

Lesson Research Proposal for 1st Years & Fundamental Principle of Counting

For the lesson on 29/01/18

At CBS Roscommon, Ms. Grennan 1st year Maths class

Instructor: Ms B. Grennan

Lesson plan developed by: B. Grennan, A. Hynes and J. Bracken, S Guinan

1. Title of the Lesson: Fundamental Principle of Counting “Are you up for the match?”

2. Brief description of the lesson

The aim is for the students to discover the Fundamental Principle of Counting through problem solving using all previous knowledge to extend their learning into the general and apply this rule to other similar problems in order to extend the learning.

3. Research Theme

At CBS Roscommon we want:

- a) Our students to reflect on their progress as learners and develop a sense of ownership of and responsibility for their learning.
- b) Our teachers to collectively develop and implement consistent and dependable formative and summative assessment practices

As a Maths department, we will actively support the achievement of these goals by paying attention to the following entry points in our everyday classes:

- a) We will encourage students will take responsibility for their own learning by using the learning resources provided to them during the class.
- b) We will build students enthusiasm for the subject by engaging them with stimulating activities.
- c) We will encourage students to reflect on their learning in class by concluding class with a 3-2-1 reflection method.
- d) We support a problem-solving mind set by introducing structured problem solving into our school and engaging actively and productively with the CPD lesson study program.

4. Background & Rationale

This lesson is aimed at first year students. The Fundamental Principle of Counting is a quick and easy way to determine the number of outcomes to two or more events. It states that if an event has m possible outcomes and another independent event has n possible outcomes, then there are $m \times n$ possible outcomes for the two events together. It can pose difficulty for weaker students as they tend to add the number of outcomes from each event instead of multiplying. A deeper understanding of the Principle will help higher Junior Certificate students by helping them to understand when to add and when to multiply in Probability. Our aim is to eliminate the use of rote learning of “And is multiply and Or is add” which students will be meeting in the senior cycle Maths.

This topic leads onto probability, which the students would have met in National school. They would have studied the strand of Chance throughout National school and in sixth class they would have listed all possible outcomes of simple random processes, studied probability and the likelihood of occurrence of events and constructed and used frequency charts and tables

Following discussion by our Maths department, we have concluded that students first introduction to The Fundamental Principle of Counting is traditional, teacher led and learning of procedure. The students are told that the number of outcomes from each event are multiplied together. Our aim in this lesson is that students will be able to identify themselves that multiplying the number of outcomes in an event by the number of outcomes in subsequent events will give the correct answer through the use of a suitable structured problem-solving question.

5. Relationship of the Unit to the Syllabus programmes

Related prior learning Outcomes	Learning outcomes for this unit	Related later learning outcomes
<p>The students would have studied a strand called chance in National school.</p> <p>In sixth class they would have discussed, identified and listed all possible outcomes of simple random processes: rolling two dice and calculating the total (2, 3, 4 ... 12) selecting two numbers at random from the numbers 1, 2, 3, 4, 5 (ten possibilities)</p> <p>In sixth class they would have studied probability and the likelihood of occurrence of events; order on a scale from 0 to 100%, 0 to 1 when tossing a coin, a head has 1 chance in 2 of occurring; thus the likelihood of a head is 1 in 2, or 1–2 or 50%, similarly for a tail</p> <p>When rolling a die, each outcome has a 1 in 6 chance of occurring—therefore the likelihood is 1–6 when drawing a cube from a bag containing 3 red and 6 blue cubes, a blue cube has 6 chances in 9 of occurring and thus has a probability of 6–9 or 2–3 ; the probability of drawing a red cube is 3–9 or 1–3 what if the bag contains 5 red, 5 blue and 5 green cubes?</p>	<p>Topic 1.1 Counting</p> <p>Listing outcomes of experiments in a systematic way, such as in a table, using sample spaces, tree diagrams.</p> <p>List all possible outcomes of an experiment – apply the fundamental principle of counting</p> <p>Encounter the language and concepts of probability.</p>	<p>Topic 1.2 – 1.3 Probability</p> <p>In second and third year students further their study of probability. Students progress from informal to formal descriptions of probability. Predicting and determining probabilities. Difference between experimental and theoretical probability.</p> <p>Students:</p> <ul style="list-style-type: none"> decide whether an everyday event is likely or unlikely to occur – recognise that probability is a measure on a scale of 0-1 of how likely an event is to occur – use set theory to discuss experiments, outcomes, sample spaces – use the language of probability to discuss events, including those with equally likely outcomes – estimate probabilities from experimental data – recognise that, if an experiment is repeated, there will be different outcomes and that increasing the number of times an experiment is repeated generally leads to better estimates of probability – associate the probability of an event with its long-run, relative frequency

