Lesson Research Proposal for 5th Years - Circle

For the lesson on 09/02/2018
At Coláiste Na Trócaire, Rathkeale, Mr O'Keeffe's class
Instructor: Mr Frank O'Keeffe
Lesson plan developed by: Frank O'Keeffe, Heather MacCarthy

1. Title of the Lesson: Going Around In Circles

2. Brief description of the lesson

In this lesson students will actively participate in solving a problem that will progress towards understanding, recognising and applying the equation of the circle in the form

$$(x-h)^2+(y-k)^2=r^2$$

3. Research Theme

At our school, we want

(a) our students to engage purposefully in meaningful learning activities,
(b) our teachers to select and use planning, preparation and assessment practices that progress students learning.

As a mathematics teachers, we will actively support the achievement of these goals by

(i) ensuring that students understand and are able to explain the purpose of the learning tasks they are engaged in and can extend and develop the activity meaningfully
(ii) designing and preparing in advance a sequence of learning tasks and activities suitable for the specific learning intentions of the lesson or series of lessons. Lesson design is flexible to allow for emerging learning opportunities.

4. Background & Rationale

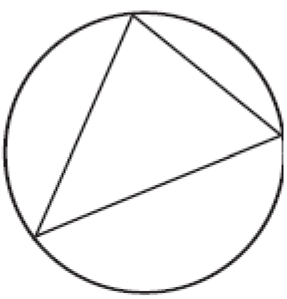
1. Why we chose this topic

In our experience of teaching this topic, we have found that students often struggle with answering questions on the circle in exams. In fact we had all seen blank answers in pre-exams and class test. Students who could answer questions on the line would fail to make the same connections with the circle. We felt that since the circle has links to many strands throughout the syllabus a new approach was needed for this topic.

2. Our research findings

Through discussions by members of the maths department we realised that our teaching of circle is imbalanced towards procedures. Therefore we have decided to commence using a problem-solving situation which naturally gives rise to the equation of the circle.
In designing the research lesson we believe it is important to engage students enthusiastically with the subject matter. The lesson proposal tries to devise creative ways to make it easier to comprehend the equation of the circle by illustrating the problem and using suitable teaching aids. The approach depends on allotting students plenty of time to think about the problem and figure it out on their own.

5. Relationship of the Unit to the Syllabus

Related prior learning Outcomes	Learning outcomes for this unit	Related later learning outcomes
In Junior Cert / Cycle pupils are introduced to the following definitions – • A circle is the set of points at a given distance (its radius) from a fixed point (its centre). Each line segment joining the centre to a point of the circle is also called a radius. • The plural of radius is radii. • A chord is the segment joining two points of the circle. The circumcircle of a triangle ΔABC is the circle that passes through its vertices (see Figure 1). Its centre is the circumcentre of the triangle, and its radius is the circumradius.  Figure 1 In Junior Cert / Cycle pupils learn about • Co-ordinating the plane. • Properties of lines and line segments including midpoint, slope, distance and the equation of a line in the form.	Students working at Leaving Cert OL Co-ordinate geometry should be able to • recognise that $(x-h)^2 + (y-k)^2 = r^2$ represents the relationship between the x and y co-ordinates of points on a circle with centre (h, k) and radius r • solve problems involving a line and a circle with centre (0, 0) Students working at Leaving Cert HL Co-ordinate geometry should be able to solve problems involving the perpendicular distance from a point to a line	Students working at Leaving Cert HL Co-ordinate geometry should be able to • recognise that $x^2 + y^2 + 2gx + 2fy + c = 0$ represents the relationship between the x and y co-ordinates of points on a circle with centre (-g,-f) and radius r where r = $\sqrt{g^2 + f^2 - c}$ • solve problems involving a line and a circle

6. Goals of the Unit

a) Students will understand the terms and definitions associated with the circle
b) Students will connect the theorems and constructions associated with the circle connecting them to the coordinated plane
c) Students will connect synthetic and co-ordinate geometry
d) Students will move between the uncoordinated and the coordinated plane, verifying geometric results using algebraic methods.
e) Students will discover how to find the centre and radius of a given circle.
f) Students will apply their prior knowledge to derive the equation of the circle.

7. The Unit Plan

Lesson	Learning goal(s) and tasks
1.	Recap of Coordinate Geometry of Line
2.	Construction16. Circumcentre and circumcircle of a given triangle, using only straight-edge and compass.
3.	Construction 17. Incentre and incircle of a given triangle, using only straight-edge and compass.
4.	Construction 21. Centroid of a triangle.
5.	Construction 22. Orthocentre of a triangle.
6.	Construction 19. Tangent to a given circle at a given point on it.
7.	Proving Circumcentre by Coordinate Geometry
8. Research Lesson	Deriving an equation for a circle in the form $(x-h)^2+(y-k)^2=r^2$

8. Goals of the Research Lesson:

(a) Mathematical Goals
Students will:

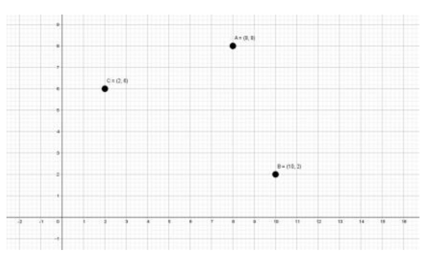
- have more than one method to solve the problem.
- be able to use appropriate language in presenting the problem to their peers
- move between the uncoordinated and the coordinated plane, verifying geometric results using algebraic methods and, hence, be able to derive the equation of a circle in the form $(x-h)^2+(y-k)^2=r^2$

