areas for further development of this lesson include having concrete examples of the various two dimensional shapes for students to engage with. It may be worthwhile to increase the time allotted to the students work especially for a 3rd year mixed ability higher level class.