<p>or 3 red, 6 blue and 6 green?</p> <p>Students would also have constructed and used frequency charts and tables after performing an experiment</p>		<p>Finding the probability of equally likely outcomes. construct sample spaces for two independent events – apply the principle that, in the case of equally likely outcomes, the probability is given by the number of outcomes of interest divided by the total number of outcomes (examples using coins, dice, spinners, containers with different coloured objects, playing cards, sports results, etc.) – use binary / counting methods to solve problems involving successive random events where only two possible outcomes apply to each event</p>
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6. Goals of the Unit

- Students will recap on their learning in National school and build on their knowledge of outcomes of simple random processes.
- Students will discover the concept of the Fundamental Principle of Counting, a basic concept and introductory topic for the study of probability.
- Students will understand how to use the Fundamental Principle of Counting when determining the number of outcomes for two or more unrelated events.
- Students will apply this understanding in future probability problems.
- Students will feel confident, enthused and happy in their Math learning.
- This lesson study will focus on assessing how the provided learning activities effectively works on the students' discovery of the fundamental principle of counting,

7. Unit Plan

Lesson	Learning goal(s) and tasks
1	Introduction to Outcomes and Listing outcomes, drawing on their previous knowledge from National school. Two Way Tables –Students to learn how to draw two-way tables and practice completing them (Define an outcome)
2	Tree Diagrams – Students learn how to draw tree diagrams and practice completing them.
3 The Research Lesson	Fundamental Principle of Counting
4	Fundamental Principle of Counting problem solving questions

8. Goals of the Research Lesson:

Mathematical Goals

- Students will discover the concept of the Fundamental Principle of Counting, a basic concept and introductory topic for the study of probability.
- Students will understand how to use the Fundamental Principle of Counting when determining the number of outcomes for two or more unrelated events.

Key Skills and Statements of Learning

In the planning and preparation of this lesson the Junior Cycle Key Skills and Statements of Learning have been taken into account and our Maths department have begun to integrate these key skills into their maths lessons.

This lesson will promote the JC Key Skills in the following way:


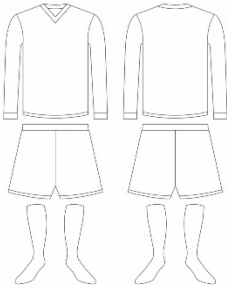
1. Being Literate: Students will have the opportunity to express their ideas clearly and accurately during Ceardaiocht.
2. Being Numerate: By providing engaging tasks students will develop a positive attitude towards problem solving.
3. Managing Myself: Students will have the opportunity to reflect on their own learning when the teacher uses the 3-2-1 reflection method at the end of class.
4. Staying Well: Students confidence and positive disposition to learning will be promoted by engaging and completing appropriate tasks.
5. Communicating: During Ceardaiocht, students will present and discuss their mathematical thinking.
6. Being Creative: Students will explore options and alternatives as they actively participate in the construction of knowledge.
7. Working with Others: Students will learn with and from each other as they discuss the different outcomes from the problem.
8. Managing Information and Thinking: Students will be encouraged to think creatively and critically.