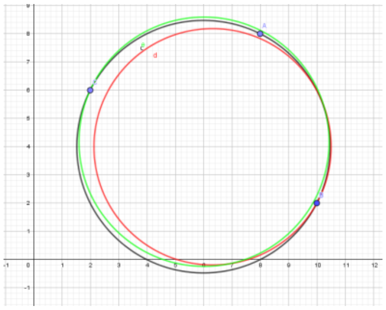
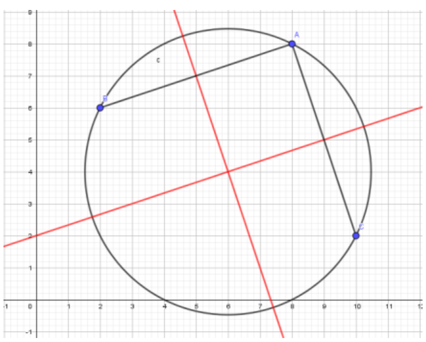
(b) Key Skills

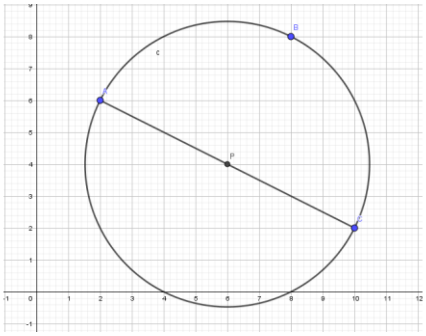
In the planning and design of this lesson the Senior Cycle Key Skills have been considered. This lesson will implement and promote those skills in the following ways:

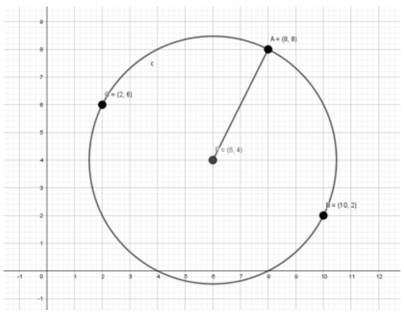
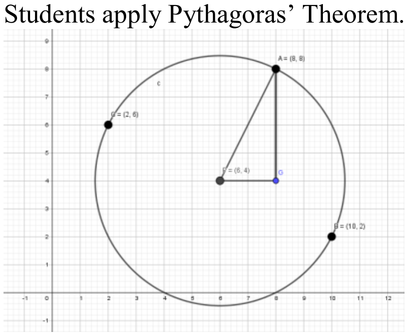
1. Information processing: Students will be presented with tasks that will require them to make sense of, or interpret, the information to which they are exposed.
2. Critical and creative thinking: Students are required to critically evaluate information and think creatively about it. Students are encouraged to solve problems in a variety of ways and are required to evaluate methods and arguments and to justify their claims and results.
3. Communicating: During the lesson students are encouraged to discuss approaches and solutions to problems and are expected to consider and listen to other viewpoints.
4. Being personally effective: Students participate in a learning environment that is open to new ideas and gain confidence in expressing their mathematical ideas and considering those of others.

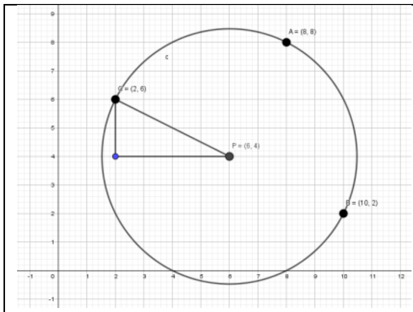
9. Flow of the Research Lesson:

Steps, Learning Activities Teacher's Questions and Expected Student Reactions	**Teacher Support**	**Assessment**
Introduction Today we are going to use our mathematical knowledge to solve a problem. We're going to try to solve the problem by ourselves and then we're going to come together as a class and use all your knowledge to learn something new…		Are students motivated?
Posing the Task **Task 1** I want you, in five minutes, to construct a circle through the given points (A, B, C). **Clarifying the problem:** Use whatever means or method you prefer and see if you can do it by using more than one method.	Present an illustration on the board to make the meaning of the problem easier to understand and hand out the problem as an A3 worksheet. 	Do students understand the task? Are students eager to solve the problem?

Student Individual Work **Task 1 – (5 Minutes)**		
<u>Student Response 1:</u> Using Trial and Improvement 	As one makes class rounds look for good examples of the various methods- manual actions, use of tables, use of arithmetic, use of algebraic equations- and note the order in which you will need to call students to the board.	Are students able to tackle the problem?
Students will make a number of attempts to find solution by guessing/ estimating the centre of the circle.	During the problem-solving stage, you might ask students who cannot figure out any solution methods: “What shape can I make with the 3 dots?” “Do know any connection between triangles and circles?”	Are students looking at alternative ways of solving the problem?
<u>Student Response 2:</u> Constructing the circumcircle  Students may construct two of the chords AB, BC and AC. Then, using a compass and straight edge, the students construct perpendicular bisectors, thus locating the circumcentre. From there the students use a compass to construct the circle.	You might ask the students who are manipulating actual objects and using a compass: “Are there any faster/ more accurate methods to solving the problem?”	Prior knowledge is required for Student Response 2. This physical construction will support students understanding for a later coordinate geometry approach.