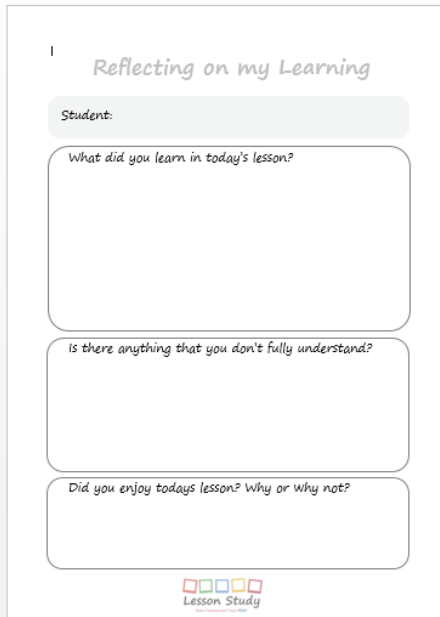
This lesson also meets the following JC Statements of Learning:

1. The student communicates effectively using a variety of means in a range of contexts.
15. The student recognises the potential uses of mathematical knowledge, skills and understanding in all areas of learning.
16. The students describes, illustrates, interprets, predicts and explains patterns and relationships.
17. The students devises and evaluates strategies for investigating and solving problems using mathematical knowledge, reasoning and skills.

9. Flow of the Research Lesson:

Steps, Learning Activities Teacher's Questions and Expected Student Reactions	Teacher Support	Assessment
Introduction Today we are going to use our mathematical knowledge to solve a problem and we are going to try to solve the problem by ourselves and then come together as a class and use all your knowledge to learn something new.	Roscommon is in the National League, how many wins, lose or draws have they had in recent weeks. How did they get on last Sunday, what do you think will happen next week.	Are students motivated?

<p>Posing the Task</p> <p>John has a match on Tuesday at home and a match on Thursday away. Write all possible results of the matches, displaying them as many ways as you can.</p> <p>How many outcomes are there?</p> 	<p>Clarify that the only outcomes available in the matches is Win, Lose or Draw.</p> <p>Problem on Board</p> <p>Jerseys given to students</p> <p>Whiteboard and markers for students to work on problem</p> <p>A4 sheet for sticking answers on whiteboard</p> 	
<p>Student Individual Work</p> <ol style="list-style-type: none"> 1. Student has two jerseys for home and away 2. Students to list outcomes 3. Students to draw a two-way table 4. Students to draw a tree diagram 5. Students to multiply 3×3 (9 outcomes) 	<p>Student will come up to the board in order of responses from least to most advanced</p> <p>If a student gets stuck avoid using keywords like tree diagram in questioning</p> <p>e.g Is there anything we did in the last few days that will help us answer this question.</p>	
<p>Ceardaíocht /Comparing and Discussing</p> <p>If a student only gets 6 outcomes, can they see where the answer can be developed?</p> <p>Which answer is the fastest? (Fundamental Principle of Counting)</p> <p>Which answer is the best? Why?</p> <p>Looking at the lists, two way table or tree diagram can we use the FPC to check our answers.</p> <p>Extending Learning – During home and away games there</p>	<p>When we get a misconception we will address this in class and use this to extend the learning on FPC.</p>	

<p>are different strips. There is two different colour jerseys, three choices of shorts and four socks. How many possible strips are there?</p> <p>In the premier league there are 20 teams, there are 11 players on each team. To pick your fantasy football team how many different teams can you pick? (20¹¹)</p> <p>Don't want to write 20 x 20 out 11 times, can anyone remember a quicker way to write that?</p>	<p>Students to answer question using whiteboards.</p>	
<p>Summing up & Reflection</p> <p>Students will fill out the reflection sheet and reflections would be stuck on the board.</p> <p>Recap of the lesson : Today we learned about the Fundamental Principal of Counting. It is a quick way to calculate the outcome from two of more events.</p> <p>Teacher will use the reflection provided by the Project Maths team</p> <div data-bbox="145 965 587 1579">  </div>	<p>Teacher will set homework and student reflection will be gathered to provide information for post lesson discussion and reflection.</p>	

NOTES re in-between desk work

Possible answers	Name of Student	Notes
Student has two jerseys for home and away		Don't over help students
Students to list outcomes		
Students to draw a two-way table		
Students to draw a tree diagram		
Students to multiply 3 x 3 (9 outcomes)		
Misconceptions 6 outcomes rather than 9 Incorrect drawing of two-way table/tree diagram.		

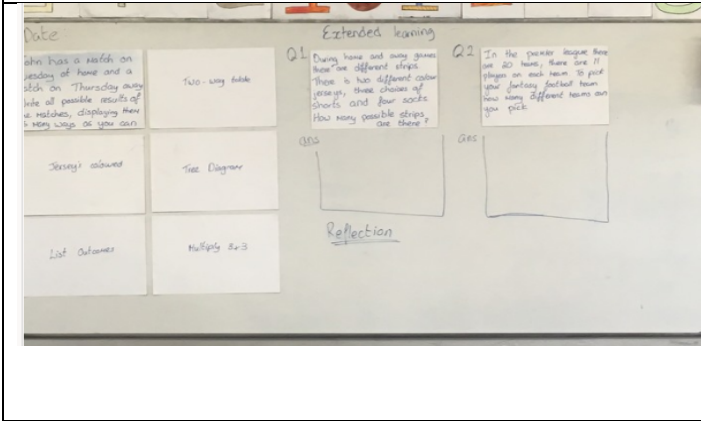
10. Board Plan

Board work was laid out pervious to the lesson and it was decided that colored card to display the questions and the extension tasks on A3 were more appealing as the room was quiet dark.

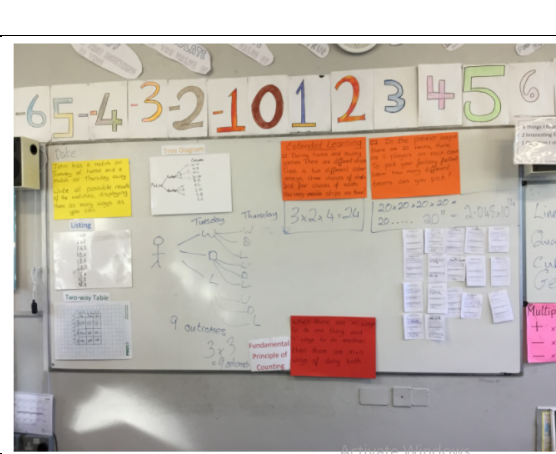
Solutions will be brought to the board from least to most sophisticated;

- 1. Student has two jerseys for home and away
- 2. Students to list outcomes
- 3. Students to draw a two-way table
- 4. Students to draw a tree diagram
- 5. Students to multiply 3 x 3 (9 outcomes)

Teachers original Board work and display in prep for lesson



Students displays after research lesson



6. Evaluation

The lesson was evaluated with the following question in mind;

‘Did the students reflect on their learning and take responsibility for their learning during class?’

There were four observers in the lesson two using iPad and using template provided earlier in the lesson plan. Using the seating plan, we assigned each teacher approximately six students, we were allocated throughout the room in a way to be least intrusive to the class and the teacher.

Notes were taken and two observers used Lesson Note the students were observed based on their interactions with each other, the teacher, the questions that they asked, any misconceptions and photographs of the students work were taken throughout the lessons on both iPads.

During the post lesson discussion, data that was collected during the lesson allowed for discussion on the following areas:

- Comments and questions that the students had
- Common misconceptions
- Were the goals of the lesson achieved
- Did the lesson promote teacher student interaction
- Did it promote student reflection on their learning
- Did students understand the general terms of F.P.C

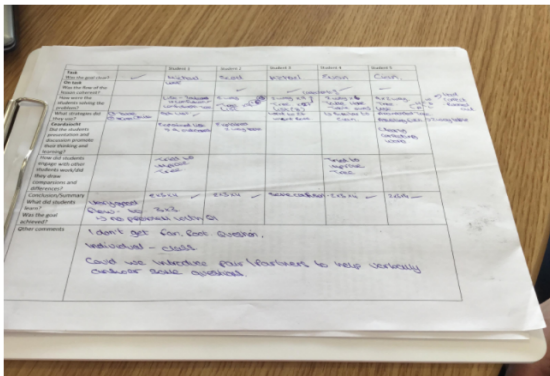
7. Reflection

Throughout the lesson the students we engaged and reflecting on their work throughout the lesson. The goals of the lesson were achieved, the teacher lad the students in Ceardaiocht to discover that 3X3 was the quickest and easiest way of discovering the outcomes.

The students demonstrated their literacy and numeracy skills, managed time and communicated with the teacher effectively, working individually and were creative in the use of their ways of solving the problems.