Student Response 3: Finding Midpoint of diameter [AC]  The students discovers that [AC] is the diameter by guess. Alternatively, realising that $\|\angle ABC\| = 90°$ the students uses prior knowledge of Theorem 19 to determine that [AC] is the diameter. Then, using construction or coordinate geometry, the students find the midpoint of AC, and from there can construct the circle.		Prior knowledge is required for this approach. Spotting that the points are vertices of a right-angled triangle simplifies the task.
Ceardaíocht /Comparing and Discussing 1 (10 Minutes) Ask specific students to come to the board and explain how they constructed the circle containing the three given points. Student Response 1: Using Trial and Improvement Student Response 2: Constructing the circumcircle Student Response 3: Finding Midpoint of diameter [AC]	. Ensure that when a student comes to the board to present work that their name is attached to it. Possible Questions: Did anybody else use this approach? Is there a problem with using this method? Why does this work? / Do you think this the most effective way of solving the problem? Why? How accurate is this? What does using this method tell us?	Response 1 will help students understand that (i) there is only one solution (ii) the problem is that we don't know the centre or radius. Comment: This method is inaccurate and inefficient. Have students acknowledged that they are capable of completing this problem based on their prior knowledge?

Student Individual Work - (5 Minutes) **Task 2** Locate the centre, P, and calculate |AP|, |BP|, |CP|. <u>Student Response 1:</u>  Measure distance with a ruler or count or estimate the number of boxes.	As one makes class rounds look for good examples of the various methods and note the order in which you will need to call students to the board. You might ask the students who are manipulating actual objects and using a compass: “Are there any faster/ more accurate methods to solving the problem?”	Are all students able to make a step in the right direction?
<u>Student Response 2:</u> Students apply Pythagoras’ Theorem.  By using the points (6,4), (8,8) and (8,4), we have the lengths 2 and 4. Radius = $\sqrt{(2^2 + 4^2}$ $= \sqrt{4 + 16}$ $= \sqrt{20}$	You might ask the students “Can you find the solution using a method we have learned previously?”	Do students notice that there are more than one method?

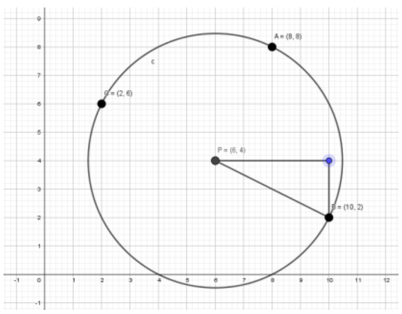


This is then repeated using the points (6,4), (2,6) and (2,4), we have the lengths 2 and 4.

Radius = $\sqrt{(2^2 + 4^2}$

= $\sqrt{4 + 16}$

= $\sqrt{20}$



This is then repeated using the points (6,4), (10,4) and (10,2), we have the lengths 2 and 4.

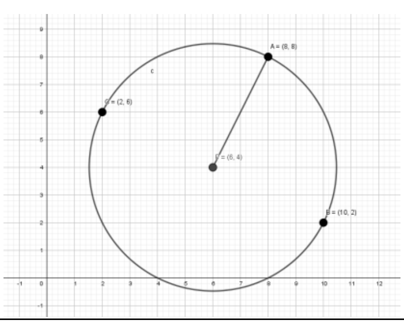
Radius = $\sqrt{(2^2 + 4^2}$

= $\sqrt{4 + 16}$

= $\sqrt{20}$

Students Response 3:

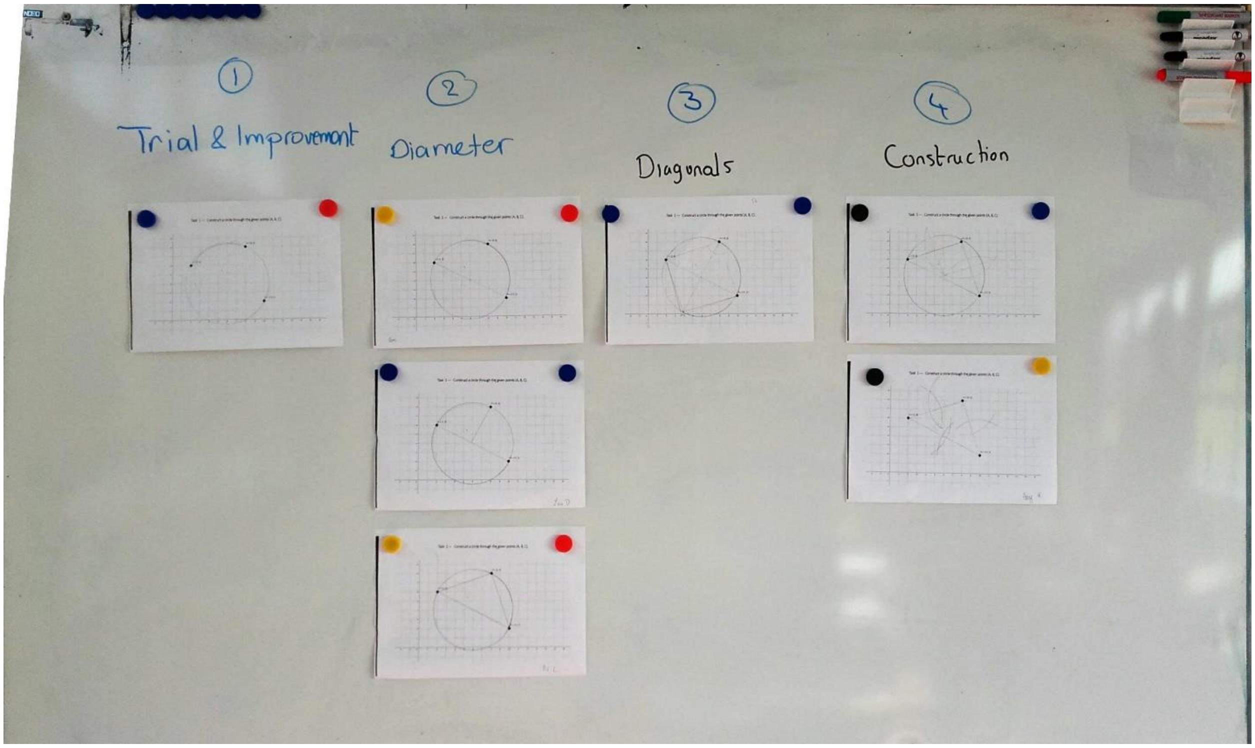
Students use the distance formula.

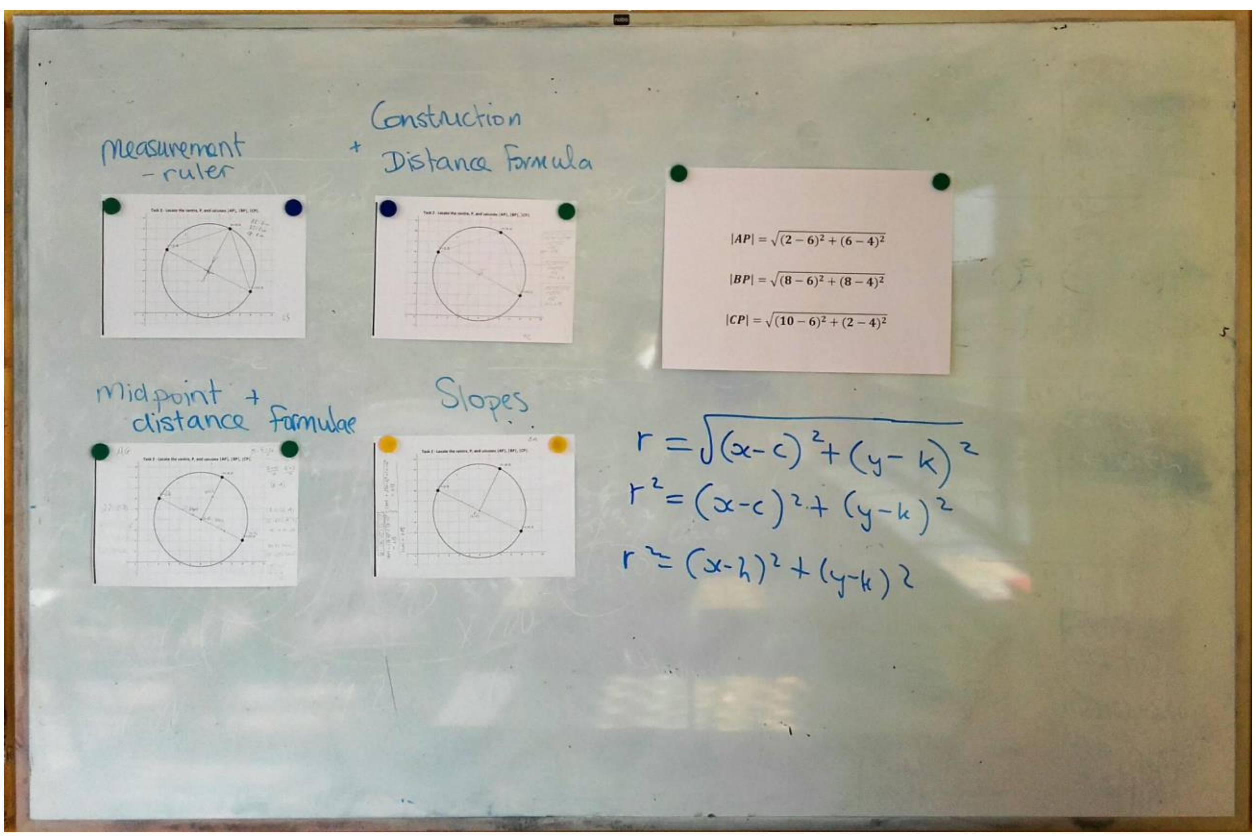


Using the points (6,4), (8,8); $\lvert BP\rvert = \sqrt{(8-6)^2+(8-4)^2}$ $= \sqrt{(2^2+4^2}$ $= \sqrt{4+16}$ $= \sqrt{20}$ This is then repeated using the points (6,4) and (10,2). $\lvert CP\rvert = \sqrt{(10-6)^2+(2-4)^2}$ $= \sqrt{(2^2+4^2}$ $= \sqrt{4+16}$ $= \sqrt{20}$ This is then repeated again using the points (6,4) and (2,6). $\lvert AP\rvert = \sqrt{(2-6)^2+(6-4)^2}$ $= \sqrt{(2^2+4^2}$ $= \sqrt{4+16}$ $= \sqrt{20}$		
Ceardaíocht **Comparing and Discussing Task 2** **(10 Minutes)** Response 1: Measure distance with a ruler or count or estimate the number of boxes. Response 2: Using Pythagoras' Theorem. Response 3: Using the distance formula.	Make sure all three lengths are written up. How accurate is this? Explain the method In the event that no students used the distance formula to solve the problem but that some used an equation begin a discussion: "Let's think about whether the problem can be represented in a mathematical statement that is even more fitting. Do you see any similarities with the values between these two methods?	Are students defending their ideas? Are they responding to each other's ideas?" How many people set up this kind of mathematical statement?

Extending students' learning Using the equations for the radius length $$\|AP\| = \sqrt{(2-6)^2 + (6-4)^2}$$ $$\|BP\| = \sqrt{(8-6)^2 + (8-4)^2}$$ $$\|CP\| = \sqrt{(10-6)^2 + (2-4)^2}$$ can the students will come up with the general formula $$(x-h)^2 + (y-k)^2 = r^2$$	"What do we call the distance from the centre to the circumference?" "Let's sub in sub in r." "How do we get rid of the square roots?" "What do notice about the numbers in each equation?" "Using (h,k) as the coordinates of the centre, can you write a general equation for a circle?"	Do all students recognise radius? Can all students remove the square root? Do students notice that 2, 8, 10 are the x coordinates of the A,B, and C and that the 6, 8, 2 are the y coordinates. Do students notice that the coordinates of the centre are present in each equation?
Summing up & Reflection Today we used our mathematical knowledge to solve a problem and in doing so we learned that the equation of a circle can be written in the form $$(x-h)^2 + (y-k)^2 = r^2$$ Ask students to write a reflection.	Using the board, reflect on what students did in class. Ask student to write a short reflection.	Do the students' reflections represent the teacher's view of the lesson?

10. Board Plan

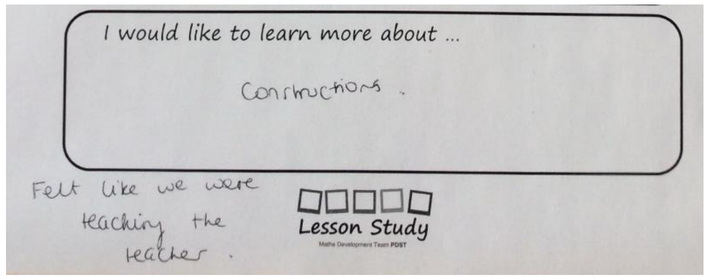




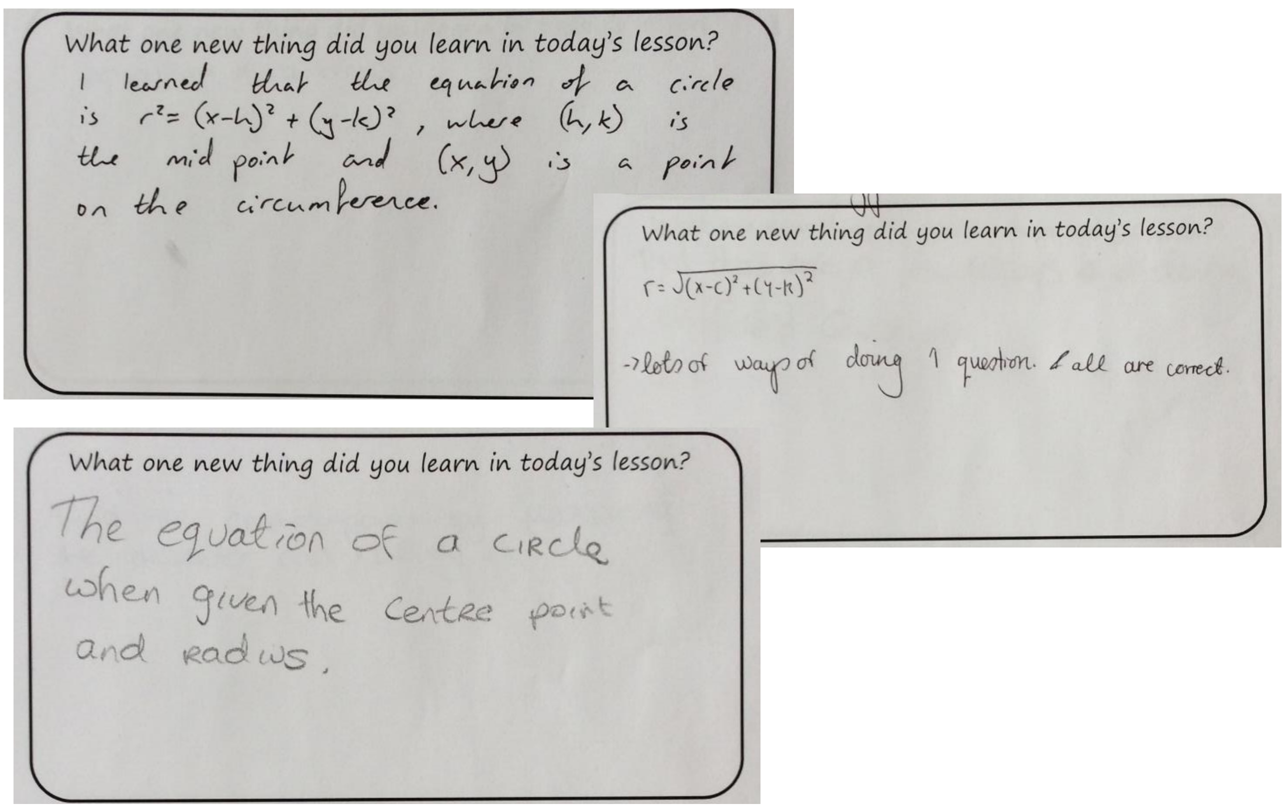
11. Evaluation

Both teacher and observers found that students engaged purposefully in the activities presented to them. We found that students understood and were able to explain the purpose of those activities and could extend and develop the activities meaningfully.

One student said that deriving the equation in this manner was “much better than the teacher just telling us the formula, because I will never forget this and I now understand where it comes from.”
Another commented that it “felt like we were teaching the teacher.”

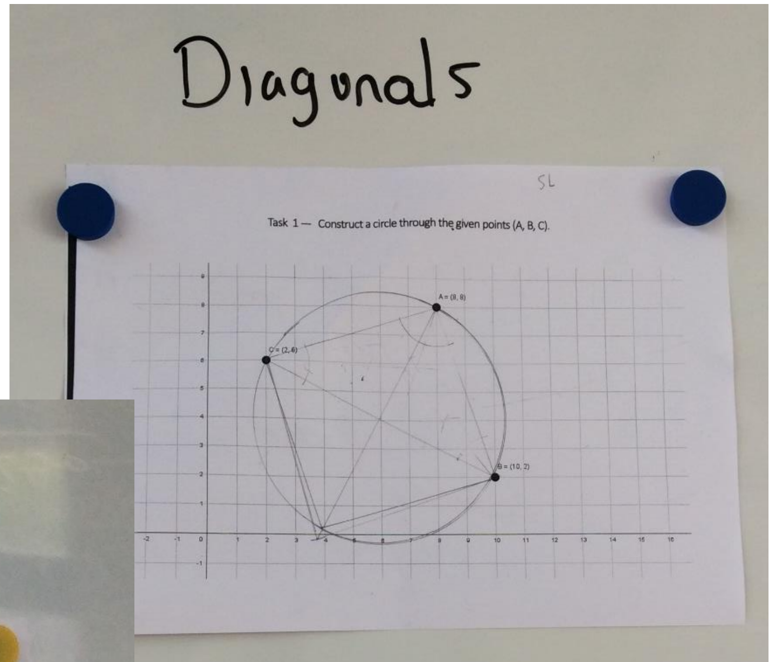


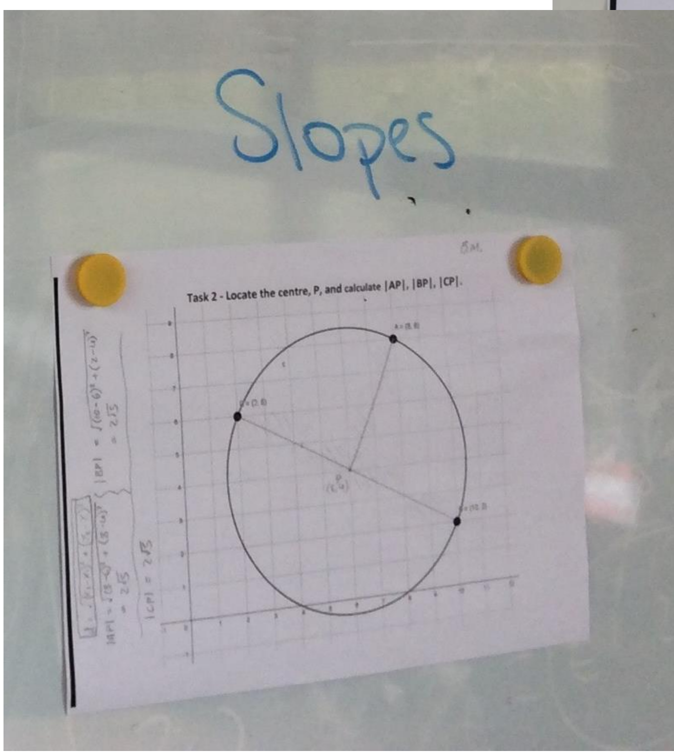
This feedback indicated that students enjoyed the process and gained confidence in their own ability to solve in a number of ways and ultimately achieving the goals of the lesson.



Interestingly, and pleasantly so, students tried more methods than we had anticipated.

In constructing the circle, one student sought the centre by constructing a square and using the intersecting diagonals to locate the centre.





In Task 2, finding the length of the three radii, no student used Pythagoras, but one used the slopes to show equality and then proceeded to use the distance formula.

In our opinion, we believe that students enjoyed and benefited from the lesson study approach and as such we feel that regular lesson study classes would benefit all students.

12. Reflection

Prior to the lesson the team had hoped that

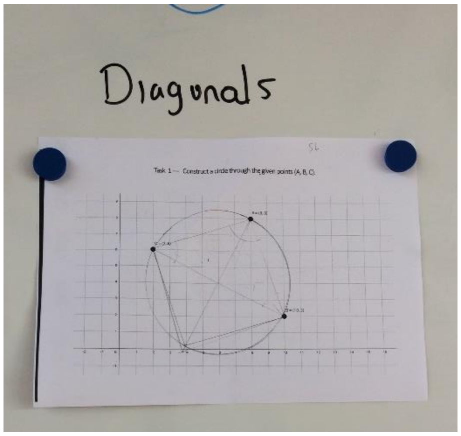
- students have more than one method to solve the problem.
- students to be able to use appropriate language in presenting the problem to their peers.
- students to move between the uncoordinated and the coordinated plane, making connections, verifying geometric results using algebraic methods and, hence, be able to derive the equation of a circle in the form $(x-h)^2+(y-k)^2=r^2$
- the lesson be a move away from learning procedures to developing an understanding.
- the class would be able to construct a circle by one of three methods and complete the second task, finding the radius length, in one of three methods.

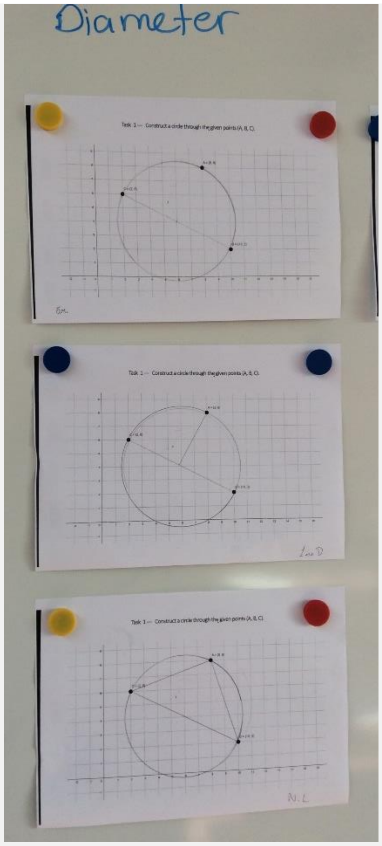
Observations

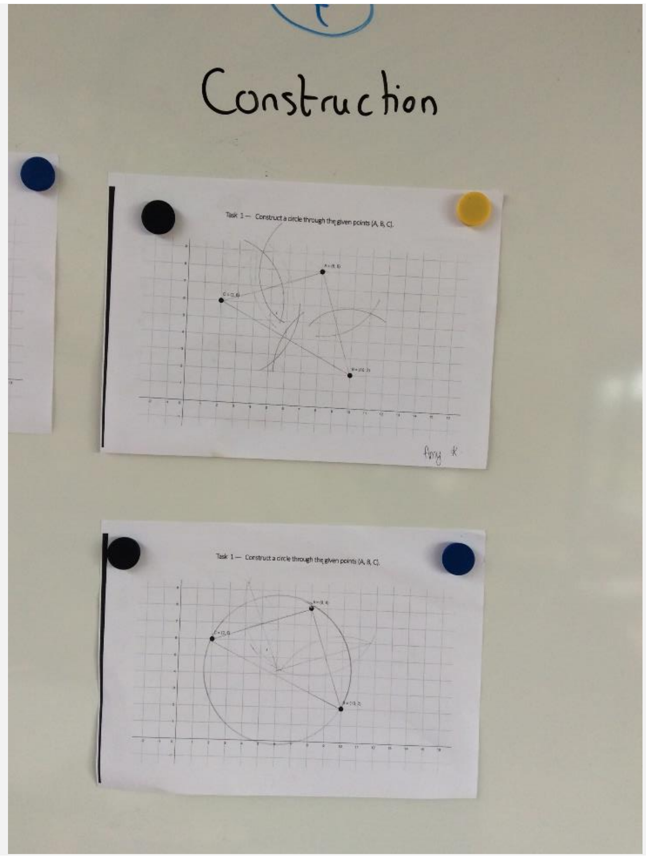
We were very happy with how the lesson went.
The teacher had a timed plan, which he wanted to stick to and he achieved this.
The class flowed very well and there was a natural progression into the second task resulting in the goal of the lesson being achieved by most students. This was measured by posing a question at the end of the lesson which all students engaged in and achieved the correct answer.

It was observed that every student was engaged and attempting both tasks. Nobody relied 100% on the teacher and there was a lot of independent learning taking place which was pleasing as the group are a lower stream class.

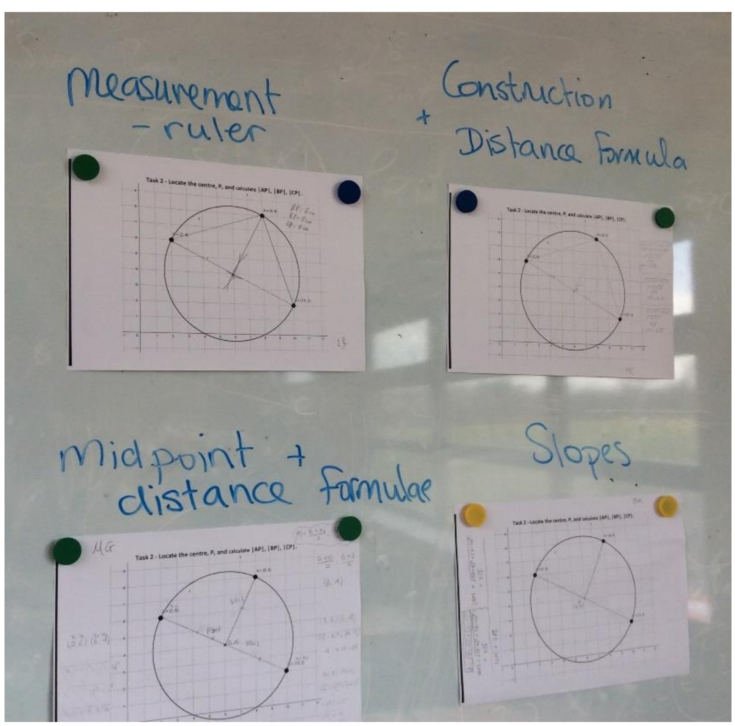
We observed that all students attempted to solve the first task using a total of four methods.



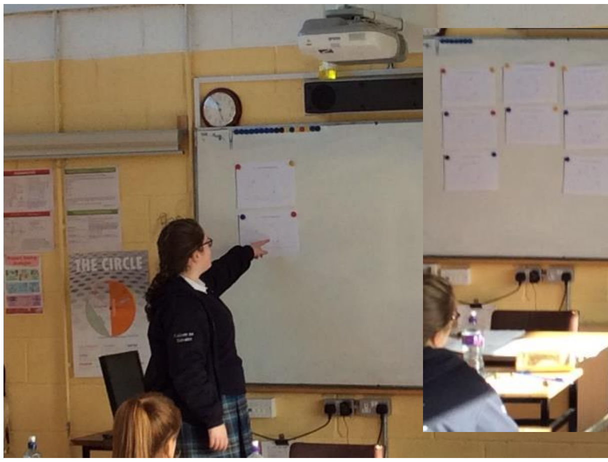


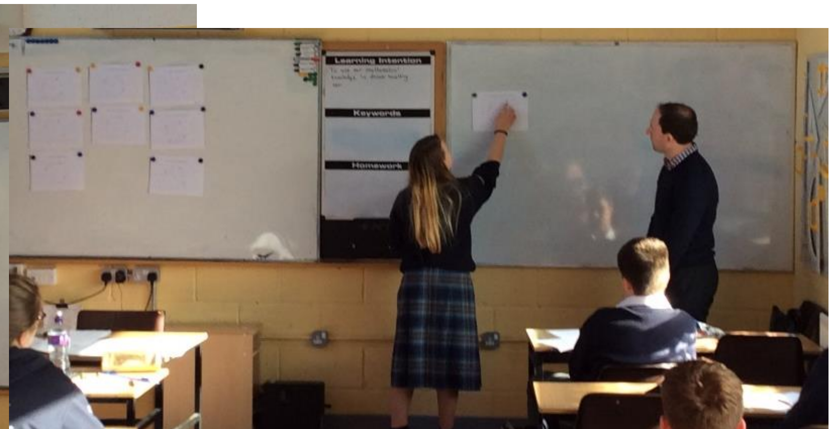


We observed that all students attempted to solve the second task using a total of four methods.

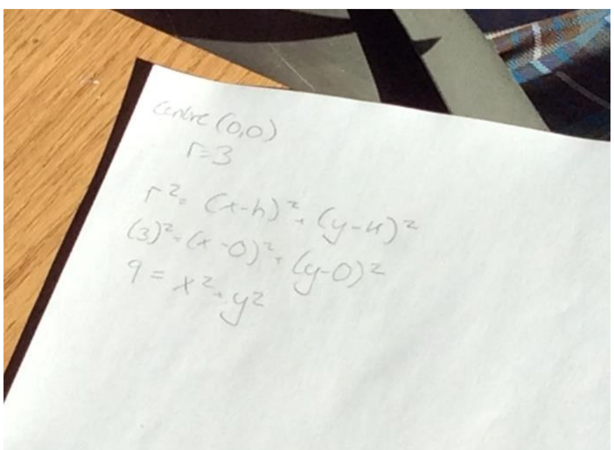


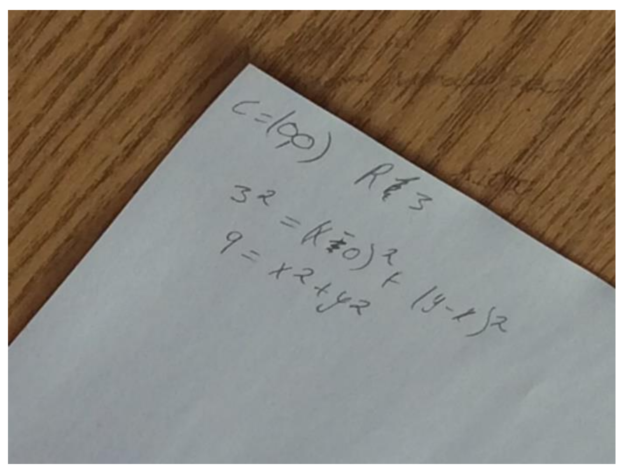
It was particularly pleasing to observe the presenting skills of the students when they displayed their work at the board. They grasped the main mathematical concepts and understood what they were doing. For both tasks, students who had attempted Trial and Improvement felt that the method was “not very good” as the method “might not always work” and was “not very reliable.”





The students were motivated throughout the lesson. All understood the tasks they had to achieve. In their reflection many students commented on how useful they found different ways of doing tasks while still getting the correct solution.





Future Study

The team suggested that the lesson could be improved if there was a greater input from the other students while the presenters were justifying their answer, and more time be spent at end to ensure goal is met by all students

Benefits of Lesson Study

Having participated in Lesson Study, we believe it is a really worthwhile activity. It benefits teachers by

- promoting collaboration with other teachers,
- increasing Subject Knowledge, and
- finding an alternative to Procedures.

Teaching can be a very solitary job and it was nice to be involved in planning a lesson as part of a team. Collaboration with other teachers allows you gain from others experience.
By picking the circle as our topic, we feel we got to see what a rich mathematical topic the circle is, how many different strands of mathematics it involves and how many skills it develops.