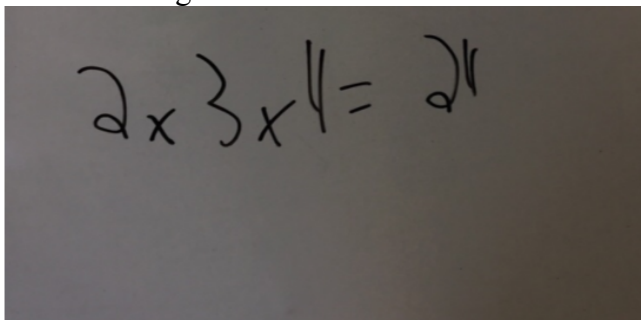
Student’s demonstrated knowledge of pervious learning within the unit as the y asked: ‘can this be done using two-way tables?’ teacher clarified the problem using the context of the game of football between Roscommon and Meath. The students discussed possible outcomes of the match and consequences for the rest of the football league.

Observer 1:



Five students in this section these are the written observations using a lesson study format to provide reflections.

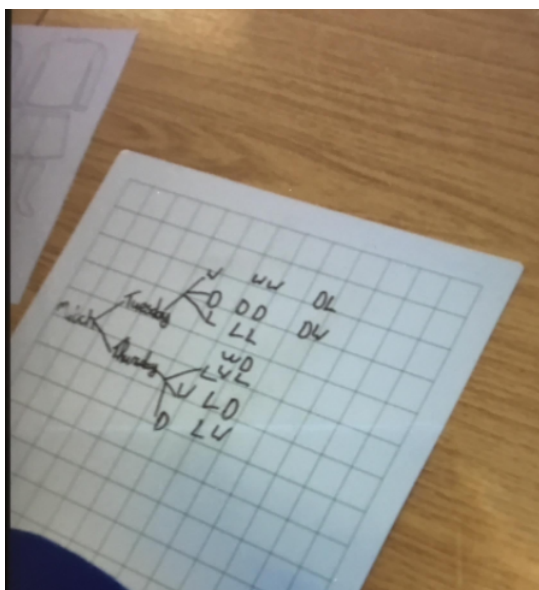
- All students began with two-way table and/or listing the outcomes.
- Only one student successfully got the correct tree diagram in the group.
- One student had the two-way table with nine outcomes and then the tree diagram had 27 outcomes
- Two students went to the board from this group to explain their answers
- Four students got the extended learning question, and all had issue with question two of the extended learn (this will be discussed at the end)
- One students got $2 \times 3 \times 4$ on the first extended learning



- As the work was displayed on the board a student was correcting his work on his own white board
- Two students worked together to come up with the two-way table.

Observer 2:

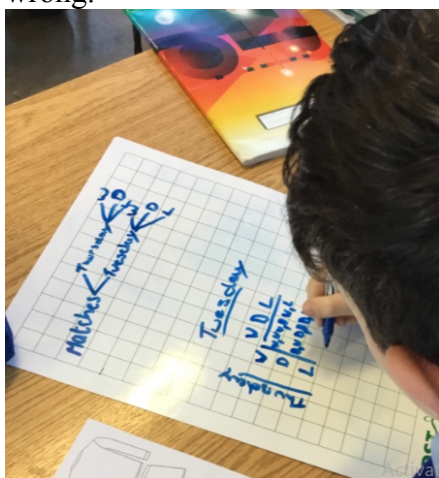
- Two students have (EAL) and these worked together to help clarify the problem.
- Teacher questioning of students to try to draw out the list and two-way table.
- Students C of the pic of the tree diagram below this was the one that was displayed on the board and then used this opportunity to explore if this was the correct display.
- Were Tuesday and Thursday, home and away potential outcomes.
- This was a common misconception possible clarification to follow.
- Student C also got question 2 was extended learning and that $20 \times 20 \times 20$ can be 20^{11}
- Teacher questioned the students to ask, 'what was the best method?' students C suggested that the two-way table was the best way to display the data even though he had drawn the tree diagram incorrectly.
- All agreed that the FPC was the fastest method to get the total number of outcomes.



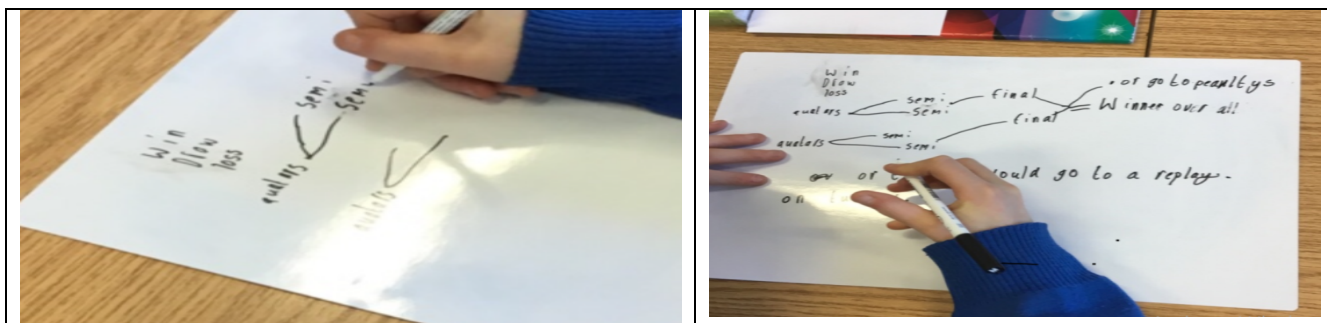
Observer 3:

Student A began writing the results of the possible matches i.e.. 0-1, 0-3 etc. the teacher clarified this problem with him individually to explain that win loose and draw was all that he needed to focus on.

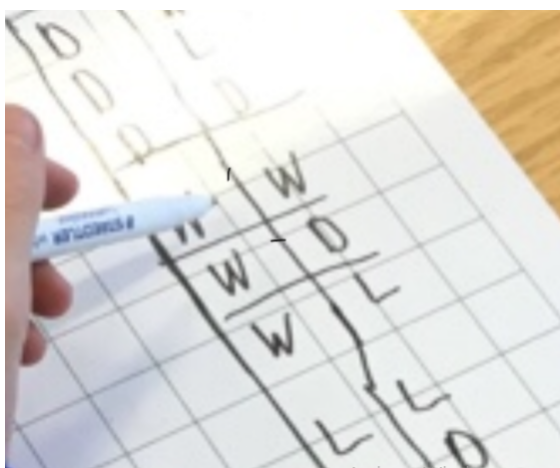
Teacher questioned the student to know why he had two different outcomes with the same results he was happy that there were 9 outcomes but was getting a different result for the tree diagram as it was wrong.



Another student in this group interpreted the question incorrectly and the teacher tried to lend support, but he produced the next diagram, as EAL interpretation of the question was difficult. Teacher did promote the student to go with win, lose or draw as potential outcomes.



Student E who is a bright boy only produced the two-way table in the time allotted with nine outcomes but attempted the extended learning.



Student Reflection:

1: What did you learn?

All students mentioned the FPC

2. Is there anything that you didn't understand?

The tree diagram and the extended learning question 2

3: Did you enjoy the lesson?

- Liked the problem solving
- Using the whiteboards
- Learned new stuff
- Wasn't just writing in our copies
- Clearly explained questions
- A better way of learning
- Everyone in the class got to share and had great ideas.

Reflecting on my Learning

Student: _____

What did you learn in today's lesson?

In today's lesson
we ~~fundamental~~
learned fundamental
principle of counting

Is there anything that you don't fully understand?

No

Did you enjoy today's lesson? Why or why not?

That everyone
had great ideas.

From the extended learning:

All students used the FPC to answer the first question however the second question is where most of the difficulty lay students went straight to 20×11 as oppose to 20^{11}

Changes to the lesson:

1. The question 2 in the extended learning needs to be altered this will be done by the team but until then it is recommended that it is left out.
2. In addition to the question of the research task we feel that an extension on to that will be "How many outcomes are possible?"
3. During the Introduction of the problem, the teacher should clarify what an outcome is so is Thursday/Tuesday an outcome in this question or id Home /Away an outcome? The may help the weaker students identify very clearly that the outcomes are of the match.
4. Teacher would need to revisit Tree diagrams in the nest lesson base on point three here to clarify the understanding.

Resources:

